Separable Differential Equations

Section 8.4

Separable Differential Equations

A **separable differential equation** is a differential equation in which we can separate the unknown function (also called the state variable) from the independent variable.

The general form of a separable equation is

$$\frac{dy}{dx} = g(x) \cdot h(y)$$

where *h(y)* is a function of *y* only, and *g(x)* is a function of *x* only.

Separable Differential Equations

It is possible to find an algebraic solution for these types of DEs.

To solve this equation, first **separate the equation** so that all *y*'s are on one side of the equation and all *x*'s are on the other side:

$$f(y)dy = g(x)dx \quad \text{where} \quad f(y) = \frac{1}{h(y)}$$

Separable Differential Equations

Next, **integrate both sides of the equation**:

$$\int f(y)dy = \int g(x)dx$$

This equation defines *y* implicitly as a function of *x*.

Finally, **solve this equation for *y* in terms of *x***, if possible.

Separation of Variables

Example:

Solve the pure-time differential equation

$$\frac{dP}{dt} = \frac{5}{1+2t}, \quad P(0) = 0$$

Separation of Variables

Example:
Solve the following autonomous differential equations.

(a) $\dfrac{dP}{dt} = 1 + P^2, \quad P(0) = 1$

(b) <u>Exponential Growth Model</u>

$$\frac{dP}{dt} = kP, \quad P(0) = P_0$$

Separation of Variables

Example:

Solve the following "mixed" differential equations.

(a) $\frac{dy}{dt} = -8ty^2, \quad y(0) = 3$

(b) $x + 3y^2\sqrt{x^2+1}\,\frac{dy}{dx} = 0, \quad y(0) = 1$

Solution of Logistic Model

Read example 8.4.5 in your textbook (starting on p. 623) which solves the logistic differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$$

Solution of Logistic Model

Result:

The solution of the logistic equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$$

with initial condition $P(0)=P_0$ is

$$P(t) = \frac{L}{1 + \frac{L-P_0}{P_0} e^{-kt}}$$

Solution of Logistic Model

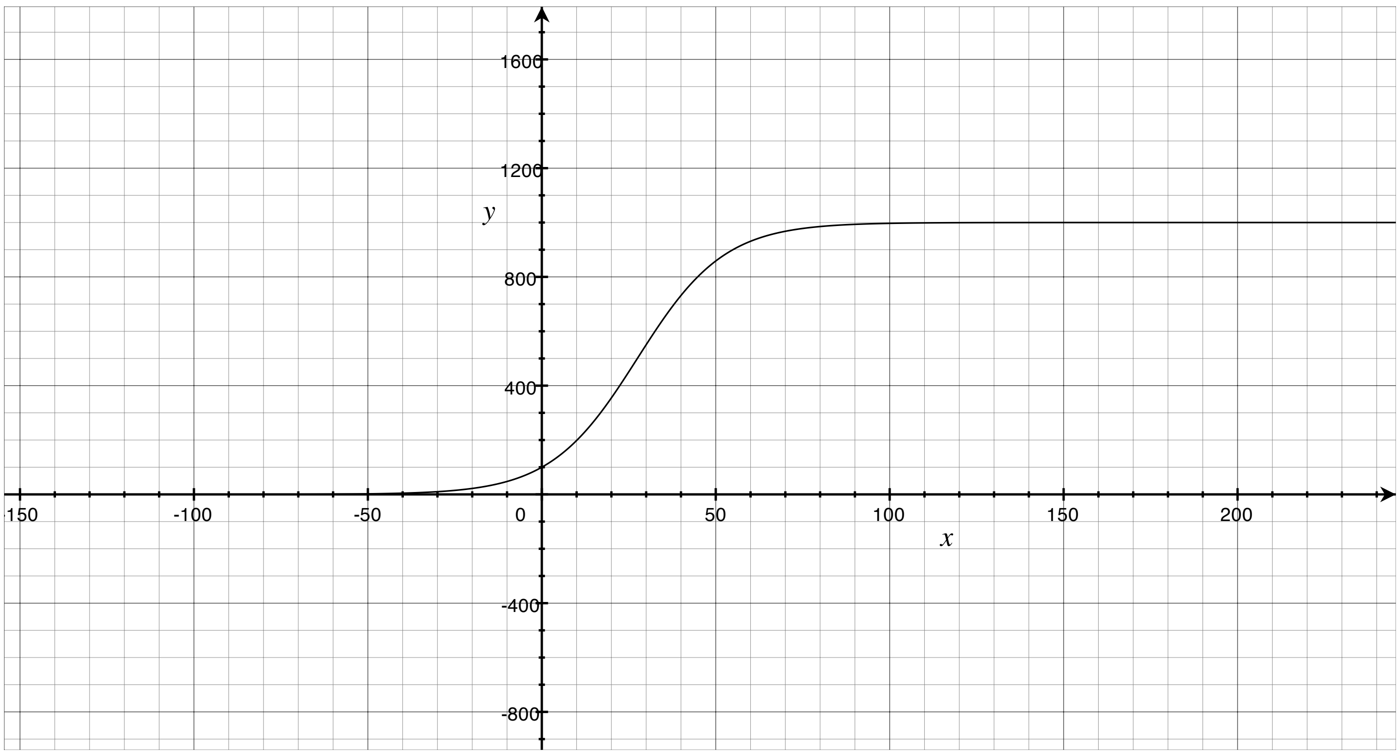
Recall Previous Example:
A population grows according to the logistic model

