Local Extreme Values

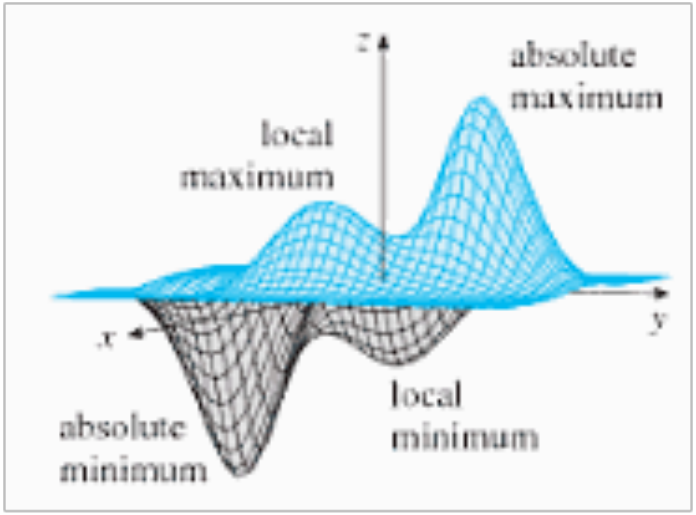
*** Note: We will not study absolute extreme values in this course.*

Section 10

Maximum and Minimum Values

Definition:

A function $f(x,y)$ has a **local maximum** at (a,b) if $f(a,b) \geq f(x,y)$ when (*x*,*y*) is near (*a*,*b*).



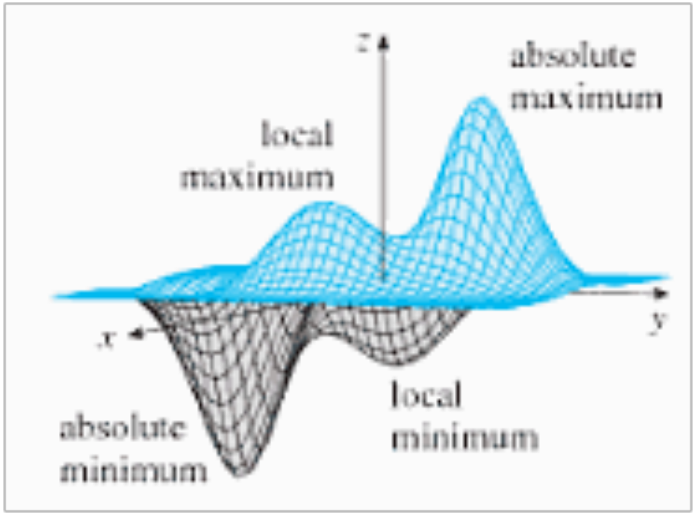
The number *f*(*a*,*b*) is called a **local maximum value.**

Maximum and Minimum Values

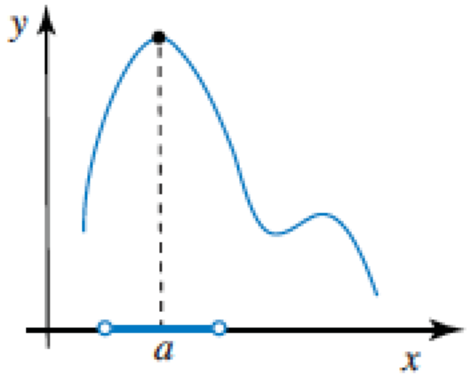
<u>Definition</u>:

