Solving Pure-Time DEs

The general form of a pure-time differential equation is

$$\frac{dF}{dx} = f(x)$$

where $F(x)$ is the unknown state variable and $f(x)$ is the measured rate of change.

Examples:

(a) $\frac{dF}{dx} = 4x^3 + 1$

(b) $\frac{dy}{dx} = 5e^x + \frac{1}{1+x^2}$

Solving Pure-Time DEs

"Guess and check" to solve each equation.

Ask yourself:

"What function has this as its derivative?"

Examples:

(a) $\frac{dF}{dx} = 4x^3 + 1$

(b) $\frac{dy}{dx} = 5e^x + \frac{1}{1+x^2}$

Antiderivatives/Indefinite Integrals

An antiderivative (or indefinite integral) of a function $f(x)$ is a function $F(x)$ with derivative equal to $f(x)$.

$$F'(x) = f(x) \quad \Leftrightarrow \quad F(x) = \int f(x)dx$$

An antiderivative $F(x)$ is a solution to the pure-time differential equation

$$\frac{dF}{dx} = f(x).$$

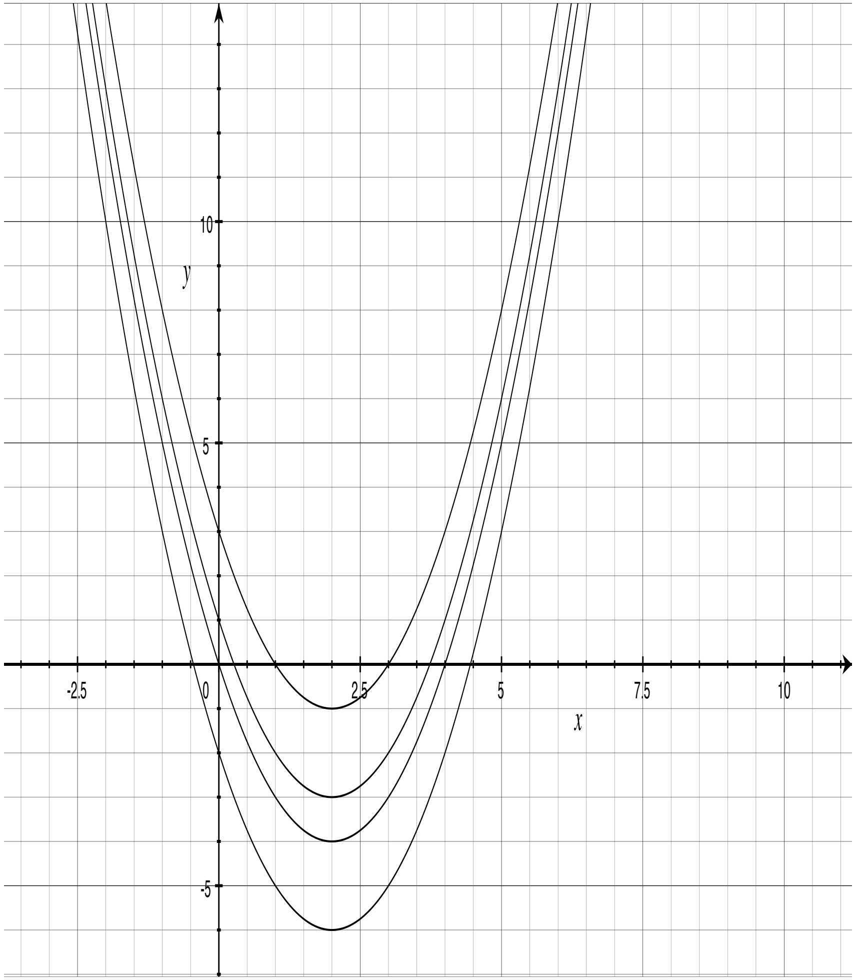
Initial Value Problems

A differential equation has a whole family of solutions.

