Stability of Equilibria

Section 8.3

Stability

Definition:

An equilibrium of an autonomous DE is **stable** if solutions that begin near the equilibrium approach the equilibrium.

An equilibrium of an autonomous DE is **unstable** if solutions that begin near the equilibrium move away from the equilibrium.

Stability Theorem for Autonomous DEs

Suppose $\frac{dm}{dt} = f(m)$

is an autonomous DE with an equilibrium at m^*.

The equilibrium at m^* is **stable** if $f'(m^*) < 0$

and **unstable** if $f'(m^*) > 0$.

Whenever $f'(m^*) = 0$ the stability theorem does not apply and we must analyze it another way.

Stability Theorem for Autonomous DEs

Examples:
Use the stability theorem to determine whether the equilibria in the following models are stable or unstable. Compare with your phase-line diagrams.

(a) $\frac{dP}{dt} = 0.08P\left(1 - \frac{P}{1000}\right)$

(b) $\frac{dx}{dt} = 1 - e^{x}$

A Model for a Disease (starts on p. 614 in your textbook)

Suppose a disease is circulating in a population. Individuals recover from this disease unharmed but are susceptible to reinfection.

Let I denote the fraction of infected individuals in a population.

Then, the rate at which the fraction of infected individuals is changing is given by

$$\frac{dI}{dt} = \alpha I(1 - I) - \mu I$$

where α and μ are positive constants.

A Model for a Disease (starts on p. 614 in your textbook)

Suppose we start with a few infected individuals, i.e., a small value of I, will this disease ever die out?