MODELLING WITH DTDSs

Bacterial Population Growth:

$$b_{t+1} = rb_t$$

The parameter r is called **per capita production**. It represents the number of new bacteria produced per bacterium.

Bacterial Population Growth in General

Solution: $b_t = b_0 r^t$

Assumption: r is constant

Reality: r will depend on the size of the population (resources are limited)

small populations $\Rightarrow$ less competition $\Rightarrow$ higher r

large populations $\Rightarrow$ more competition $\Rightarrow$ lower r

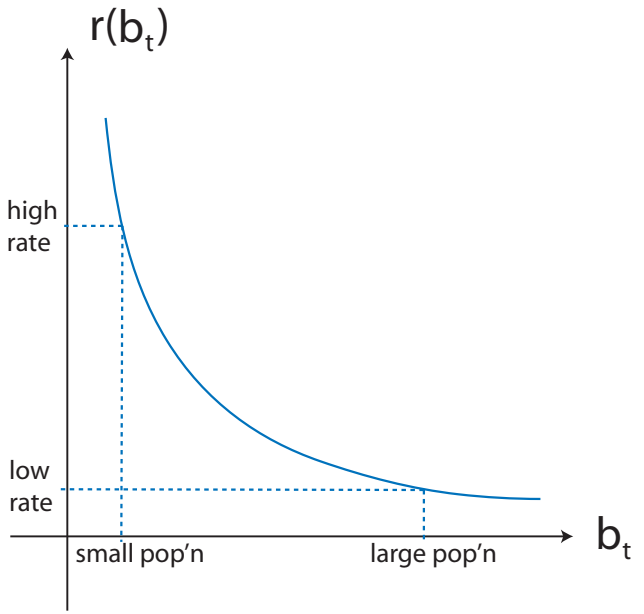
MODELLING WITH DTDSs

Model for **Limited** Bacterial Population Growth:

$$b_{t+1} = r(b_t) \cdot b_t$$

Replace the constant r by a function which matches natural observations:

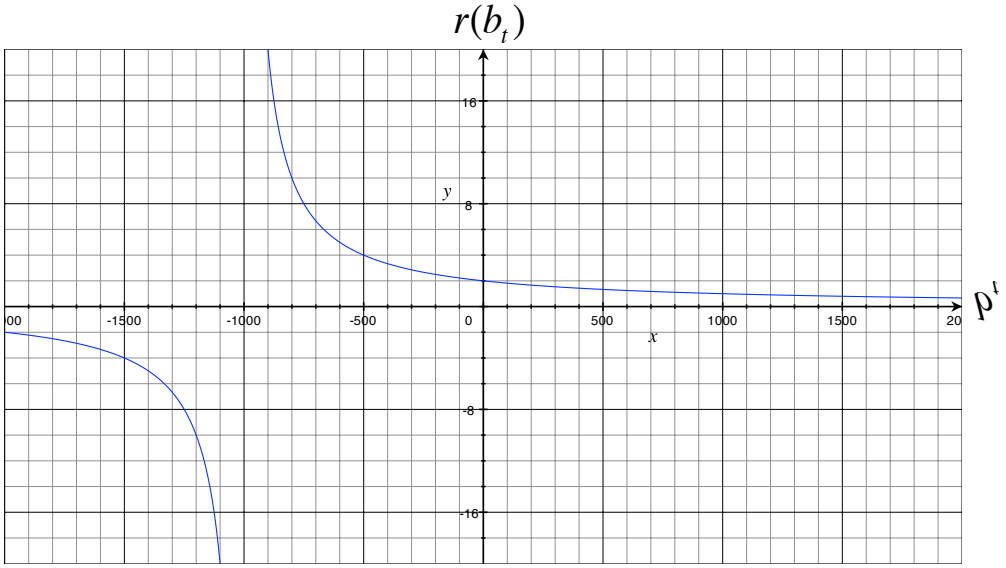
$$r \, \alpha \frac{1}{b_t} \quad \Rightarrow \quad r(b_t) = k \cdot \frac{1}{b_t}$$



