Discrete Random Variables

Section 6

Random Variables

Definition:

Assume that S is a sample space of a random experiment. A *random variable X* is a function from S into the set of real numbers.

Random Variables

Examples:

1. Let *X* be the random variable that records the sum of two dice.

2. Let *Y* be the random variable that counts the number of tosses of a coin required to obtain "tails."

3. Let *T* to be the amount of time it took for a student to write a 1 hour test.

Discrete and Continuous Random Variables

Definition:

If the range of a random variable X is a finite or a countable set, then X is called a *discrete random variable*. Otherwise, if its range is an uncountable set, a random variable X is called a *continuous random variable.*

Probability Mass Function

Definition:

Let X be a discrete random variable. The function

$$p(x) = P(X = x)$$

is called the *probability mass function.*

The function p(x) is said to describe a *probability distribution* of X.

Disappearance and Recurrence of a Virus

Example # 12.

Consider a virus that appears and disappears over time within a population with seemingly no pattern.

Suppose that if the virus is present within the population, then it will be present in the following month with probability 0.75. If the virus is absent from the population, then it will be absent the following month with probability 0.8.

Assume that at this moment the virus is present in the population. Find the probability mass function for the random variable X="number of virus-free months in the 2-month period from now."

Probability Mass Function

Properties:

Let X be a discrete random variable with probability mass function p(x)=P(X=x). The function p(x) satisfies the following:

(a) $0 \le p(x) \le 1$ for all x in the range of X.

(b) $\sum_{x} p(x) = 1$ for all x in the range of X.

Cumulative Distribution Function

Definition:

Let X be a random variable. The *cumulative distribution function* F(x) of X is defined as

$$F(x) = P(X \leq x)$$

Disappearance and Recurrence of a Virus

Example # 12 continued....
Find the cumulative distribution function for the random variable X="number of virus-free months in the 2-month period from now." Sketch its graph.

x	p(x)
0	0.5625
1	0.2375
2	0.2

Cumulative Distribution Function

Properties:

Let X be a discrete random variable with probability mass function p(x)=P(X=x). The cumulative distribution function F(x) satisfies the following:

(a) $0 \leq F(x) \leq 1$ for all $x \in R$.

(b) F(x) is non-decreasing (i.e., constant or increasing) for all $x \in R$.

(c) F(x) has jump discontinuities of size p(x) at those x where p(x)>0 (i.e., at all x in the range of X where P(X=x)>0).

(d) F(x) is right-continuous at all points x where p(x)>0.

(e) $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$.