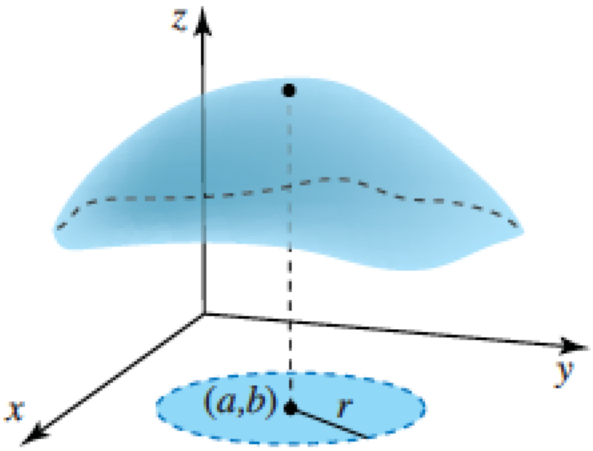
A function $f(x,y)$ has a **local minimum** at (*a*,*b*) if $f(a,b) \leq f(x,y)$ when (*x*,*y*) is near (*a*,*b*).

The number *f*(*a*,*b*) is called a **local minimum value.**



The Meaning of “Near”





Fermat's Theorem

If a function $f(x,y)$ has a local minimum or a local maximum at (a,b), then (a,b) is a *critical point* of f.

Critical Points

Definition:

A point (a,b) in the domain of a function $f(x,y)$ is called a **critical point** if either

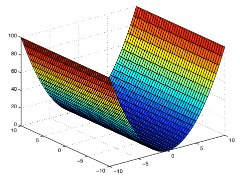
(a) $f_x(a,b)=0$ and $f_y(a,b)=0$, or

(b) at least one of $f_x(a,b)$ or $f_y(a,b)$ does not exist.

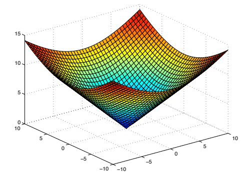
Critical Points

Some interesting cases:

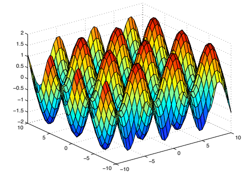
(a) $f(x,y) = x^2$



(b) $f(x,y) = \sqrt{x^2 + y^2}$



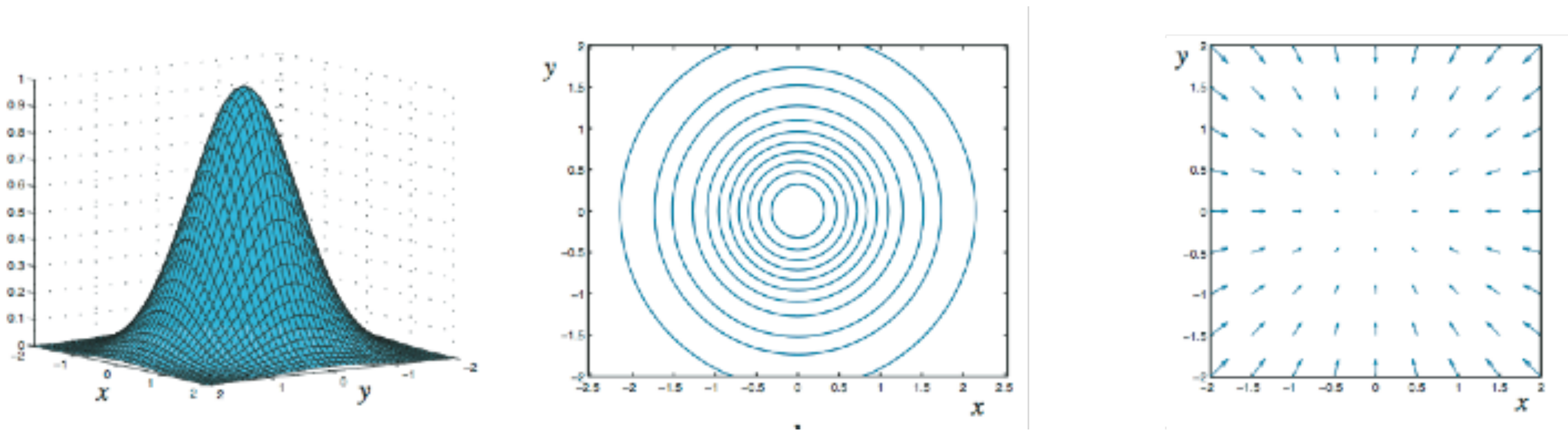
(c) $f(x,y) = \sin x - \sin y$



Critical Points

Some interesting cases:

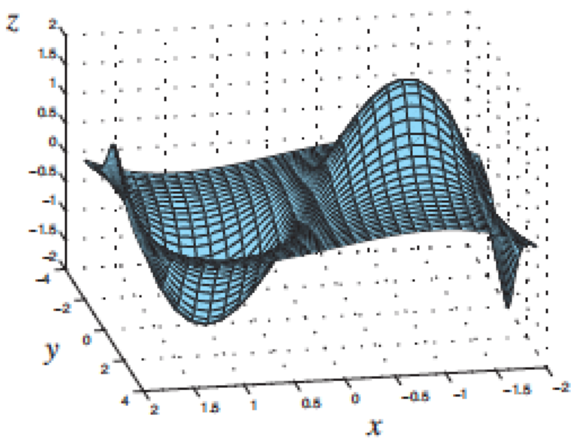
(d) $f(x,y) = e^{-x^2 - y^2}$

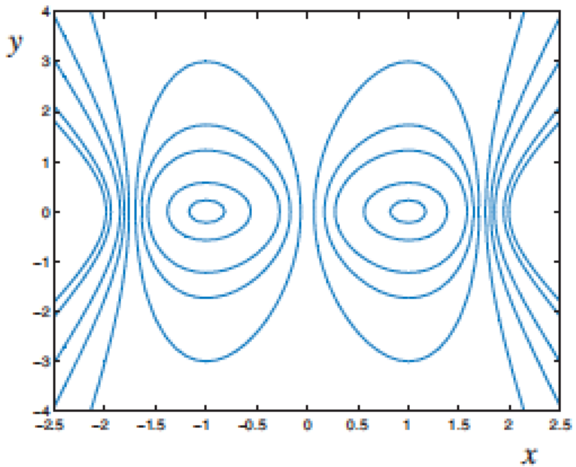


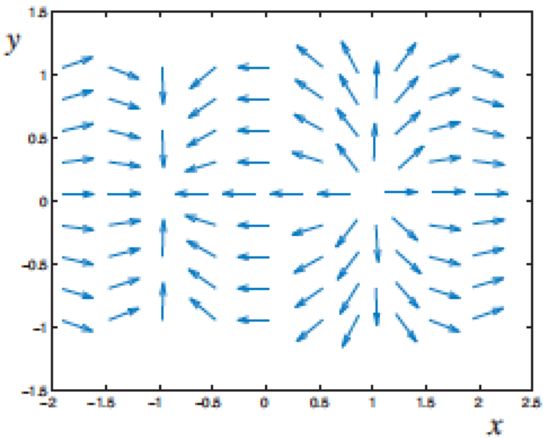
Critical Points

Some interesting cases:

