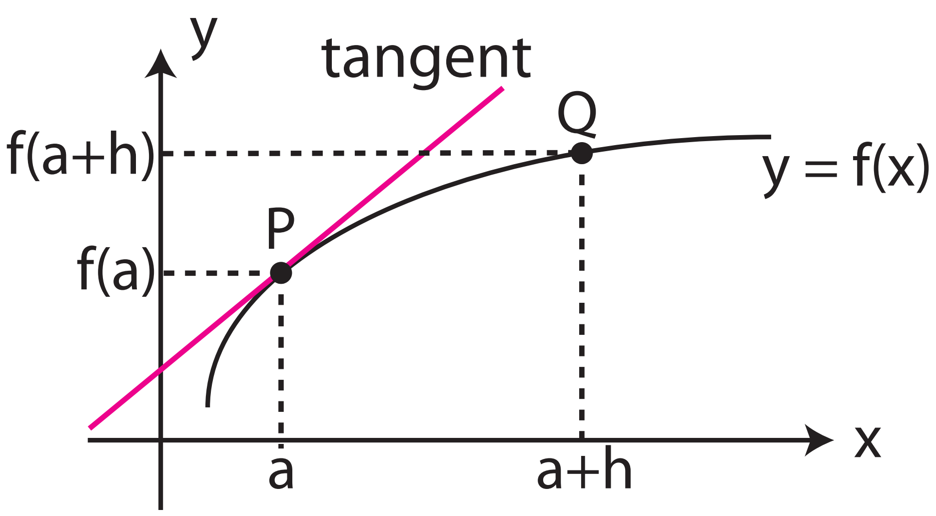
The Derivative

Recall:
The **instantaneous rate of change** of the function *f*(*x*) at *x*=*a* is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(provided this limit exists).

Geometrically, this number represents the **slope of the tangent** to the curve at (*a*, *f*(*a*)).



$$m_P = \lim_{Q \to P} m_{PQ}$$

The Derivative

Definition:

Given a function $f(x)$, the **derivative of *f* with respect to *x*** is the *function* $f'(x)$ defined by

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The ***domain*** of this function is the set of all x-values for which the limit exists.

$$domain(f') \subseteq domain(f)$$

The Derivative

Interpretations of f':

1. The function $f'(x)$ tells us the **instantaneous rate of change** of $f(x)$ with respect to x for all x-values in the domain of $f'(x)$.

2. The function $f'(x)$ tells us the **slope of the tangent** to the graph of $f(x)$ at every point $(x, f(x))$, provided x is in the domain of $f'(x)$.

The Derivative

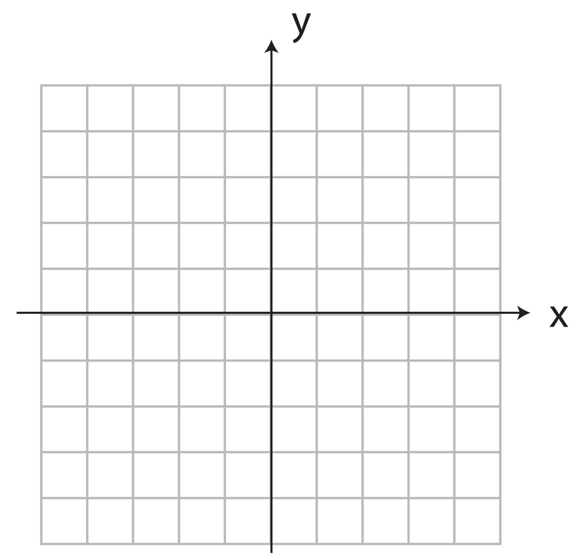
Example:

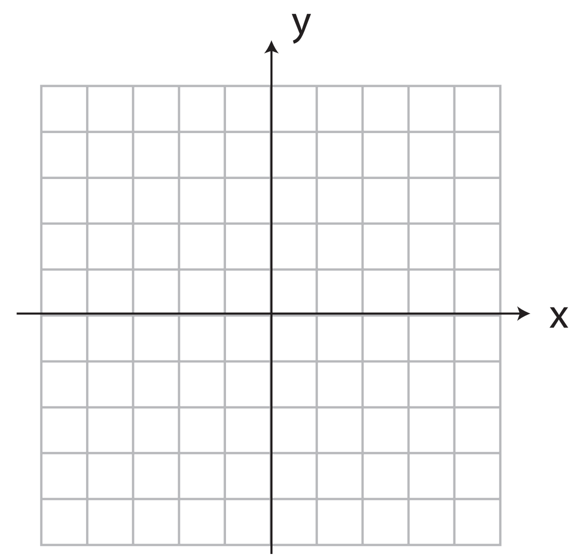
Find the derivative of

$$f(x) = x^2 + 2x.$$

and use it to calculate the instantaneous rate of change of *f*(*x*) at *x*=1.

