MATHEMATICS 1LT3 TEST 1

Day Class — Dr. E. Clements
Duration of Test: 50 minutes
McMaster University — 16 February 2022

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 6 PAGES AND 4 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 28. Marks are indicated next to the problem number in square brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Question 1.

Problem	Points	Mark
1	8	
2	8	
3	7	
4	5	
TOTAL	28	

MATH 1LT3 * Test 1 * 16 February 2022 Name:_____

Student No.: _____

1. **Multiple choice questions: circle ONE answer. No justification is needed**.

(a) [2] Consider the differential equation $dy/dt = y^3 - y$. Which of the following statements is/are true?

(I) $dy/dt = y^3 - y$ is an autonomous differential equation.

(II) $y^* = 0$ is a stable equilibrium.

(III) $y^* = 1$ is an unstable equilibrium.

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(b) [2] Let $y(x)$ be the solution to the initial value problem $\dfrac{dy}{dx} = 1 + y^2$, where $y(0) = 1$. Use the separation of variables technique to find $y(2)$, correct to three decimal places.

(A) -0.138 (B) 1.657 (C) -1.728 (D) 2.883

(E) 5.341 (F) 0.432 (G) -0.372 (H) none of these

(c) [2] The following pair of equations represent the population growth of two different species where one is the predator and the other is the prey.

$$\frac{dA}{dt} = 0.1A - 0.005AB, \quad \frac{dB}{dt} = -0.05B + 0.0001AB$$

Which of the following statements is/are true?

(I) The variable A represents the prey population.

(II) The per capita growth rate of of species B is $0.0001A$

(III) $(A, B) = (20, 50)$ is an equilibrium of this system.

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(d) [2] Determine the range of $z = e^{-x^2-y^2}$.

(A) $(-\infty, \infty)$ (B) $[1, e]$ (C) $(0, e)$ (D) $(-\infty, 1]$

(E) $(-\infty, 0)$ (F) $(0, \infty)$ (G) $(0, 1]$ (H) none of these

2. A population of caribou is modelled by $\frac{dP}{dt} = 0.7P\left(1 - \frac{P}{480}\right)$.

(a) [2] Find the equilibria of this equation. What does the larger equilibrium represent?

(b) [2] Graph $\frac{dP}{dt}$ as a function of P.

(c) [2] Draw a phase-line diagram for $\frac{dP}{dt} = 0.7P\left(1 - \frac{P}{480}\right)$.

(d) [2] Suppose that initially there are 80 caribou. Sketch the solution curve $P(t)$.

3. Consider the modified competition equations

$$\frac{da}{dt} = 0.1\left(1 - \frac{b}{200}\right)a, \quad \frac{db}{dt} = 0.2\left(1 - \frac{a}{300}\right)b$$

(a) [2] Find and graph the nullclines in the phase plane.

(b) [1] Identify the equilibria.

(c) [2] Add direction arrows to your phase-plane diagram in part (a). Include direction arrows in each region as well as on the nullclines.

(d) [2] Sketch phase-plane trajectories starting from (i) $(a, b) = (150, 100)$ and (ii) $(a, b) = (400, 300)$.

4. (a) [2] Find and sketch the domain of $f(x, y) = \ln(x - y^3)$.

(b) [3] Create a contour map for $g(x, y) = 2\arctan(xy)$. Include level curves corresponding to $k = -2$, $k = 0$, and $k = 2$.