(e) $f(x,y)=\dfrac{x^3-3x}{1+y^2}$



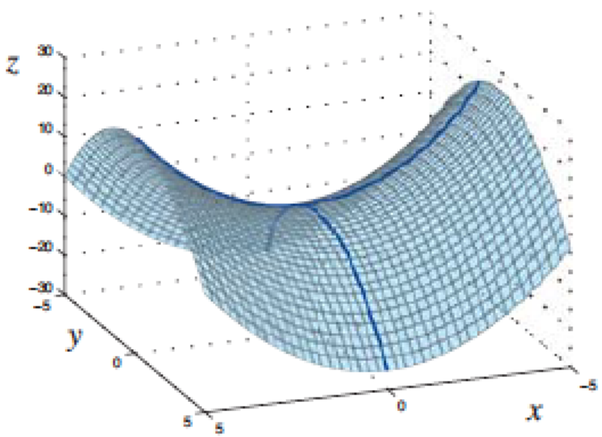


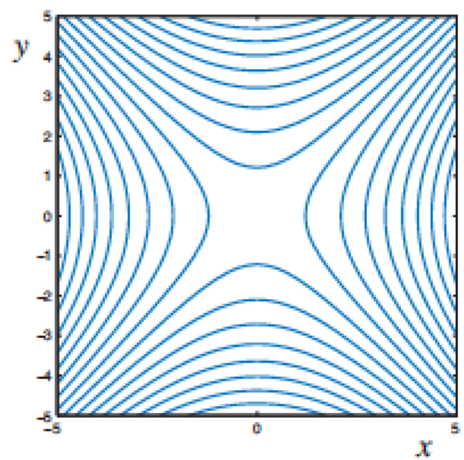


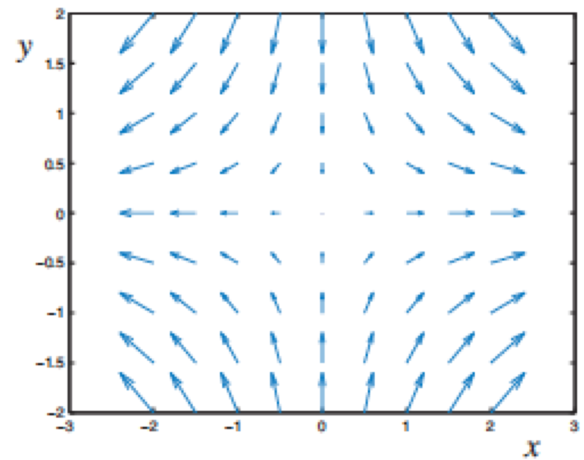
Critical Points

Some interesting cases:

(f) $f(x,y) = x^2 - y^2$

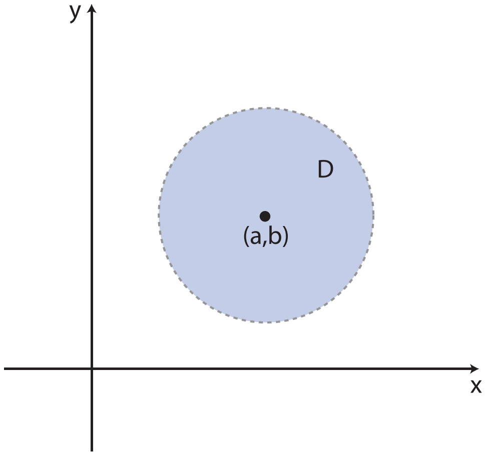






Second Derivatives Test

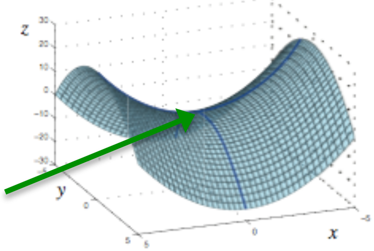
Suppose the second partial derivatives of f are continuous on a disk with centre (a,b) and suppose that $f_x(a,b)=0$ and $f_y(a,b)=0$.



Second Derivatives Test

Let $D = D(a,b) = f_{xx}(a,b)\,f_{yy}(a,b) - \left[f_{xy}(a,b) \right]^2.$

(a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local min.
(b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local max.
(c) If $D < 0$, then $f(a,b)$ is not a local max or min and we say (a,b) is a saddle point of *f*.



(0,0) is a saddle point

<u>*Note*</u>:
If *D*=0, the test gives no information: *f* could have a local max or min at (*a*,*b*) or (*a*,*b*) could be a saddle point of *f*.

Second Derivatives Test

Example:

Find the local minimum and maximum values and saddle points (if any) of $f(x, y) = x^2 + y^2 + 2xy^2$.

When The Second Derivatives Test Does Not Apply

Example:

For the function to the right, it can be shown that (0,0) is the only critical point of *f* and that D(0,0)=0 and so the second derivatives test is inconclusive.

What is (0,0)?

$$f(x,y) = x^3 - 3xy^2$$