Basic Differentiation Rules

- All rules are proved using the **definition of the derivative**:

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- The derivative exists (i.e. a function is differentiable) at all values of *x* for which this limit exists.

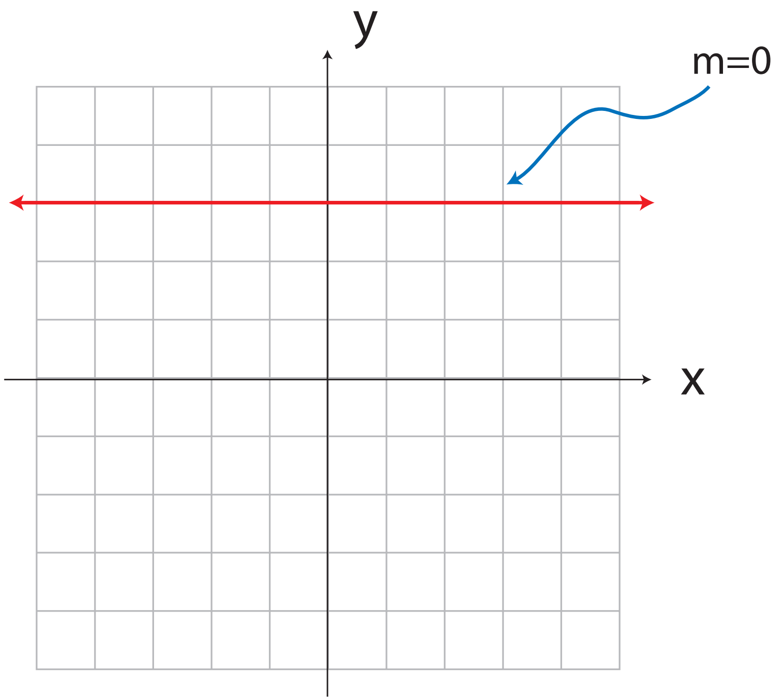
The Constant Function Rule

If $f(x) = k$, where k is a constant, then $f'(x) = 0$.

Example:

$$f(x) = 3$$

$$f'(x) = 0$$



The Power Rule

If $f(x) = x^n$, where $n \in R$, then $f'(x) = nx^{n-1}$.

Example:

Differentiate the following.

(a) $f(x) = x$

(b) $g(x) = x^{100}$

(c) $h(x) = \frac{1}{x^6}$

(d) $s(t) = \sqrt{t}$

The Constant Multiple Rule

Let k be a constant.

Then $$\frac{d}{dx}\left[k \cdot f(x)\right] = k \cdot \frac{d}{dx}\left[f(x)\right].$$

Example:

Find each derivative.

(a) $\frac{d}{dK}\left[\frac{Kl(\gamma+1)^2}{D^4}\right]$ (b) $\frac{d}{dD}\left[\frac{Kl(\gamma+1)^2}{D^4}\right]$

The Sum/Difference Rule

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

provided f and g are differentiable functions.

Examples:

Differentiate.

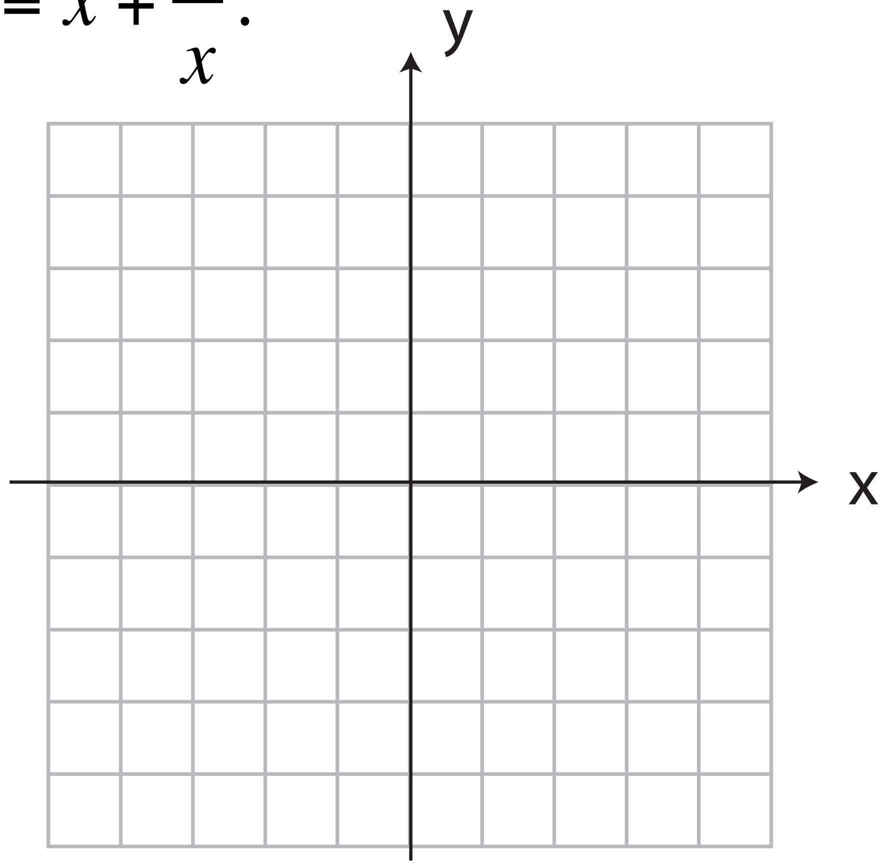
(a) $f(x) = 3x^4 + 5x^2 - 10$

(b) $g(x) = \sqrt{5x} + \dfrac{x}{\sqrt{5}} - \pi^2$

Using the Derivative to Sketch the Graph of a Function

Example:

