Partial Derivatives

Section 4

Derivative of y=f(x)

Recall:

Definition of the Derivative in Single Variable Calculus:

$$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

instantaneous rate of change of *f* with respect to *x*

Partial Derivatives of z=f(x,y)

The **partial derivative** of a function of several variables is a way to measure the rate of change of the function as **<u>one</u>** of its variables changes.

Partial Derivatives of z=f(x,y)

Example: Dynamics of Prey Consumption

Consider the *type-2 functional response* model

$$c(N,T_h) = \frac{aN}{1 + aT_h N}$$

where $c(N,T_h)$ is the number of prey captured (in some fixed time interval), T_h is the handling time, and N is the density of prey.

How does the number of rabbits captured depend on the handling time and the density?

Partial Derivatives of z=f(x,y)

The **partial derivative of *f* with respect to *x*** is the real-valued function $\partial f / \partial x$ defined by

$$\frac{\partial f}{\partial x}(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

provided that the limit exists.

This function tells us the rate of change of *f* in the *x*-direction at all points (*x*,*y*) for which the limit exists.

Partial Derivatives of z=f(x,y)

The **partial derivative of *f* with respect to *y*** is the real-valued function $\partial f/\partial y$ defined by

$$\frac{\partial f}{\partial y}(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

provided that the limit exists.

This function tells us the rate of change of *f* in the *y*-direction at all points (*x*,*y*) for which the limit exists.

Partial Derivatives of z=f(x,y)

Example:

Using the definitions, compute $\partial f / \partial x$ and $\partial f / \partial y$ for $f(x) = x^2 - y.$

Partial Derivatives of z=f(x,y)

Rule for finding partial derivatives of $z=f(x,y)$:

1. To find f_x, treat y as a constant and differentiate $f(x,y)$ with respect to x.

2. To find f_y, treat x as a constant and differentiate $f(x,y)$ with respect to y.

Partial Derivatives of z=f(x,y)

Example:

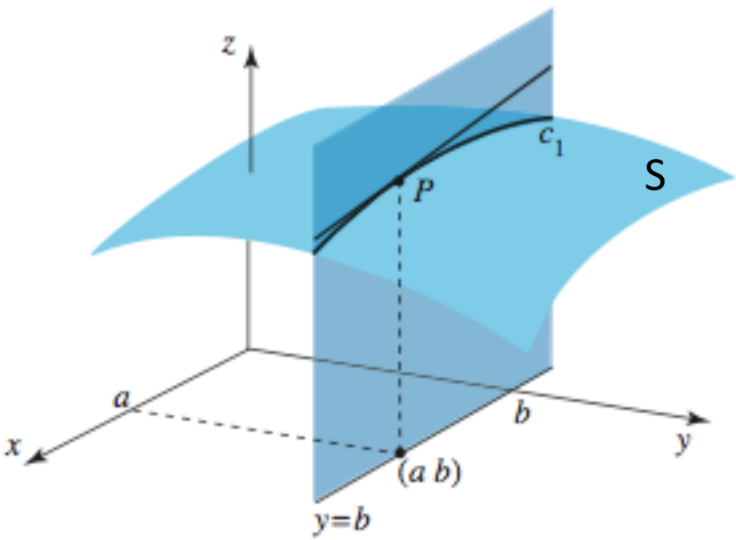
Find the first partial derivatives of the following functions.

(a) $f(x,y) = x^4 y^3 + 8x^2 y$

(b) $z = x^y$

(c) $z = \arctan\left(\frac{y}{x}\right)$

Geometric Interpretation of the Partial Derivatives of z=f(x,y)

