Cobwebbing is a graphical technique used to determine the *behaviour* of solutions to a DTDS without calculations.

This technique allows us to sketch the graph of the solution (a set of discrete points) directly from the graph of the updating function.

#### Algorithm:

1. Graph the updating function and the diagonal.

2. Plot the initial value  $m_0$  on the horizontal axis. From this point, move vertically to the updating function to obtain the next value of the measurement. The coordinates of this point are  $(m_0, m_1)$ .

3. Move horizontally to the point  $(m_1, m_1)$  on the diagonal. Plot the value  $m_1$  on the horizontal axis. This is the next value of the solution.

4. From the point  $(m_1, m_1)$  on the diagonal, move vertically to the updating function to obtain the point  $(m_1, m_2)$  and then horizontally to the point  $(m_2, m_2)$  on the diagonal. Plot the point  $m_2$  on the horizontal axis.

5. Continue alternating (or "cobwebbing") between the updating function and the diagonal to obtain a set of solution points plotted along the horizontal axis.

Example:

Starting with the initial condition  $b_0 = 1$ , sketch the graph of the solution to the system  $b_{t+1} = 2b_t$ by cobwebbing 3 steps.



#### **A Solution From Cobwebbing**



#### Example:

Consider the DTDS for the methadone concentration in a patient's blood:

$$M_{t+1} = \frac{1}{2}M_t + 1$$

Cobweb for 3 steps starting from

(i) 
$$M_0 = 1$$
  
(ii)  $M_0 = 5$   
(iii)  $M_0 = 2$ 



# Equilibria

Definition:

A point  $m^*$  is called an equilibrium of the DTDS

$$m_{t+1} = f(m_t)$$

if  $f(m^*) = m^*$ .

Geometrically, the equilibria correspond to points where the updating function intersects the diagonal.

# Equilibria



# Equilibria



# Solving for Equilibria

#### Algorithm:

- 1. Write the equation for the equilibrium.
- 2. Solve for  $m^*$ .
- 3. Think about the results.

# Solving for Equilibria

#### Examples:

Find the equilibria, if they exist, for each of the following systems.

(a) 
$$M_{t+1} = \frac{1}{2}M_t + 1$$
 (b)  $x_{t+1} = \frac{ax_t}{1+x_t}$ 

#### Example:

Consider the DTDS for a population of codfish

$$n_{t+1} = -0.6n_t + 5.3$$

where  $n_t$  is the number of codfish in millions and t is time.

Suppose that initially there are 1 million codfish. Determine the equilibria and the behaviour of the population over time by cobwebbing.



### A Solution From Cobwebbing



# Stability of Equilibria

An equilibrium is <u>stable</u> if solutions that start near the equilibrium move closer to the equilibrium.

An equilibrium is <u>unstable</u> if solutions that start near the equilibrium move away from the equilibrium.