MATHEMATICS 1LT3 TEST 1

Evening Class — Dr. E. Clements
Duration of Test: 60 minutes
McMaster University — 7 July 2022

FIRST NAME (please print): ____________________

FAMILY NAME (please print): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 34. Marks are indicated next to the problem number in square brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Question 1.

Problem	Points	Mark
1	8	
2	8	
3	7	
4	5	
5	3	
6	3	
TOTAL	34	

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1. **Multiple choice questions: circle ONE answer. No justification is needed**.

(a) [2] Suppose the temperature of an object changes according to $dT/dt = 0.4(20 - T)$, where $T(0) = 15$. Which of the following statements is/are true?

(I) dT/dt is a decreasing function of T.

(II) In the phase-line diagram, arrows point right when $T < 20$.

(III) $T(t) = 15e^{-0.4t}$ is the solution of the initial value problem.

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(b) [2] Consider the differential equation, $\dfrac{dy}{dx} = ye^{-\beta y} - \alpha y$, where α and β are parameters. Which of the following statements is/are true?

(I) $\dfrac{dy}{dx} = ye^{-\beta y} - \alpha y$ is an autonomous differential equation

(II) $y^* = -\dfrac{\ln \alpha}{\beta}$ is an equilibrium

(III) $y^* = 0$ is a stable equilibrium when $\alpha > 1$

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(c) [2] The following pair of equations represent the population growth of two different species where one is the predator and the other is the prey.

$$\frac{dA}{dt} = 0.1A - 0.005AB, \quad \frac{dB}{dt} = -0.05B + 0.0001AB$$

Which of the following statements is/are true?

(I) The variable A represents the prey population.

(II) The per capita growth rate of of species B is $0.0001A$

(III) $(A, B) = (20, 50)$ is an equilibrium of this system.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(d) [2] Determine which of the following statements is/are true.

(I) The top half of the unit sphere is described by the equation $z = \sqrt{1 - x^2 - y^2}$.

(II) The level curves for $z = \arctan(y/x)$ are linear.

(III) The range of $z = e^{-x^2-y^2}$ is $(0, 1]$.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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2. A population of caribou is modelled by $\dfrac{dP}{dt} = 0.7P\left(1 - \dfrac{P}{480}\right)$.

(a) [2] Find the equilibria of this equation. What does the larger equilibrium represent?

(b) [2] Graph $\dfrac{dP}{dt}$ as a function of P.

(c) [2] Draw a phase-line diagram for $\dfrac{dP}{dt} = 0.7P\left(1 - \dfrac{P}{480}\right)$.

(d) [2] Suppose that initially there are 80 caribou. Sketch the solution curve $P(t)$.

3. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{2xy^2}{1+x^2}$ and initial condition $y(0) = 1$.

(a) [2] Using Euler's Method with a step size of 0.5, estimate the value of $y(1)$.

(b) [3] Using the separation of variables technique, find a formula for $y(x)$ and use it to find the true value of $y(1)$. Round your answer to two decimal places.

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4. Consider the modified competition equations

$$\frac{da}{dt} = 0.1\left(1 - \frac{b}{200}\right)a, \quad \frac{db}{dt} = 0.2\left(1 - \frac{a}{300}\right)b$$

(a) [2] Find and graph the nullclines in the phase plane. Please use different colours for each nullcline.

(b) [1] Identify the equilibria.

(c) [2] Add direction arrows to your phase-plane diagram in part (a). Include direction arrows in each region as well as on the nullclines.

(d) [2] Sketch phase-plane trajectories starting from (i) $(a, b) = (150, 100)$ and (ii) $(a, b) = (400, 300)$.

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5. [3] Find and sketch the domain of $f(x, y) = \sqrt{x + y} + \ln(x - y)$.

6. [3] Show that $\lim_{(x,y)\to(0,0)} \dfrac{5y\sin(7x)}{4x^2 + 6y^2}$ does not exist.

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ROUGH WORK

THE END