Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

"derivative of the outer function evaluated at the inner function times the derivative of the inner function"

Example:
Differentiate the following.

(a) $f(x) = (4x^3 + 1)^{10}$

(b) $g(x) = \dfrac{1}{\sqrt{x^2 + e^{3x}}}$

Derivatives of General Exponential Functions

If $f(x) = a^x$, then $f'(x) = a^x \cdot \ln a$.

Example:

Differentiate.

(a) $f(x) = e^x$ (b) $g(x) = 7^x + x^7$ (c) $h(x) = 2^{5x^2+1}$

Derivatives of General Exponential Functions

Example:

Show that $y = 3e^{4x}$ satisfies the differential equation $y' = 4y$.

Derivatives of Logarithmic Functions

If $f(x) = \log_a x$, then $f'(x) = \dfrac{1}{x \cdot \ln a}$.

Example 1:
Differentiate.
(a) $f(x) = \ln x$ (b) $g(x) = \log_4(x^2 + 5x + 6)$

Example 2:
Determine the equation of the tangent line to the curve $f(x) = \dfrac{\ln x}{x}$ at the point $P(1,0)$.