Rates of Change

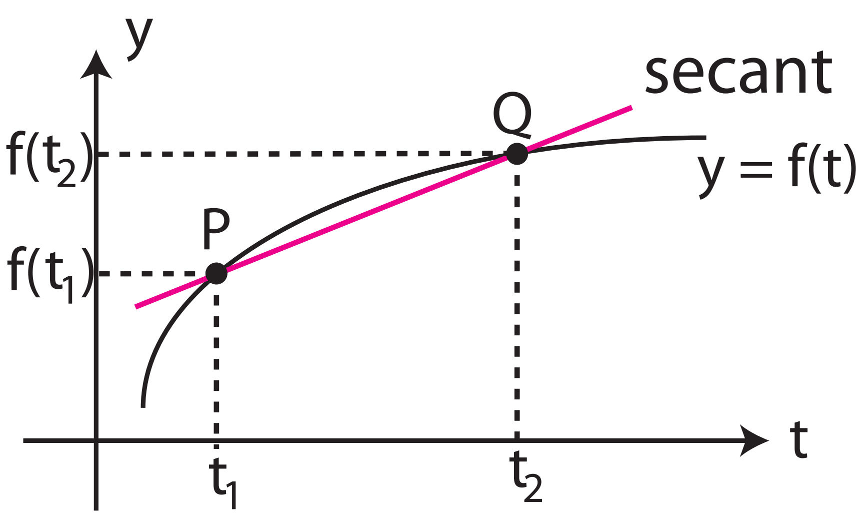
The rate of change of a function tells us how the dependent variable changes when there is a change in the independent variable.

Geometrically, the **rate of change** of a function corresponds to the **slope** of its graph.

Average Rate of Change = Slope of Secant Line

The average rate of change of $f(t)$ from $t=t_1$ to $t=t_2$ corresponds to the slope of the secant line PQ.

$$m_{PQ} = \frac{\Delta f}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

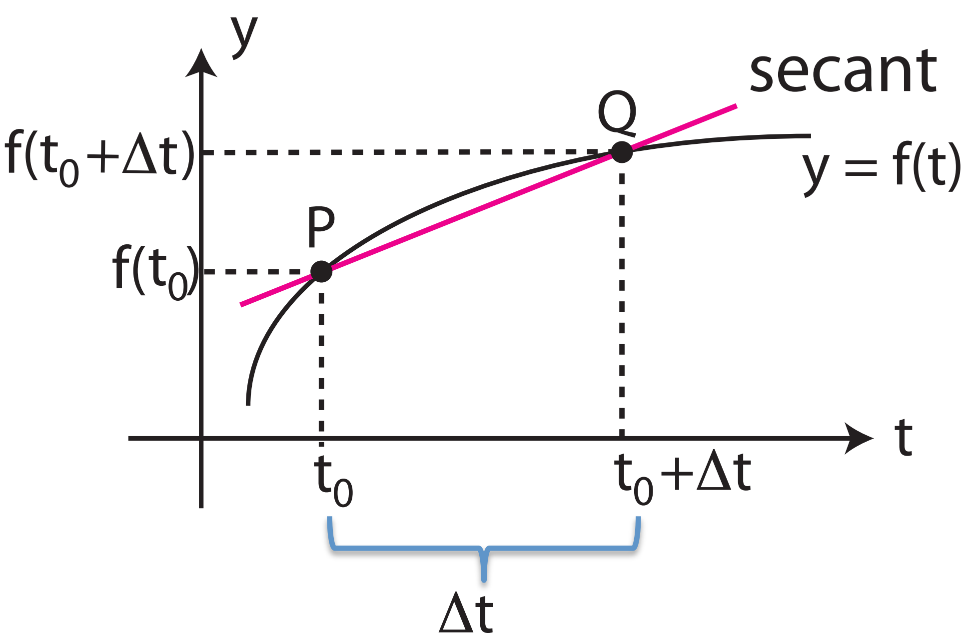


Average Rate of Change = Slope of Secant Line

Alternative Notation:

The average rate of change of f(t) from the base point $t=t_0$ to $t=t_0+\Delta t$ is

$$m_{PQ} = \frac{\Delta f}{\Delta t} = \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}$$



Average Rate of Change = Slope of Secant Line

Example:
Find the average rate of change of the function

$$s(t) = \ln t$$

starting from time t=1 and lasting 1, 0.1, and 0.01 units of time.

