Introduction to Stochastic Models

Sections 1+2

Deterministic Models

So far, we have been studying idealized, *deterministic* models where the outcome is always *certain*.

Stochastic Models

Now, we study models which account for *random* or *chance* factors. These factors are unpredictable, unknown events which have an effect on the process we are studying.

Stochastic Model

Definition:
A *stochastic model* is a mathematical model that describes processes (such as biological processes) that are driven by chance (random) events.

Stochastic Population Models

Example: Lion Population with Immigration

Suppose that a population p_t of lions at time t is described by the stochastic dynamical system

$$p_{t+1} = p_t + I_t$$

where the term I_t represents *possible* immigration of lions and where time t=0,1,2,… is measured in years.

Stochastic Population Models

Example: Lion Population with Immigration

Suppose that initially there are 160 lions and that the immigration term is defined as

$$I_t = \begin{cases} 12 & \text{with 50\% chance} \\ 0 & \text{with 50\% chance} \end{cases}$$

Analyze *possible* dynamics of this population over the next three years.

Stochastic Population Models

Example: Lion Population with Immigration

Trials/simulations:

t	I_t	p_t
0		160
1		
2		
3		

t	I_t	p_t
0		160
1		
2		
3		

Stochastic Population Models

```
# lion population with immigration (It = 12 with a 50% chance; It=0 with a 50% chance)

numyears = 3

pt = 160 # initial population

for i in range(0,numyears):
    a = ra.randrange(1,3) # generates a random integer between 1 and 2, so, either 1 or 2
    if a == 1.:
        It = 0
    if a == 2.:
        It = 12
    pt = pt + It
    print("year:", i+1,"population:",pt)

print("The lion population size after", numyears, "years is", pt)
```

```
year: 1 population: 172
year: 2 population: 184
year: 3 population: 196
The lion population size after 3 years is 196
```

```
# lion population with immigration; multiple simulations

numyears = 3
numsims = 10
pt_end = [] # population size after numyears

for simulation in range(0, numsims):
    pt = 160
    for i in range(0,numyears):
        a = ra.randrange(1,3)
        if a == 1.:
            It = 0
        if a == 2.:
            It = 12
        pt = pt + It
    pt_end.append(pt)


print('Final population sizes:', pt_end)
print('Largest population size is', max(pt_end))
print('Smallest population size is', min(pt_end))
```

```
Final population sizes: [184, 172, 172, 184, 160, 172, 196, 160, 184, 184]
Largest population size is 196
Smallest population size is 160
```

Random Experiment and Sample Space

Definition:
A *random experiment* is an experiment that is repeatable but has an uncertain outcome.

The set of all possible outcomes of a random experiment is called the *sample space* of that experiment.

Stochastic Population Models

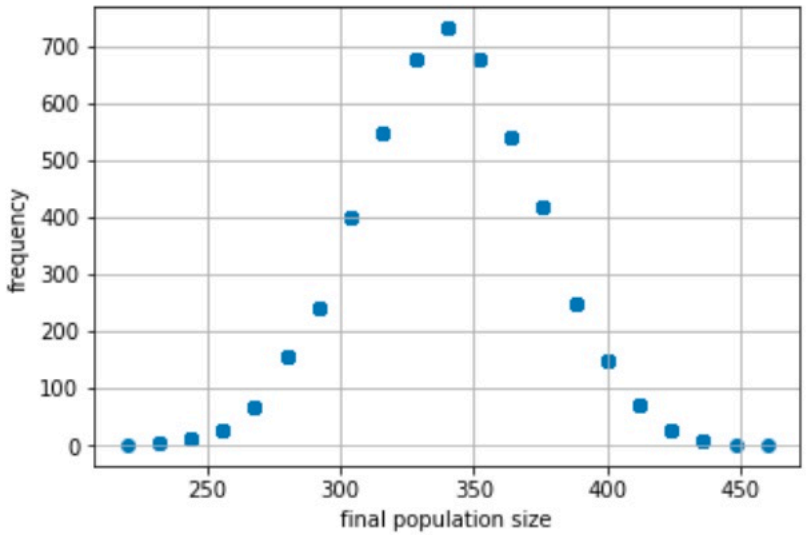
Example: Lion Population with Immigration

Sample Space:
At the end of 3 years, what are the possible sizes of the lion population? Do all occur with equal likelihood?

Expectations:
What is your prediction for the population of lions in the next 3 years?

Stochastic Population Models

```
# lion population with immigration; counting frequency

numyears = 30
numsims = 5000
pt_end = []

for simulation in range(0, numsims):
    pt = 160
    for i in range(0,numyears):
        a = ra.randrange(1,3)
        if a == 1.:
            It = 0
        if a == 2.:
            It = 12
        pt = pt + It
    pt_end.append(pt)

# recording frequency of events

size = []
frequency = []

for j in range(0, len(pt_end)):
    freq = 0
    for k in range(0, len(pt_end)):
        if pt_end[j] == pt_end[k] :
            freq = freq + 1
    size.append(pt_end[j])
    frequency.append(freq)

# plot

plt.scatter(size, frequency)
plt.xlabel("final population size")
plt.ylabel("frequency")
plt.grid()
plt.show()
```



Sample Spaces and Events

Definition: Simple Event and Event
A single outcome of a random experiment is called a *simple event*. An *event* is a collection (or set) of simple events.

Statistic

Definition:
A *statistic* is a set of numerical values that can summarize the outcomes of a random experiment.

Stochastic Models

Example #6.

A population of leopards p_t is modelled by

$$p_{t+1} = p_t + I_t \quad \text{where} \quad I_t = \begin{cases} 10 & \text{with a } 90\% \text{ chance} \\ -100 & \text{with a } 10\% \text{ chance} \end{cases}$$

What is more likely to happen to the number of leopards over time – a net increase or decrease? Or will the population remain at about the same size? Explain.