MATHEMATICS 1LT3 TEST 2

Day Class — Dr. E. Clements
Duration of Test: 50 minutes
McMaster University — 30 March 2022

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 6 PAGES AND 5 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 26. Marks are indicated next to the problem number in square brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Question 1.

Problem	Points	Mark
1	8	
2	6	
3	7	
4	3	
5	2	
TOTAL	26	

1. **Multiple choice questions: circle ONE answer. No justification is needed**.

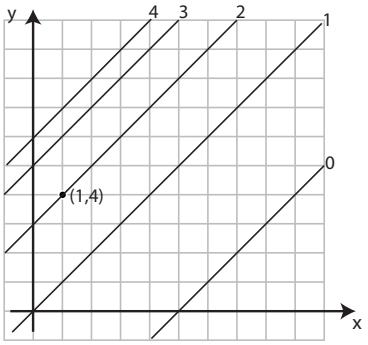
(a) [2] Consider the function $f(x, y) = e^{y/x}$. Which of the following statements is/are true?

(I) f is continuous at $(2, 1)$.
(II) f_x and f_y are continuous on $B_{1.5}(2, 1)$.
(III) f is differentiable at $(2, 1)$.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b) [2] For the contour diagram of $f(x, y)$ below, which of the following is/are true?

(I) $f_{yy}(1, 4) > 0$
(II) $\bigtriangledown f(1, 4)$ points in the same direction as $\mathbf{v_1} = -\mathbf{i} + \mathbf{j}$
(III) $D_{\mathbf{v_2}} f(1, 4) = 0$ when $\mathbf{v_2} = \mathbf{i} + \mathbf{j}$



(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(c) [2] Consider the random experiment of rolling a fair, six-sided die. Let A be the event of rolling an even number and let B be the event of rolling a number greater than 2. Which of the following is/are true?

(I) $P(A \cap B) = \frac{1}{3}$ (II) $P(A \cup B) = \frac{5}{6}$ (III) $P(A^C) = \frac{1}{6}$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(d) [2] The Centers for Disease Control estimates that 1 in every 20,000 eggs are contaminated with Salmonella (a bacteria which commonly causes foodborne illness). What is the probability that in a sample of 1644 eggs, at least one is contaminated with Salmonella?

(A) 0.0107 (B) 0.3128 (C) 0.0112 (D) 0.0018

(E) 0.7998 (F) 0.1201 (G) 0.0789 (H) none of these

2. (a) [3] Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{4y\sin(5x)}{6x^2+8y^2}$ does not exist.

(b) [3] Find the linearization of the function $f(x,y)=\sqrt{x^2+3y^2}$ at the point $(1,1)$ and use it to approximate $f(0.9,1.1)$.

3. Consider the function $f(x, y) = x^3 - 6xy + y^2$.

(a) [2] $(0, 0)$ is a critical point of $f(x, y)$. Find the other critical point.

(b) [3] Using the second derivatives test, classify the critical points from part (a).

(c) [2] Find the directional derivative of $f(x, y)$ at the point $(2, 1)$ in the direction specified by $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

4. Consider a population of 250 moose. Suppose that within any given year, there is a 60% chance that the population will increase by 12 and a 40% chance that it will stay the same.

(a) [2] Write the sample space for the population of moose after 3 years.

(b) [1] What is the probability that the population will have increased after 3 years?

5. [2] The incidence of asthma in young adults is 6.4% for females and 4.5% for males. Consider a group of young adults consisting of 30 females and 40 males. What is the probability that a young adult from this group who has asthma is female? Round your answer to three decimal places.

THE END