MODELLING WITH DTDSs

Model for **Limited** Bacterial Population Growth:

Example: $r(b_t) = \dfrac{2}{1 + 0.001 b_t}$

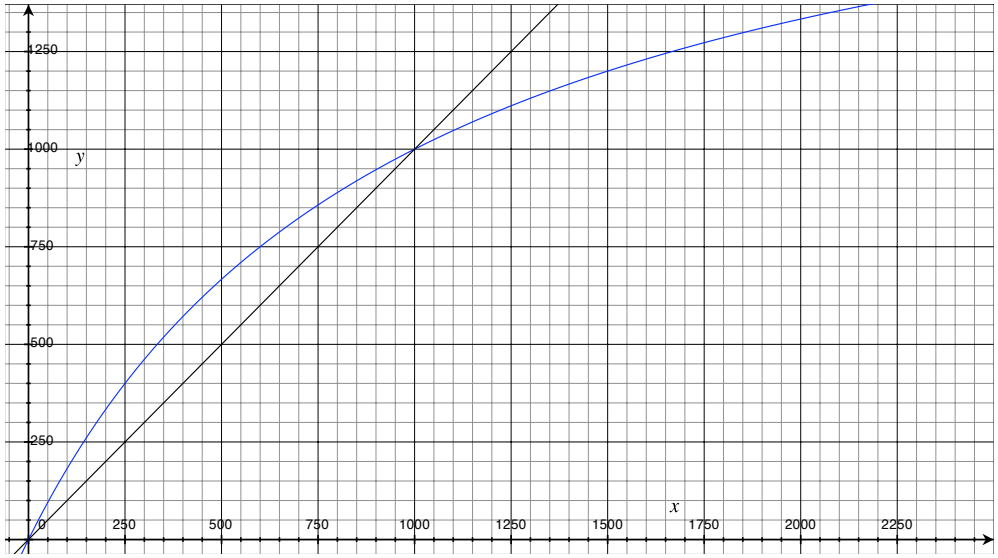


MODELLING WITH DTDSs

Model for **Limited** Bacterial Population Growth:

Example:

$$b_{t+1} = \left(\frac{2}{1 + 0.001 b_t} \right) \cdot b_t$$



Determine equilibria and behaviour of nearby solutions by cobwebbing.

elimination of chemicals

*** filtration by kidneys (kidneys break down constant amount per hour … caffeine)

*** breaking down the chemicals using enzymes from the liver (amount of chemical broken down depends on the amount present … alcohol)

Substance Absorption (Elimination) and Replacement (Consumption) Models

Absorption of Caffeine:

Our bodies eliminate caffeine at a **constant rate** of 13% per hour.

DTDS: $$c_{t+1} = 0.87c_t + d$$

amount of caffeine (mg) 1 hour later

amount of caffeine now

amount of "new" caffeine consumed at time t+1

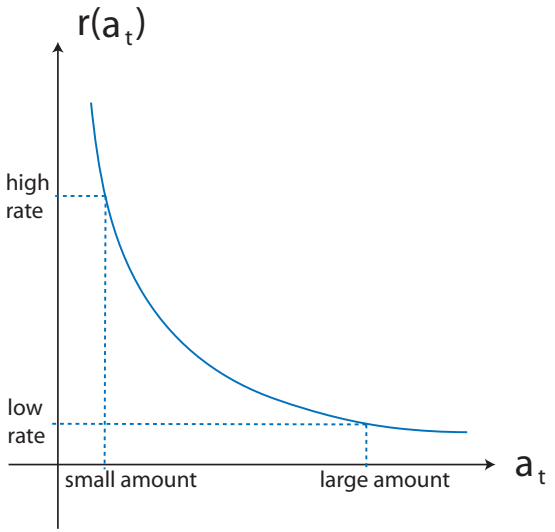
* Similar to "methadone" example

Substance Absorption (Elimination) and Replacement (Consumption) Models

<u>Elimination of Alcohol:</u>

The amount of alcohol that is broken down by the liver depends on the amount of alcohol present in the body.