Sketch the graph of *f*(*x*) and the graph of *f'*(*x*).





The Derivative

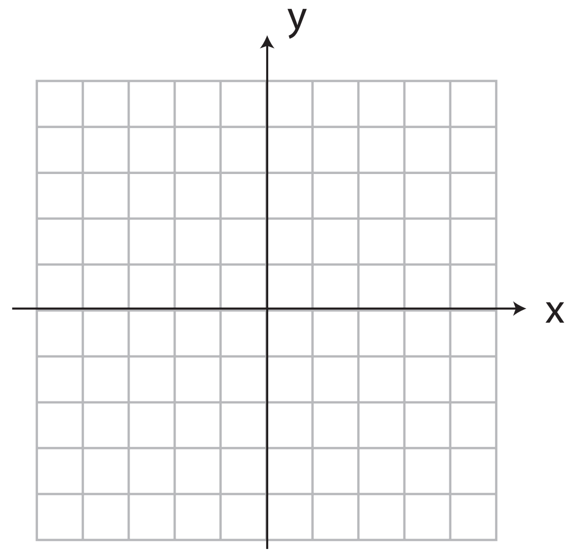
Example:

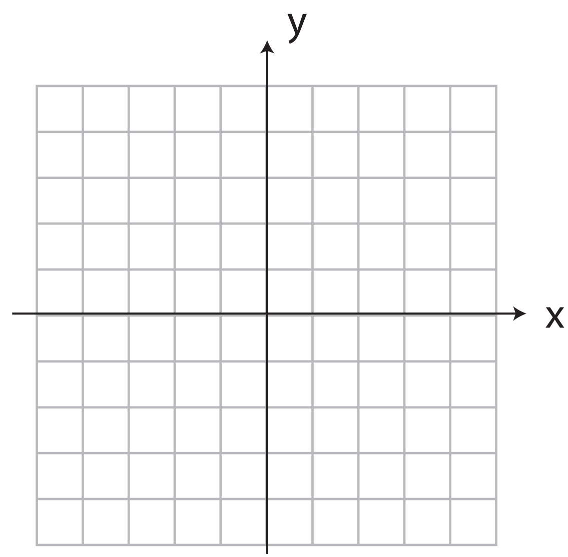
Find the derivative of

$$f(x) = \sqrt{x + 3}$$

and state the domain of both $f(x)$ and $f'(x)$.

Sketch the graph of $f(x)$ and the graph of $f'(x)$.





Relationship between *f'* and *f*

<u>If *f* is increasing on an interval (*c*,*d*):</u>
The derivative *f'* is positive on (*c*,*d*).
The rate of change of *f* is positive for all *x* in (*c*,*d*).
The slope of the tangent is positive for all *x* in (*c*,*d*).

<u>If *f* is decreasing on an interval (*c*,*d*):</u>
The derivative *f'* is negative on (*c*,*d*).
The rate of change of *f* is negative for all *x* in (*c*,*d*).
The slope of the tangent is negative for all *x* in (*c*,*d*).

Critical Numbers

<u>Definition:</u>

c is a **critical number** of *f* if *c* is in the domain of *f* and either *f'(c)=0* or *f'(c)* D.N.E.

Example:

Identify the critical numbers of $f(x) = x^2 + 2x$ and $f(x) = \sqrt{x+3}$.

