Maximum and Minimum Values

$f(c)$ is a <u>*global (absolute) maximum*</u> of f if
$f(c) \geq f(x)$ for all x in the domain of f.

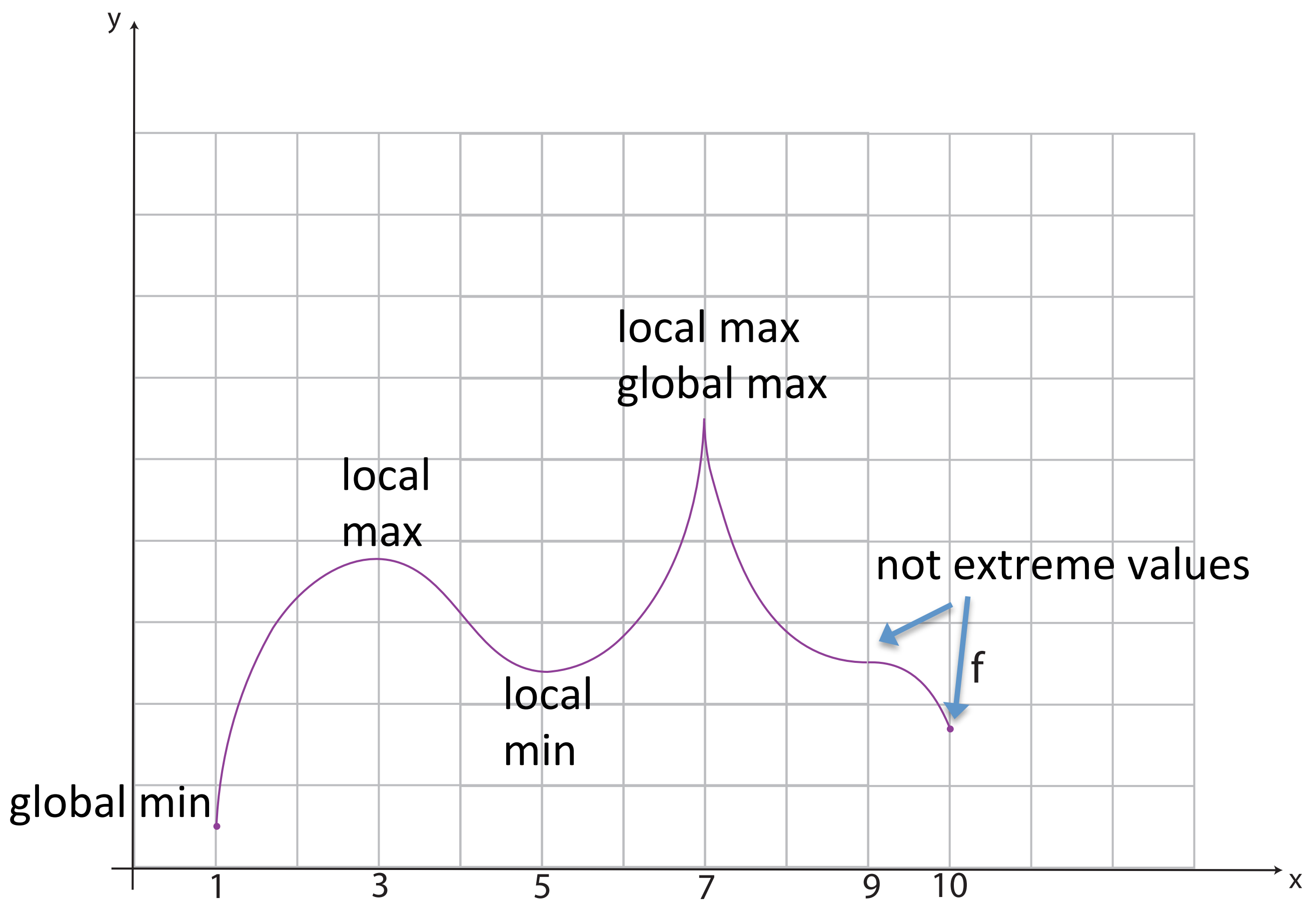
$f(c)$ is a <u>*local (relative) maximum*</u> of f if
$f(c) \geq f(x)$ for all x in some interval around c.