t_0	Δt	$t_0+\Delta t$	$\frac{f(t_0+\Delta t) - f(t_0)}{\Delta t}$
1	1	2	
1	0.1	1.1	
1	0.01	1.01	

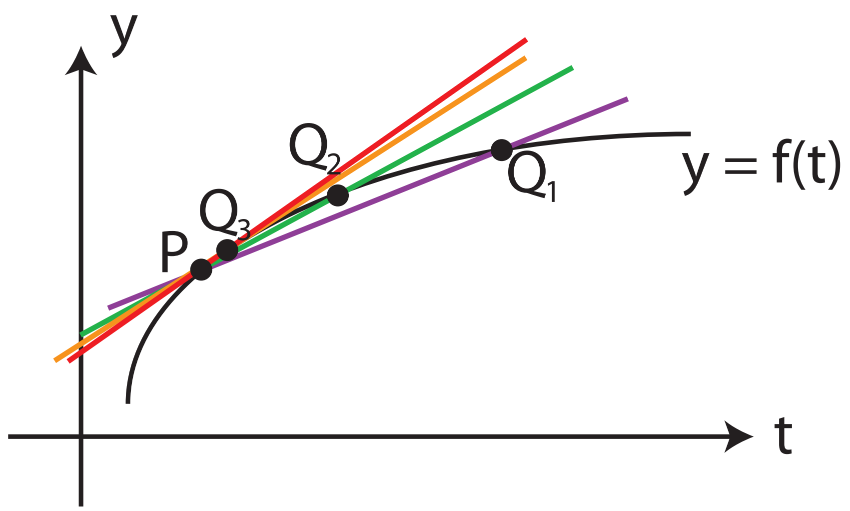
Estimating the Slope of the Tangent

Steps:

1. Approximate the tangent at P using a secant line intersecting P and a nearby point Q.

2. Obtain a better approximation to the tangent at P by moving Q closer to P, but $Q \neq P$.

3. Define the slope of the tangent at P to be the limit of the slopes of secants PQ as Q approaches P.

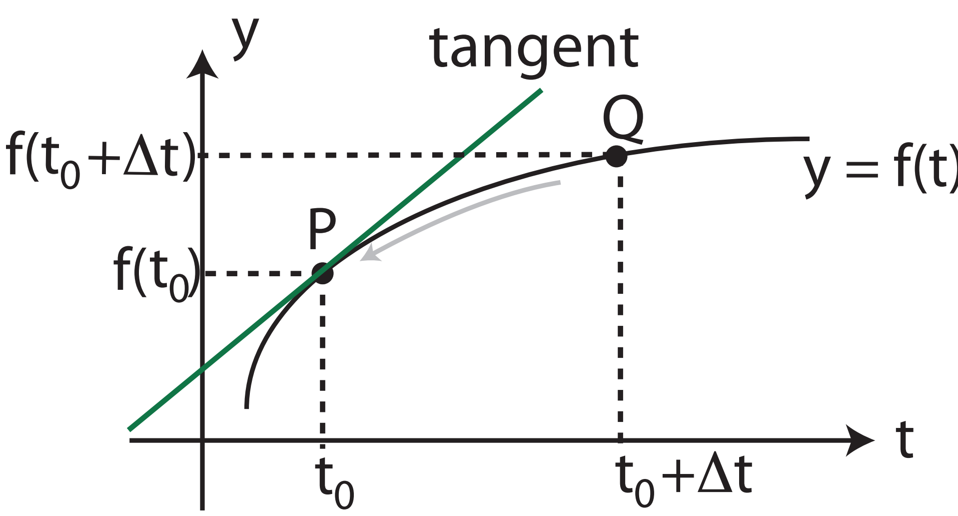


Instantaneous Rate of Change = Slope of Tangent Line

The instantaneous rate of change of f(t) at $t=t_0$ corresponds to the slope of the tangent line at $t=t_0$.

$$f'(t_0) = \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}$$



Note:
The slope of the curve y=f(t) at P is the slope of its tangent line at P.

Instantaneous Rate of Change = Slope of Tangent Line

Example:

$$s(t) = \ln t$$

t_0	Δt	$t_0+\Delta t$	$\frac{f(t_0+\Delta t) - f(t_0)}{\Delta t}$
1	0.1	1.1	0.9531
1	0.01	1.01	0.9950
1	0.001	1.001	0.9995

(a) Guess the limit of the slopes of the secants as $\Delta t \to 0$.

(b) Use this to find the equation of the tangent line to s(t) when t=1.