Example:

$$y' = 2x - 4$$

$$y = x^2 - 4x + C$$



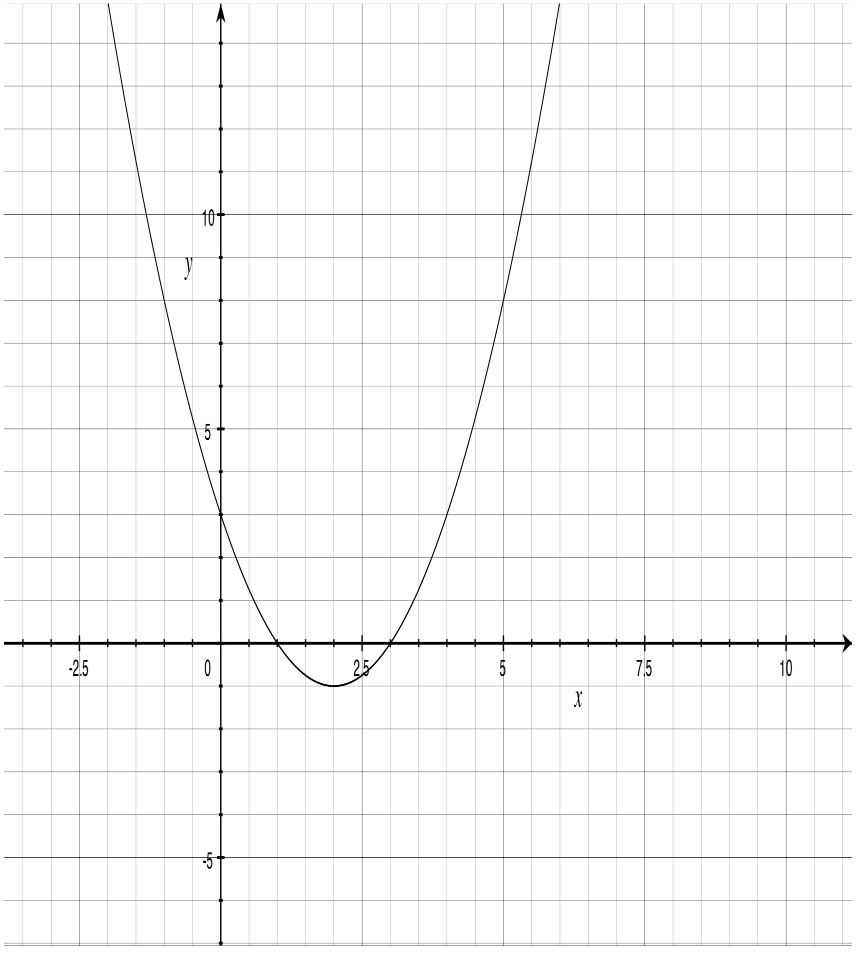
Initial Value Problems

An initial value problem provides an initial condition so you can find a particular solution.

Example:

$$y' = 2x - 4, \quad y(0) = 3$$

$$y = x^2 - 4x + 3$$



Antiderivatives/Indefinite Integrals

Theorem 7.2.1:
If $F(x)$ is an antiderivative of $f(x)$, then the most general antiderivative of $f(x)$ is $F(x)+C$; i.e.,

$$\int f(x)dx = F(x) + C$$

where C is a real number.

If an initial value of the solution is given, then we can solve for C to find a specific or particular antiderivative of $f(x)$.

Rules for Antiderivatives

The Power Rule for Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad for\ n \neq -1$$

Example:

Integrate each.

(a) $\int x^7 dx$ (b) $\int \frac{1}{t^4} dt$ (c) $\int \sqrt{x}\, dx$

Rules for Antiderivatives

The Constant Multiple Rule for Integrals

Suppose $\int f(x)dx = F(x) + C.$

Then $\int af(x)dx = aF(x) + C'.$ for any constant $a.$

The Sum Rule for Integrals

Suppose $\int f(x)dx = F(x) + C \quad and \quad \int g(x)dx = G(x) + C'.$

Then

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx = F(x) + G(x) + C''.$$

Examples

Example 1:
Integrate.

(a) $\int (5x^4 - \frac{3}{x})dx$ (b) $\int (\sec^2 x + e^{4x})dx$

Example 2:
Solve the differential equation

$$f'(x) = 2^x + \frac{1}{2\sqrt{x}}$$

with initial condition $f(0) = 0$.

More Examples

Example 1:

Integrate.

(a) $\int \frac{Kl(\gamma+1)^2}{D^4}\,dD$ (b) $\int \frac{Kl(\gamma+1)^2}{D^4}\,d\gamma$

Example 2:

True or False? Explain.

(a) $\int \arctan x\,dx = \frac{1}{1+x^2}$ (b) $\int e^{x^2}\,dx = \frac{e^{x^2}}{2x} + \mathrm{C}$

Summary Of Some Basic Integration Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad for\ n \neq -1$$

$$\int \cos x dx = \sin x + C$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int e^x dx = e^x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$