Types of Integrals

• Indefinite Integral

function of x

$$\int f(x) dx = F(x) + C$$
antiderivative of f

• Definite Integral

$$\int_{a}^{b} f(x) dx = net area$$

The Fundamental Theorem of Calculus

If f is **continuous** on [a,b], then

$$\int_{a}^{b} f(x) dx = F(x) \Big|_{a}^{b} = F(b) - F(a)$$

where F is <u>any</u> antiderivative of f, i.e., F'=f.

Evaluating Definite Integrals

Example:

Evaluate each definite integral using the FTC.





Evaluating Definite Integrals

Example:

Try to evaluate the following definite integral using the FTC. What is the problem?

$$\int_{1}^{4} \frac{1}{(x-2)^2} dx$$

Differentiation and Integration as Inverse Processes

If *f* is integrated and then differentiated, we arrive back at the original function *f*.

$$\frac{d}{dx}\int_{a}^{x} f(t) dt = f(x)$$
FTC I

If *F* is differentiated and then integrated, we arrive back at the original function *F*.

$$\int_{a}^{b} \frac{d}{dx} F(x) dx = F(x) \Big|_{a}^{b}$$
FTC II

The Definite Integral - Total Change

Interpretation:

The definite integral represents the total amount of change during some period of time.

Total change in *F* between times *a* and *b*:

$$F(b) - F(a) = \int_{a}^{b} \frac{dF}{dt} dt$$
value at end value at start rate of change

Application – Total Change

Example:



Suppose that the growth rate of a fish is given by the differential equation

$$\frac{dL}{dt} = 6.48e^{-0.09t}$$

where *t* is measured in years and *L* is measured in centimetres and the fish was 0.0 cm at age *t*=0 (time measured from fertilization).

Application – Total Change

(a) Determine the amountthe fish grows between 2and 5 years of age.

(b) At approximately what age will the fish reach45cm?