$$\frac{dP}{dt} = 0.08P\left(1 - \frac{P}{1000}\right)$$

with initial population $P(0) = 100.$

Solution of Logistic Model

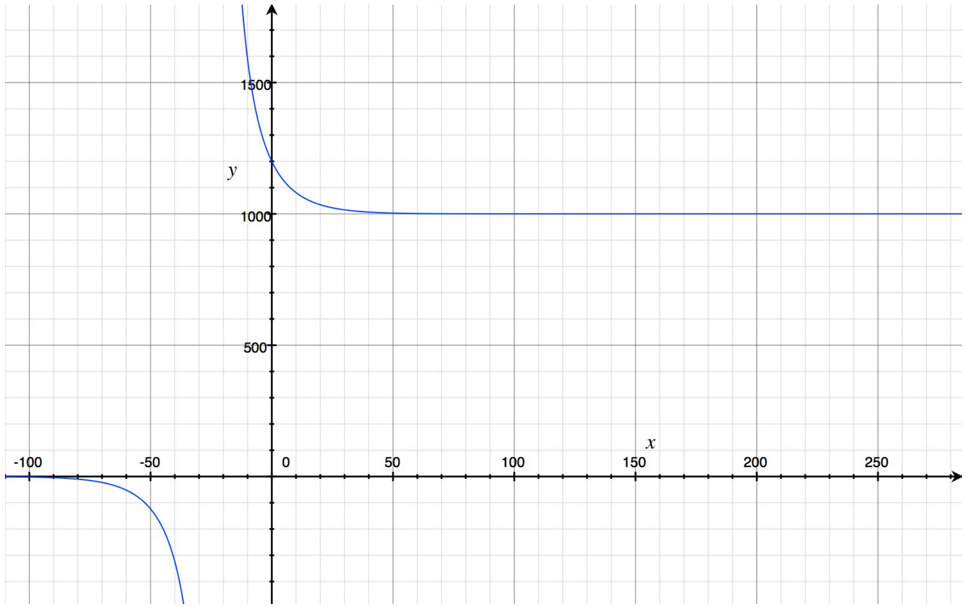
The solution to this IVP is $P(t) = \dfrac{1000}{1 + \frac{1000-100}{100} e^{-0.08t}}$



Solution of Logistic Model

What do solutions look like when $P(0) = 1200$?

$$P(t) = \frac{1000}{1 + \frac{1000-1200}{1200} e^{-0.08t}} = \frac{1000}{1 - \frac{1}{6} e^{-0.08t}}$$



Solution of Selection Model

Example p. 628, #24.
The rate of spread of a virus is proportional to the product of the fraction, *I(t)*, of the population who are infected and the fraction of those who are not yet infected, where *t* is the time in days.

(a) Write an autonomous DE for *I(t)*.
(b) Suppose that initially 15 people of the total population of 500 were infected and that 3 days later, 125 people were infected. Solve the equation.