Sketch the graph of $F(x) = x + \frac{1}{x}.$



Derivative of the Natural Exponential Function

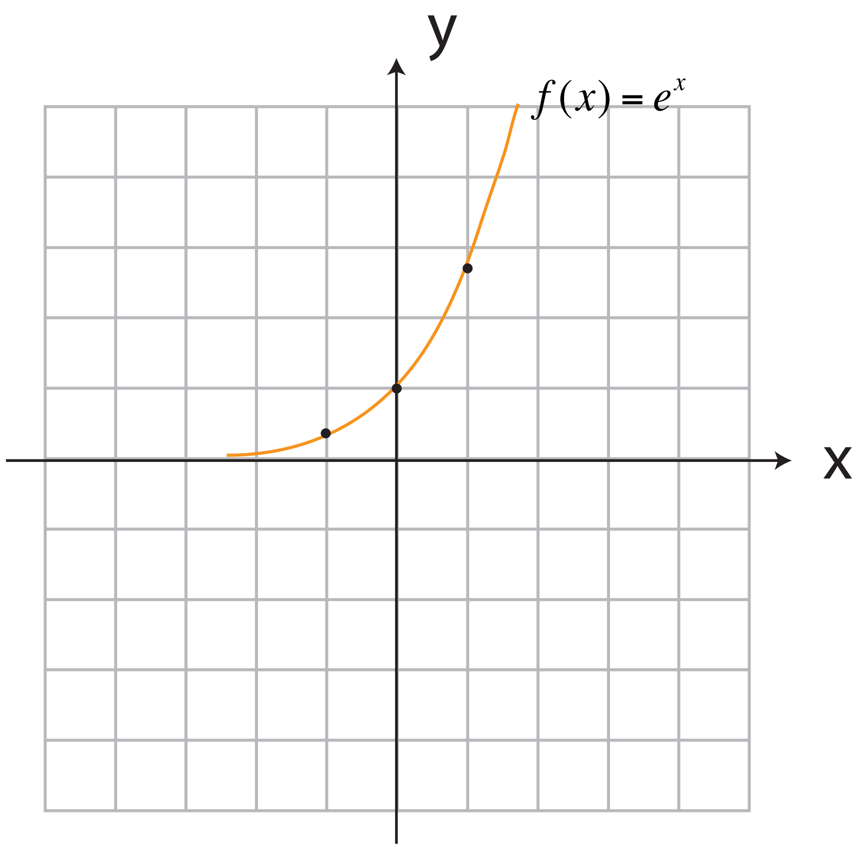
Definition:

The number *e* is the number for which

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

Natural Exponential Function:

$$f(x) = e^x$$



Derivative of the Natural Exponential Function

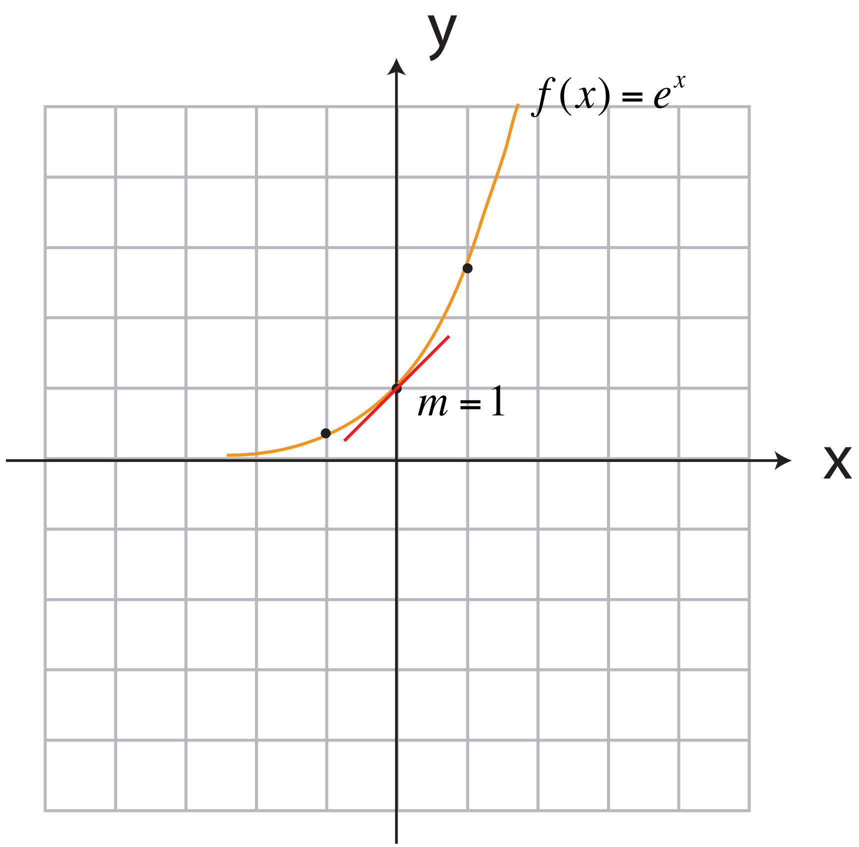
Note:

This definition states that the slope of the tangent to the curve at (0,1) is exactly 1, i.e.

$$f'(0) = \lim_{h \to 0} \frac{e^{0+h} - e^{0}}{h}$$

$$= \lim_{h \to 0} \frac{e^{h} - 1}{h}$$

$$= 1$$



Derivative of the Natural Exponential Function

$$\text{If } f(x) = e^x, \text{ then } f'(x) = e^x.$$

In words:
The slope of the tangent line to the curve $f(x) = e^x$ at the point P is equal to the value of the function at P.