Graphs, Level Curves + Contour Maps

Section 2

In general, sketching the graphs of functions of two variables (surfaces) is difficult so instead we sketch 2-dimensional representations of these surfaces in R² called <u>contour maps</u>.





Level Curves:

The level curves of a function *f* of two variables are the curves with equations

$$f(x,y) = k$$

where k is a constant in the **<u>RANGE</u>** of the function.

A level curve f(x,y) = k is a curve in the domain of falong which the graph of f has height k.

Contour Maps:

A contour map is a collection of level curves.

To visualize the graph of *f* from the contour map, imagine raising each level curve to the indicated height.

The surface is steep where the level curves are close together and it is flatter where they are farther apart.

Examples:

Draw a contour map for the following functions showing several level curves.

(a)
$$f(x,y) = 6 - 3x - 2y$$

(b)
$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

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Questions:

1. Why is it not possible for the level curves of two different values to intersect each other?

2. If the level curves of a function are parallel lines, can we conclude that the function is linear?

3. Compare the contour maps of a paraboloid, a cone, and the top (or bottom) half of a sphere. How are they the same? How are they different?