Review of Differential Equations and Integration Techniques

Sections 7.1, 7.2, and 7.5

Differential Equations

A **differential equation (DE)** is an equation that involves an unknown function and one or more of its derivatives.

Examples:

$y' = 2 + y$ $\qquad$ $y' + 2xy = x^2$ $\qquad$ $y' = x^2 + e^x$

Differential Equations

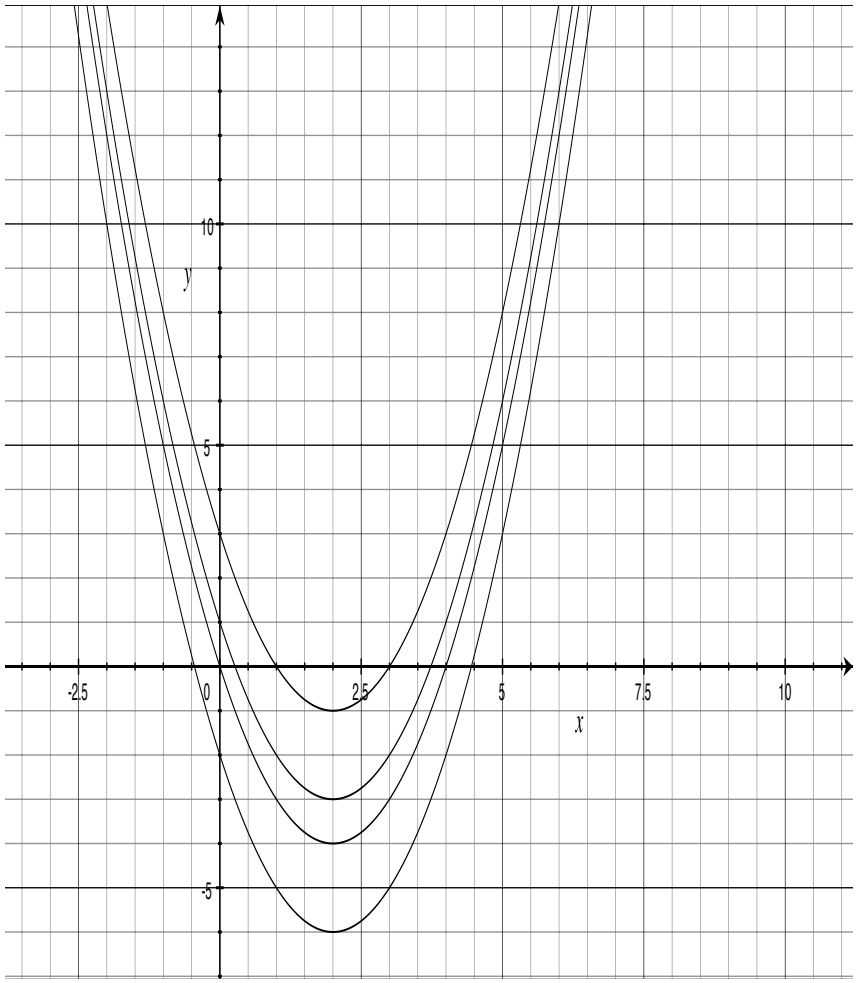
A **solution** of a differential equation is a *function* that, along with its derivatives, satisfies the DE.

Example:
Show that $z(t) = 1 + \sqrt{1+2t}$ is the solution of the differential equation $\frac{dz}{dt} = \frac{1}{z-1}$ with initial condition $z(0) = 2$.

Differential Equations

In general, a differential equation has a whole *family* of solutions.

