Checking Stability of Equilibria

To determine stability, we can use:

1. Cobwebbing
2. "Graphical Criteria" (if the the updating function is increasing at the equilibrium)
3. "Slope Criteria" i.e. the Stability Theorem (provided the slope at the equilibrium isn't exactly -1 or 1)

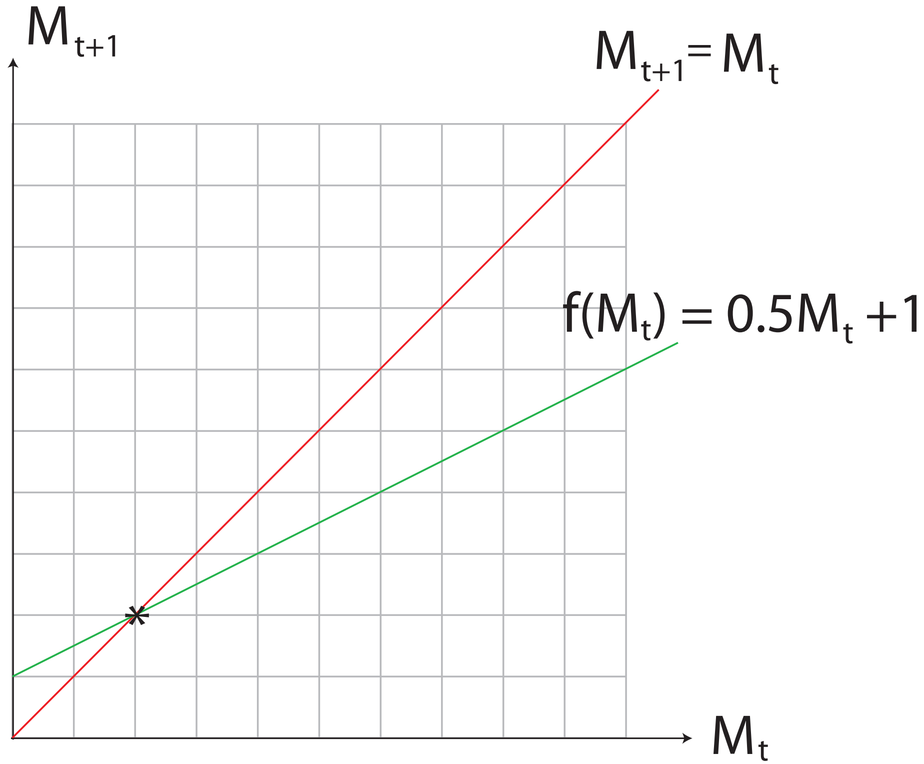
Stability Theorem for DTDSs

- An equilibrium is **stable** if the absolute value of the derivative of the updating function is **< 1** at the equilibrium, i.e.,

$$\left|f'(m^*)\right| < 1$$

Example:

$$M_{t+1} = \frac{1}{2} M_t + 1$$



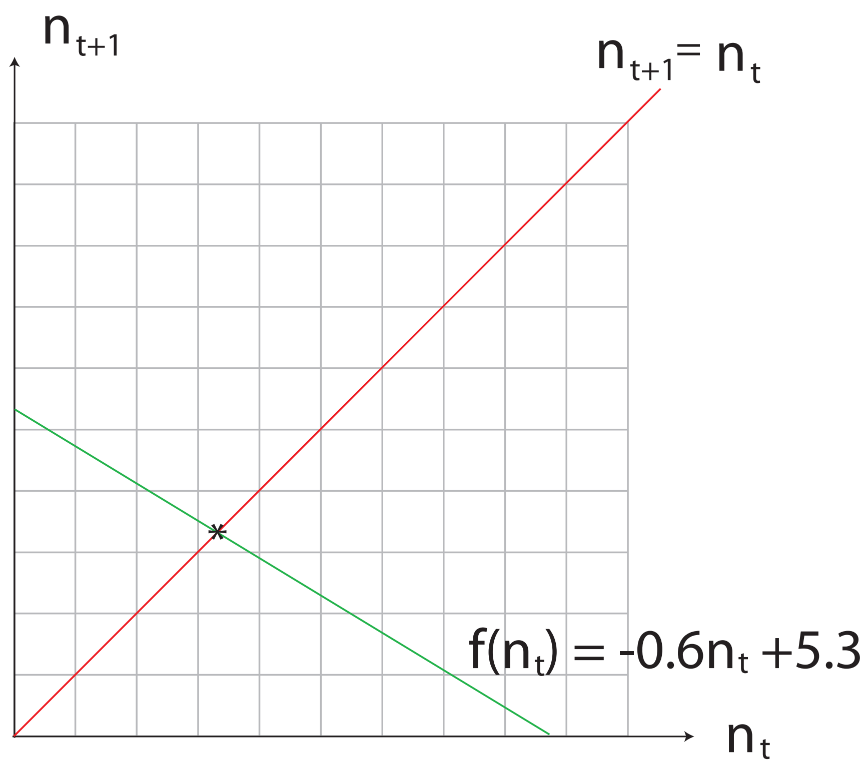
Stability Theorem for DTDSs

- An equilibrium is **stable** if the absolute value of the derivative of the updating function is **< 1** at the equilibrium, i.e.,

$$\left|f'(m^*)\right| < 1$$

Example:

$$n_{t+1} = -0.6n_t + 5.3$$



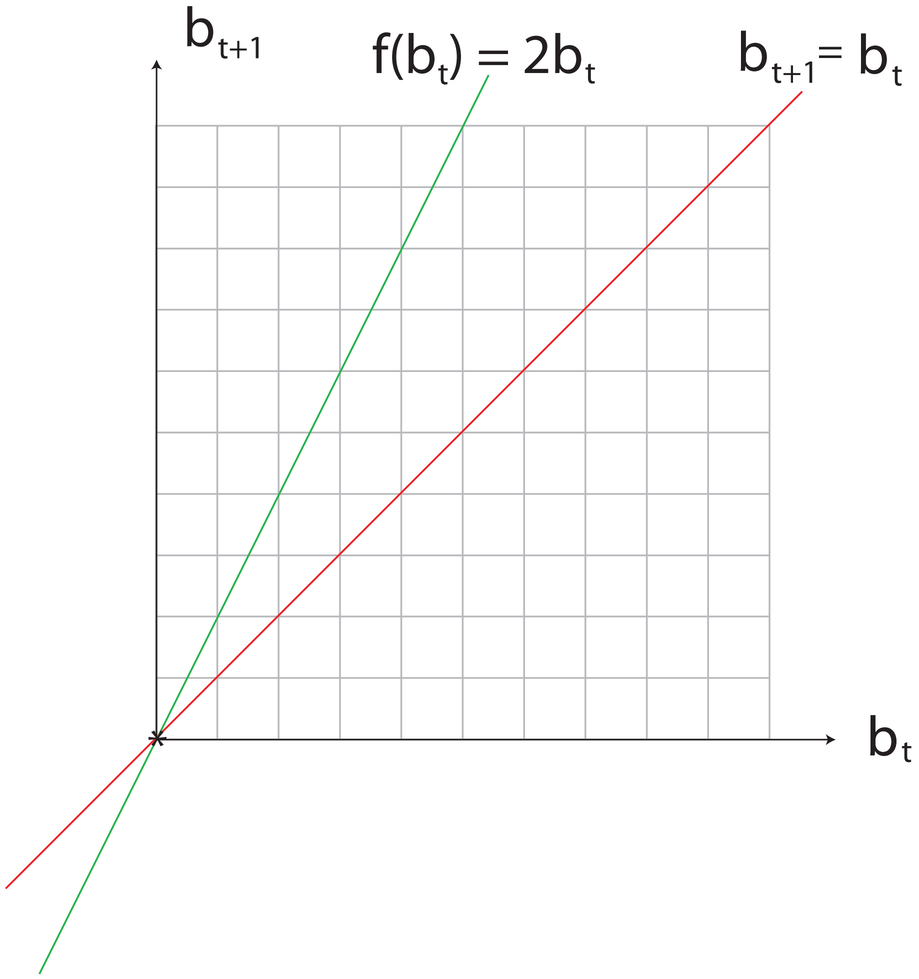
Stability Theorem for DTDSs

- An equilibrium is **unstable** if the absolute value of the derivative of the updating function is **> 1** at the equilibrium, i.e.,

