L'Hopital's Rule

Another application of derivatives is to help evaluate limits of the form

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

where either $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$

or $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$.

Idea:
Instead of comparing the functions f(x) and g(x), compare their derivatives (rates) f'(x) and g'(x).

L'Hopital's Rule

Suppose that *f* and *g* are differentiable functions such that

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is an **indeterminate form** of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

If $g'(x) \neq 0$ near a (could be 0 at a) then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

L’Hopital’s Rule

Evaluate the following limits using L’Hopital’s Rule, if it applies.

(a) $\lim_{x \to \infty} \frac{\ln x}{\sqrt[3]{x}}$

(b) $\lim_{x \to 0} \frac{\sin x}{x}$

(c) $\lim_{x \to 0} \frac{\tan x - x}{x^3}$

L'Hopital's Rule

Evaluate the following limits using L'Hopital's Rule, if it applies.

(a) $\lim_{x \to \infty} x^2 e^{-3x}$

(b) $\lim_{x \to \infty} x^{\frac{1}{x}}$