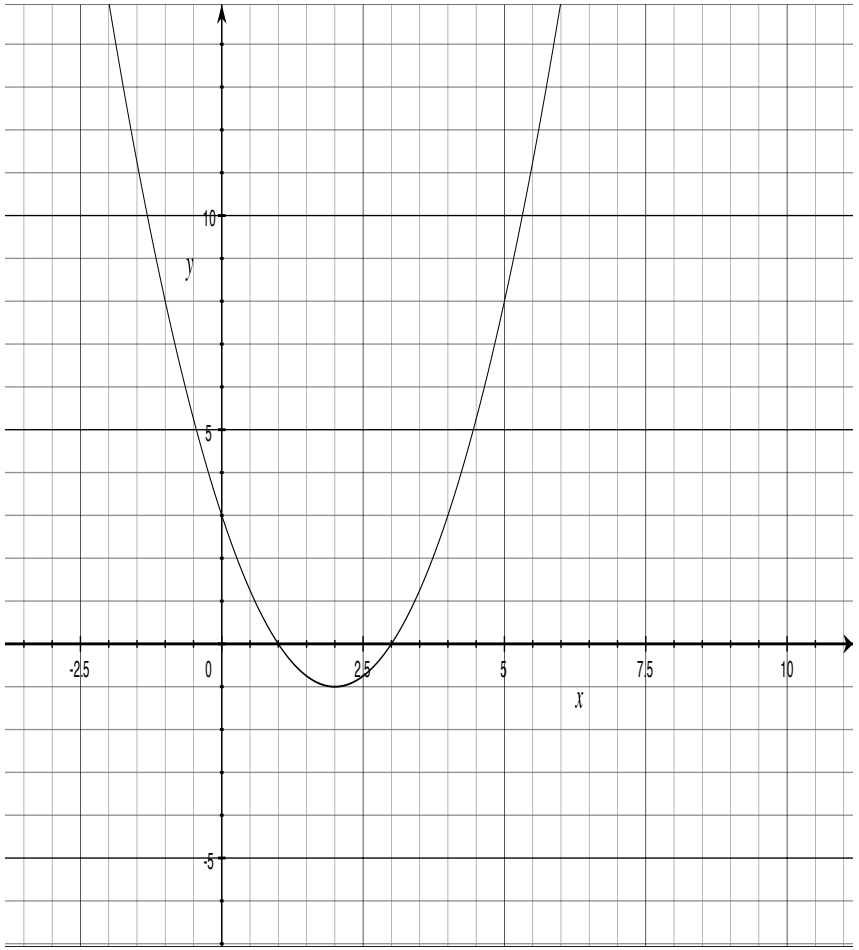
Example:
Find the general solution of the DE $y' = 2x - 4$.



Differential Equations

An **initial value problem (IVP)** provides an initial condition so you can find a particular solution.

Example:
Find the unique solution of the IVP $y' = 2x - 4, \quad y(0) = 3.$



Modeling: Verbal Descriptions ➡ IVPs

Example:
Write a differential equation and an initial condition to describe the following event:

The relative rate of change of the population of wild foxes in an ecosystem is 0.75 baby foxes per fox per month. Initially, the population is 74 thousand.

Solutions for General DEs

- **Algebraic Solutions**
  - an explicit formula or algorithm for the solution (often, impossible to find)
- **Geometric Solutions**
  - a sketch of the solution obtained from analyzing the DE
- **Numeric Solutions**
  - an approximation of the solution using technology and and some estimation method, such as Euler's method

Algebraic Solutions

Example 1:

Find the general solution of the pure-time DE

$$\frac{dy}{dx} = 5e^{10x} + \frac{1}{1+25x^2}$$

Example 2:

Find the general solution of the pure-time DE

$$y' = \ln x$$

Algebraic Solutions

Example 3:

Find the solution of the autonomous DE $\frac{dF}{dt} = 0.75F$ with initial condition $F(0) = 74\,000.$

More Integration Practice

Example:

(a) $\int \frac{x}{1+x^2}\,dx$

(b) $\int \frac{x^2}{1+x^2}\,dx$

(c) $\int xe^{0.2x}dx$

(d) $\int xe^{-x^2}dx$

(e) $\int \frac{1}{x\ln x}\,dx$

(f) $\int x\ln x\;dx$

** To check your answers, try using an online integral evaluator. For example:* https://www.integral-calculator.com/

Geometric Solutions

Example:

Sketch the graph of the solution to the DE

$$y' = \arctan x$$

given an initial condition of $y(0) = 1$.

Euler's Method

Algorithm:

$$t_{n+1} = t_n + h$$

$$y_{n+1} = y_n + F(t_n, y_n)h$$

Algorithm In Words:

next time step = previous time step + step size

next approximation = previous approximation + rate of change of the function x step size

Euler's Method

Example:
Consider the IVP

$$\frac{dP}{dt} = e^{-t^2}, \quad P(0) = 5$$

Approximate P(1) using Euler's method and a step size of h=0.5.

<u>Note:</u> We are not able to find an exact solution for this IVP.

Euler's Method

Example:

Calculations:

Table of Approximate Values for the Solution P(t) of the IVP

$t_n = t_{n-1} + h$	P_n = approx. value of solution at t_n
$t_0 = 0$	$P_0 = 5$