Maximum and Minimum Values

$f(c)$ is a *global (absolute) minimum* of f if
$f(c) \leq f(x)$ for all x in the domain of f.

$f(c)$ is a *local (relative) minimum* of f if
$f(c) \leq f(x)$ for all x in some interval around c.

Extrema



y
local max
global max
local
max
not extreme values
f
local
min
global min
1
3
5
7
9
10
x

Extreme Values

Notice:

Extreme values occur at either a critical number of f or at an endpoint of the domain.

(However, not all critical numbers and endpoints correspond to an extreme value.)

Also note:

By definition, ***relative*** extreme values do not occur at endpoints.

Finding Local Maxima and Minima (First Derivative Test)

Assume that f is continuous at c, where c is a critical number of f.

If f' changes from + to - at $x=c$, then f changes from increasing to decreasing at $x=c$ and $f(c)$ is a local maximum value.

If f' changes from - to + at $x=c$, then f changes from decreasing to increasing at $x=c$ and $f(c)$ is a local minimum value.

If f' does not change sign at $x=c$, then f doesn't have an extreme value at $x=c$.

Finding Local Maxima and Minima (First Derivative Test)

Example:

Find the **local** extrema of $f(x)=\dfrac{\ln x}{x}$.

Finding Local Maxima and Minima (Second Derivative Test)

Assume that f'' is continuous near c and $f'(c)=0$.

If $f''(c)>0$ then the graph of f is concave up at $x=c$ and $f(c)$ is a local minimum value.

If $f''(c)<0$ then the graph of f is concave down at $x=c$ and $f(c)$ is a local maximum value.

If $f''(c)=0$ or $f''(c)$ D.N.E. then the second derivative test doesn't apply and you have to use the other method.

Application

Assignment 53, #1 (modified):

Consider the function $f(t) = Ate^{-\beta t}$, where $A, \beta > 0$.

(a) Find the critical number of *f*.