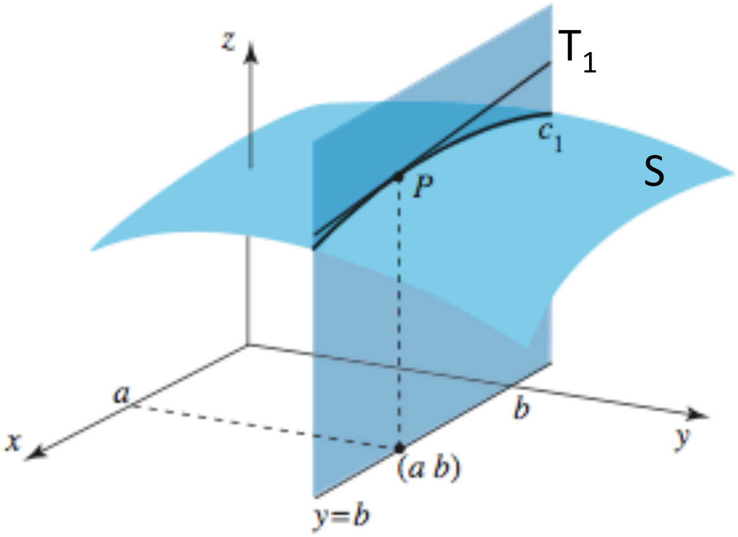


Let $z=f(x,y)$ be a function of two variables whose graph is the surface S.

Fix $y=b$ (constant) and let x vary.

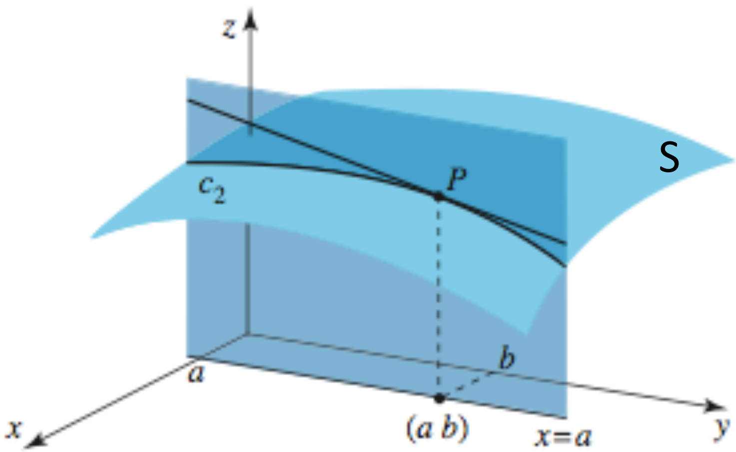
The curve c_1 on the surface S is defined by $z=f(x,b)$. (Note: this is now only a function of the variable x)

Geometric Interpretation of the Partial Derivatives of z=f(x,y)



The partial derivative of f with respect to x at (a,b) is the slope of the tangent T_1 to the curve c_1 at the point P.

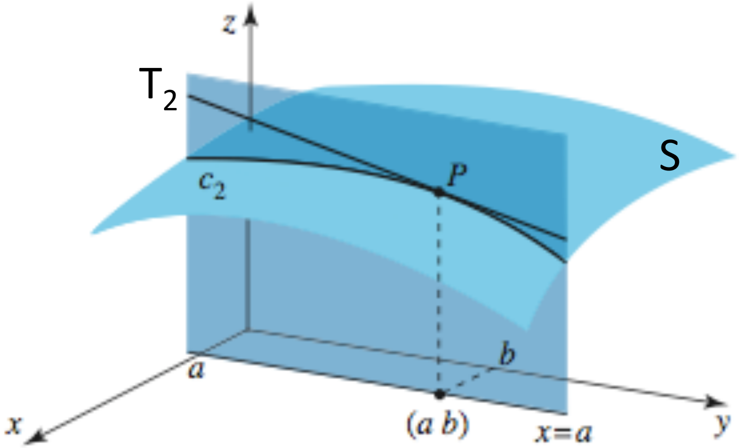
Geometric Interpretation of the Partial Derivatives of z=f(x,y)



Now, fix $x=a$ (constant) and let y vary.