The larger the amount, the smaller the proportion of alcohol being eliminated.

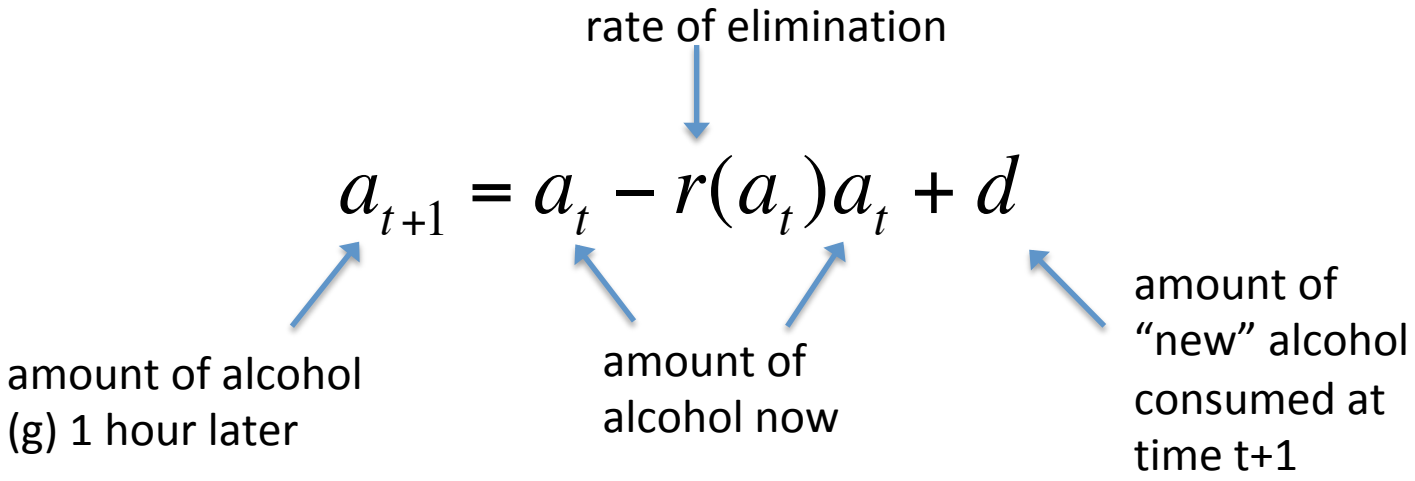


*Similar to the limited growth population model

Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

DTDS:

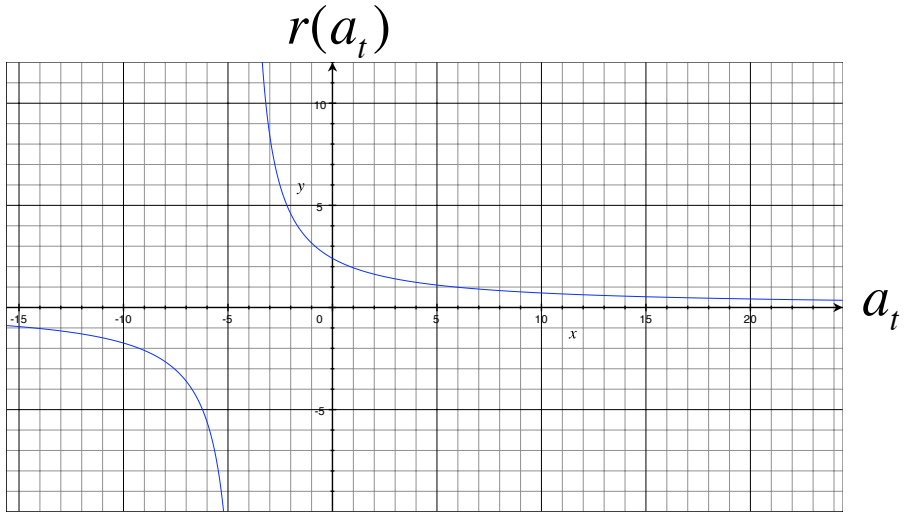


Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

Example:

Rate of Elimination: $r(a_t) = \dfrac{10.1}{4.2 + a_t}$

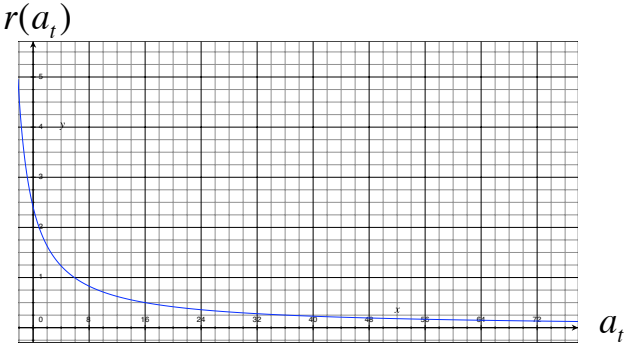


Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

Example:

Rate of Elimination: $r(a_t) = \dfrac{10.1}{4.2 + a_t}$



DTDS: $a_{t+1} = a_t - \left(\dfrac{10.1}{4.2 + a_t} \right) a_t + d$

definition

one drink = 14 grams of alcohol

* 5 ounces of wine, or
* 12 ounces of beer, or
* 1.5 ounces of 80 proof (vodka, rum, gin, etc.)

Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

Example:

A standard drink contains 14g of alcohol.

Compare what happens over time for the following situations:

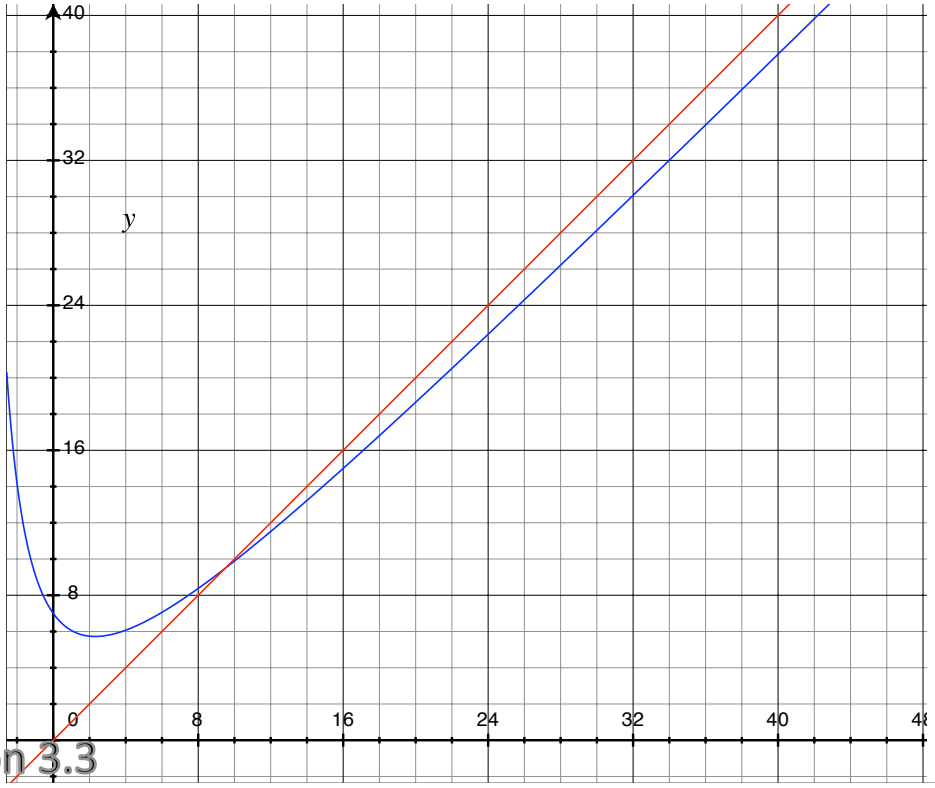
(a) You consume two drinks right away and continue to have half of a drink every hour