Differentiable Functions

A function *f(x)* is said to be **differentiable** at *x=a* if we are able to calculate the derivative of the function at that point, i.e., *f(x)* is differentiable at *x=a* if

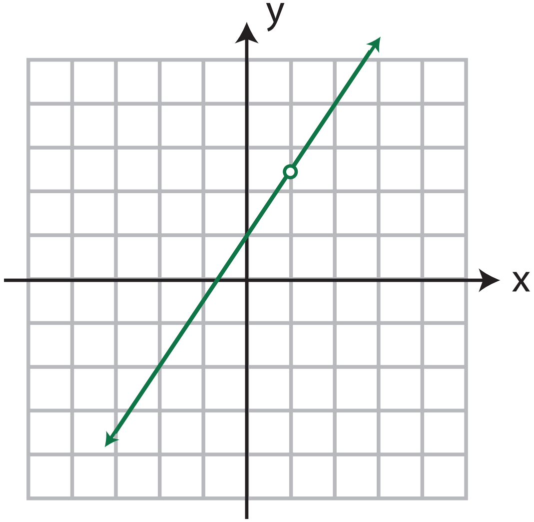
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

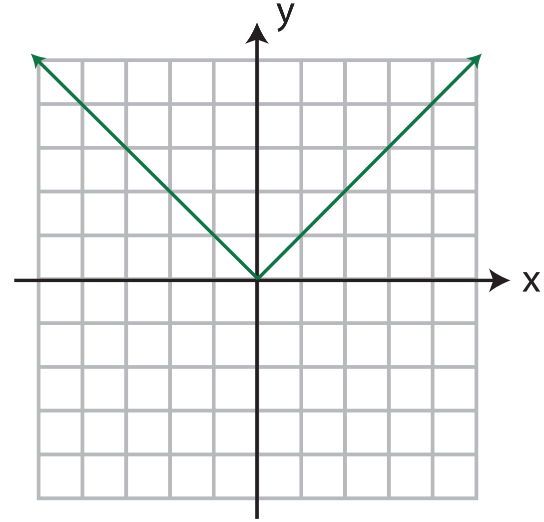
exists.

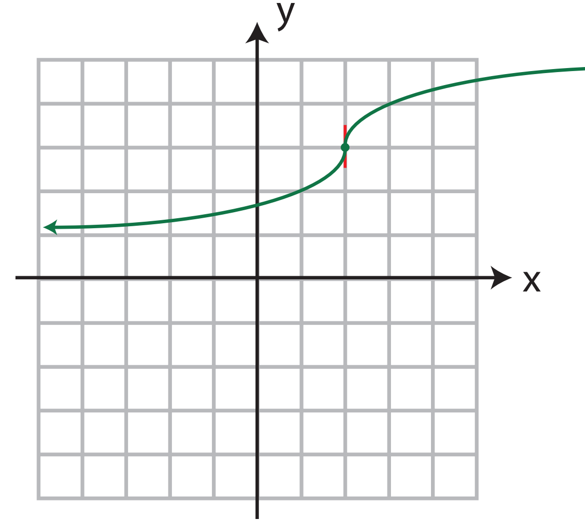
Differentiable Functions

Geometrically, a function is differentiable at a point if its graph has a unique tangent line with a well-defined slope at that point.

3 Ways a Function Can **<u>Fail</u>** to be Differentiable:



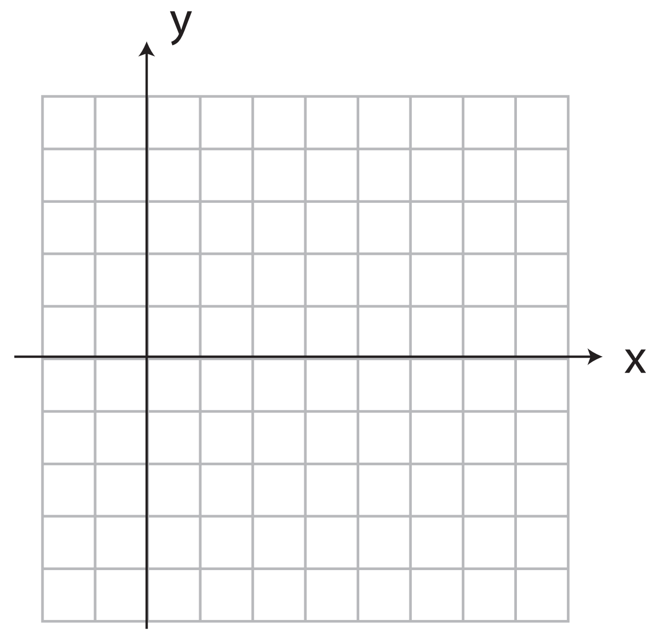




Graphs

Example:

(a) Sketch the graph of $f(x) = \left|x^2 - 2x\right|$.



(b) By looking at the graph of *f*, sketch the graph of $f'(x)$.