## Graphs, Level Curves + Contour Maps

## Section 2

## Contour Maps and Level Curves

In general, sketching the graphs of functions of two variables (surfaces) is difficult so instead we sketch 2-dimensional representations of these surfaces in $\mathrm{R}^{2}$ called contour maps.


## Contour Maps and Level Curves

## Level Curves:

The level curves of a function $f$ of two variables are the curves with equations

$$
f(x, y)=k
$$

where $k$ is a constant in the RANGE of the function.

A level curve $f(x, y)=k$ is a curve in the domain of $f$ along which the graph of $f$ has height $k$.

## Contour Maps and Level Curves

Contour Maps:
A contour map is a collection of level curves.
To visualize the graph of $f$ from the contour map, imagine raising each level curve to the indicated height.

The surface is steep where the level curves are close together and it is flatter where they are farther apart.

## Contour Maps and Level Curves

Examples:
Draw a contour map for the following functions showing several level curves.
(a) $f(x, y)=6-3 x-2 y$
(b) $f(x, y)=\sqrt{4-x^{2}-y^{2}}$

## Contour Maps and Level Curves

## Questions:

1. Why is it not possible for the level curves of two different values to intersect each other?
2. If the level curves of a function are parallel lines, can we conclude that the function is linear?
3. Compare the contour maps of a paraboloid, a cone, and the top (or bottom) half of a sphere. How are they the same? How are they different?