(b) You consume one drink every hour

Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

(a) You consume two drinks right away and continue to have half of a drink every hour

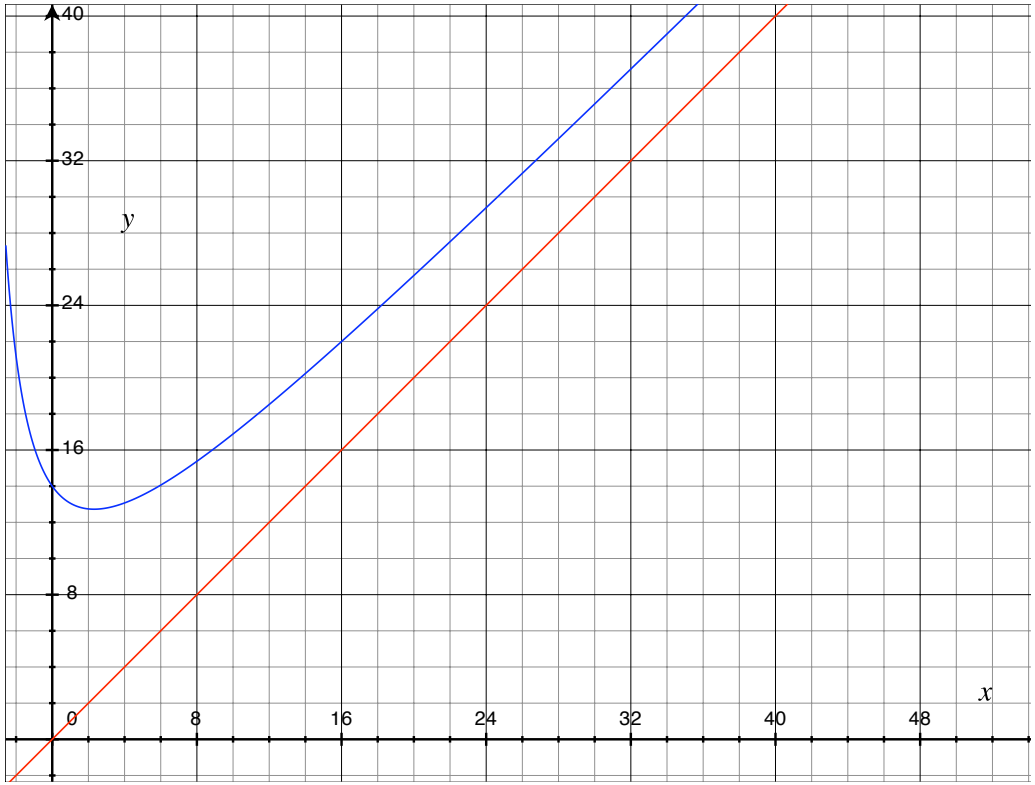


$$f(a_t) = a_t - \left(\frac{10.1}{4.2 + a_t}\right)a_t + 7, \quad a_0 = 28$$

Substance Absorption (Elimination) and Replacement (Consumption) Models

Elimination of Alcohol:

(b) You consume one drink every hour



$$f(a_t) = a_t - \left(\frac{10.1}{4.2 + a_t}\right)a_t + 14, \quad a_0 = 0$$

so … how much alcohol is in the body

** 2 rapid drinks, then 1/2 drink every hour … decreases, stabilizes at 9.5 grams

** one drink every hour … increases, after 5 hours reaches 41 grams. keeps increasing, no limit