$$\left| f'(m^*) \right| > 1$$

Example:

$$b_{t+1} = 2b_t$$



section 6.8

Stability Theorem for DTDSs

- If the slope of the updating function is exactly **1 or -1** at the equilibrium, i.e.,

$$|f'(m^*)| = 1$$

then the equilibrium could be **stable**, **unstable**, or **half-stable**.

Example:

Stability Theorem for DTDSs

Example:

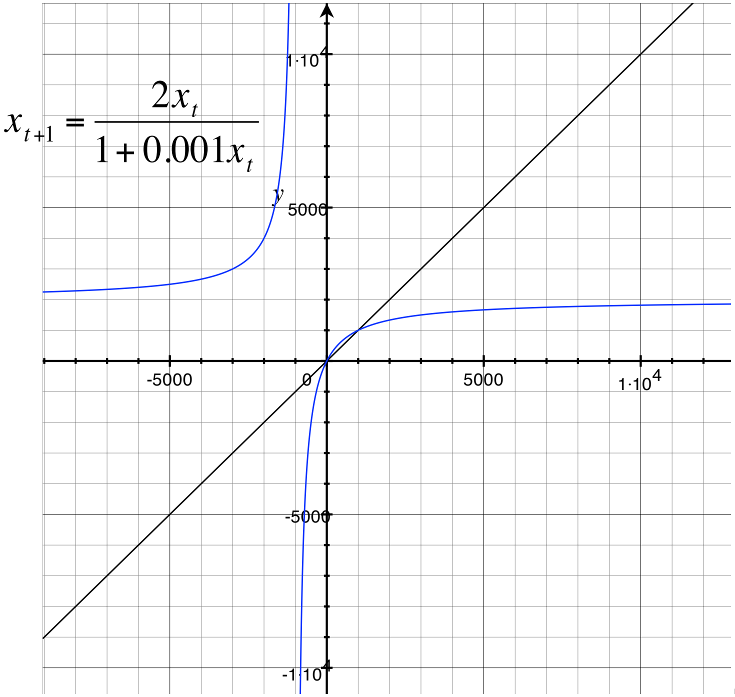
DTDS for a limited population

$$x_{t+1} = \frac{2x_t}{1 + 0.001x_t}$$

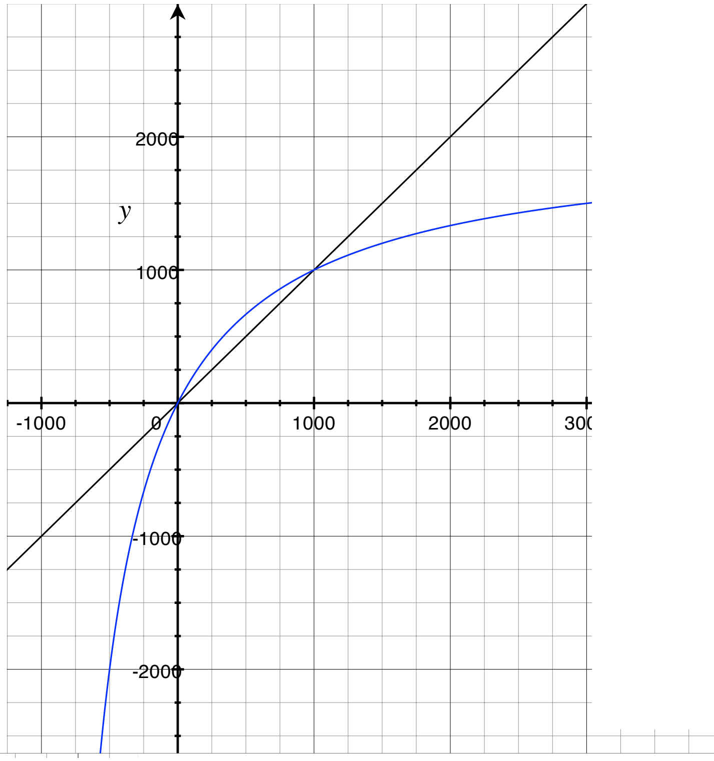
Stability Theorem for DTDSs

Example:

DTDS for a limited population



Zoom In



Stability Theorem for DTDSs

Example:

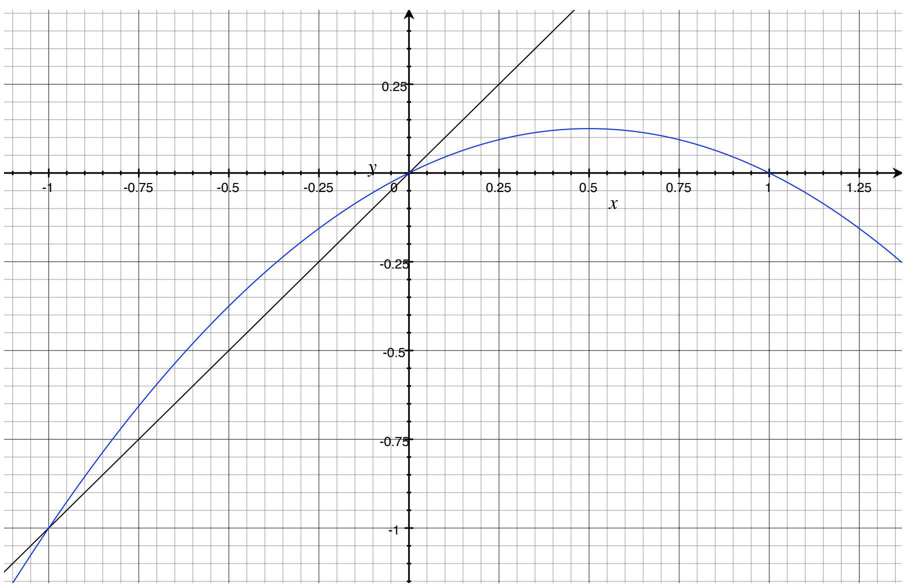
logistic dynamical system

$$x_{t+1} = rx_t(1 - x_t)$$

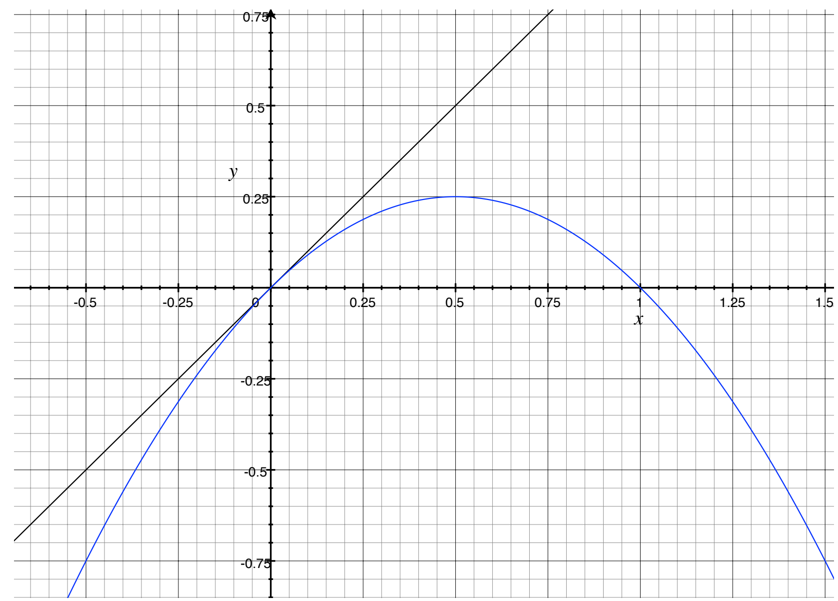
Stability Theorem for DTDSs

Example:

logistic dynamical system



$$x_{t+1} = 0.5x_t(1 - x_t)$$



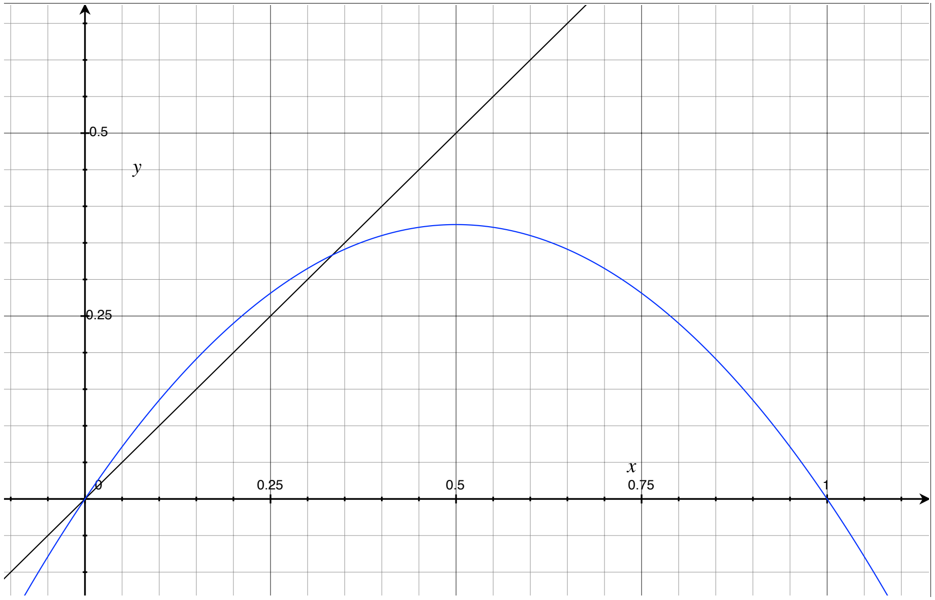
$$x_{t+1} = x_t(1 - x_t)$$

section 6.8

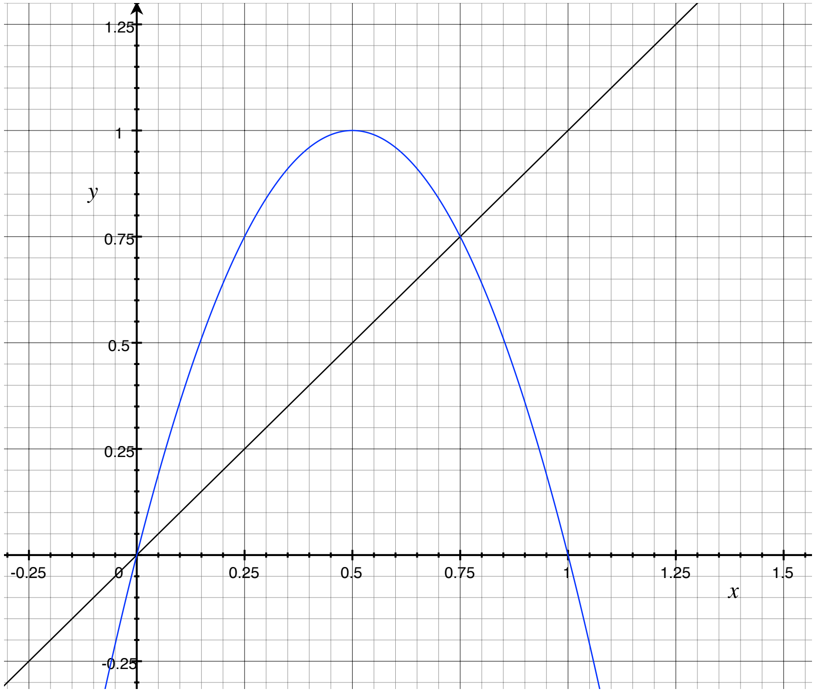
Stability Theorem for DTDSs

Example:

[logistic dynamical system]



$$x_{t+1} = 1.5 x_t (1 - x_t)$$



$$x_{t+1} = 3.5 x_t (1 - x_t)$$