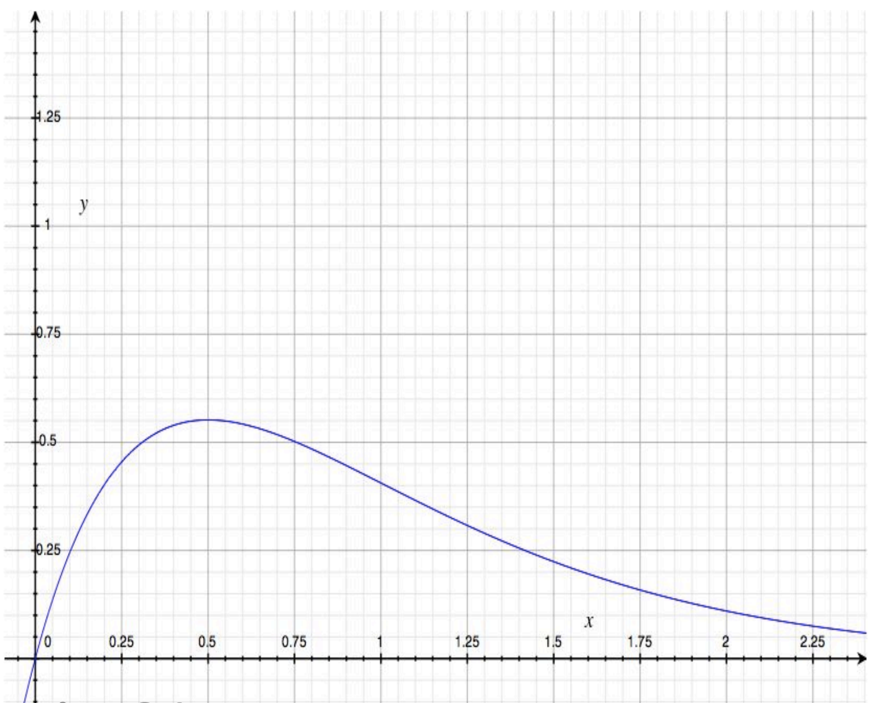
Application

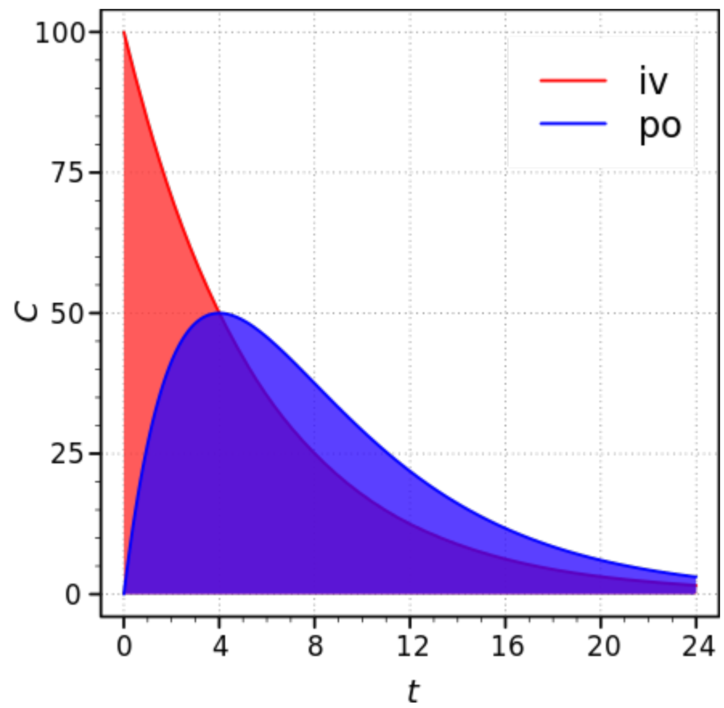
Assignment 53, #1 (modified):
(b) Use the second derivative test to determine if the critical number in part (a) corresponds to a local maximum, local minimum, or neither.

Application

Assignment 53, #1 (modified):

(c) Determine the values of A and β such that f describes the graph given below.





Extreme Value Theorem

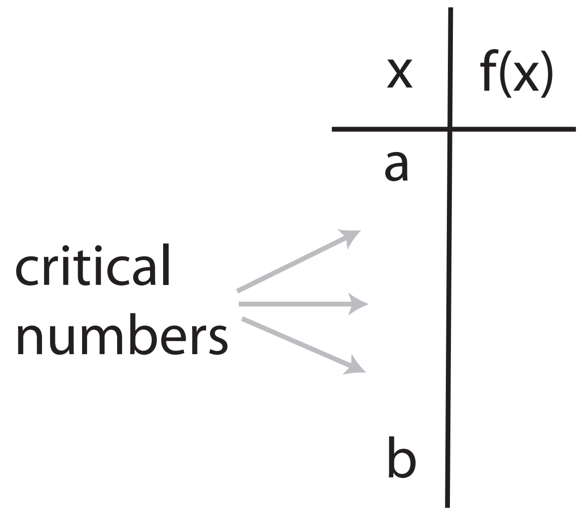
If $f(x)$ is continuous for all $x \in [a,b]$, then there are points $c_1, c_2 \in [a,b]$ such that $f(c_1)$ is the global minimum and $f(c_2)$ is the global maximum of $f(x)$ on $[a,b]$.

In words:

If a function is *continuous* on a *closed, finite interval*, then it has a global maximum and a global minimum on that interval.

Finding Absolute Extreme Values on a Closed Interval [a,b]

1. Find all critical numbers in the interval.
2. Make a table of values.



The largest value of f(x) is the absolute maximum and the smallest value is the absolute minimum.

Finding Absolute Extreme Values on a Closed Interval [a,b]

Example:

Find the **absolute** extrema of $g(x) = x^{\frac{1}{3}}(x-2)^2$ on $[-1, 1]$.