The curve c_2 on the surface S is defined by $z=f(a,y)$. (Note: this is now only a function of the variable y)

Geometric Interpretation of the Partial Derivatives of z=f(x,y)

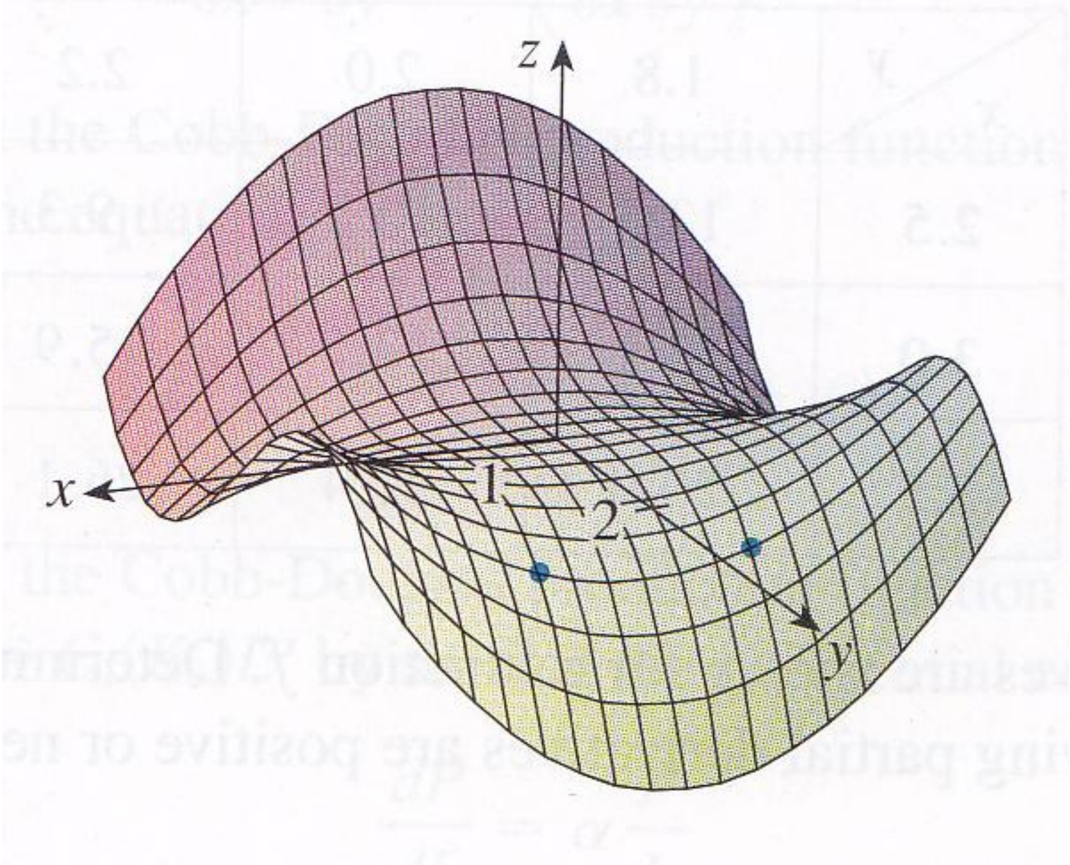


The partial derivative of *f* with respect to *y* at (*a*,*b*) is the slope of the tangent T_2 to the curve c_2 at the point P.

Geometric Interpretation of the Partial Derivatives of z=f(x,y)

Example:

Determine the signs of $f_x(1,2)$ and $f_y(1,2)$ on the graph below.



Partial Derivatives of z=f(x,y)

Example:

If $f(x,y)=\sqrt{4-x^2-y^2}$, find $f_x(1,0)$ and $f_y(1,0)$ and interpret geometrically.

Partial Derivatives of z=f(x,y)

Example: *Wind Chill*
The table below contains values of the wind chill index, or simply *wind chill*, $W(T,v)$ based on measurements of air temperature T (in degrees Celsius) and wind speed v (in kilometres per hour).

	T=-25	T=-20	T=-15	T=-10
v=40	-40.8	-34.1	-27.4	-20.8
v=30	-39.1	-32.6	-26.0	-19.5
v=20	-36.8	-30.5	-24.2	-17.9

Estimate $W_T(-20, 30)$ and interpret the result.