MATHEMATICS 1LT3   TEST 2

Evening Class   Dr. E. Clements
Duration of Test: 60 minutes
McMaster University   26 July 2022

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR EN-
SURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 34. Marks are indicated next to the problem number in square
brackets. You may use the McMaster standard calculator, Casio fx991 MS+, on this test.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT
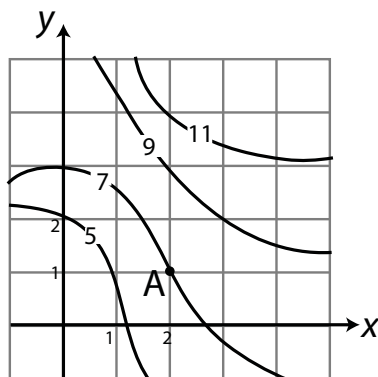BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Question 1.

Problem	Points	Mark
1	8	
2	7	
3	7	
4	5	
5	4	
6	3	
TOTAL	34	

1. **Multiple Choice**. Clearly **circle** the one correct answer.

(a) [2] Determine which of the following is/are true for the function $f(x, y)$ whose contour map is given below.

(I) $f_y(2, 1) > 0$ (II) $f_{yy}(2, 1) > 0$ (III) $D_{\mathbf{v}}f(2, 1) \approx 2$ when $\mathbf{v} = \mathbf{i} + \mathbf{j}$

(A) none   (B) I only   (C) II only   (D) III only

(E) I and II   (F) I and III   (G) II and III   (H) all three

(b) [2] Use the linearization of $f(x, y) = \sqrt{3x + 2y}$ at $(1, 3)$ to estimate the value of $f(1.1, 3)$.

(A) 3.05   (B) 2.99   (C) 3.01   (D) 2.81

(E) 3.15   (F) 3.12   (G) 2.86   (H) none of these

(c) [2] Consider the function $f(x,y) = 6ye^{2x} + 3y^7$. Evaluate $f_{yx}(0,2)$.

(A) 36   (B) $9e$ (C) 16   (D) 144

(E) 33   (F) 12   (G) 156   (H) none of these

(d) [2] Consider the sample space $S = \{1,2,3,4\}$, where $P(1) = 0.2$, $P(2) = 0.1$, and $P(3) = 0.3$. If $A = \{1,2\}$ and $B = \{2,3\}$, determine which of the following is/are true.

(I) $P(4) = 0.4$ (II) $A^C \cap B^C = \{4\}$ (III) $(A \cap B)^C = A^C \cup B^C$

(A) none   (B) I only   (C) II only   (D) III only

(E) I and II   (F) I and III   (G) II and III   (H) all three

2. Consider the function $f(x, y) = \arctan(x/y)$.

(a) [3] Show that $f(x, y) = \arctan(x/y)$ is differentiable at $(3, 2)$. Make sure to specify an appropriate radius r for $B_r(3, 2)$. [Remember that the domain of a partial derivative must always be a subset of the domain of the function f!]

(b) [2] Compute the directional derivative of the function $f(x, y) = \arctan(x/y)$ at the point $(3, 2)$ in the direction specified by $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$.

(c) [2] What is the maximum rate of change of $f(x, y) = \arctan(x/y)$ at $(3, 2)$?

3. Consider the function $f(x, y) = \dfrac{x^3 - 12x}{9 - y^2}$.

(a) [4] Find the critical points of $f(x, y)$.

(b) [3] Using the second derivatives test, classify the critical points from part (a).

Note that $f_{xx} = \dfrac{6x}{9 - y^2}$, $f_{xy} = \dfrac{6y(x^2 - 4)}{(9 - y^2)^2}$, and $f_{yy} = \dfrac{6(x^3 - 12x)(y^2 + 3)}{(9 - y^2)^3}$.

4. Consider a population of 400 frogs. Suppose that within any given year, there is a 60% chance that the population will increase by 50 and a 40% chance that it will stay the same.

(a) [2] Write the sample space for the population of frogs after 3 years.

(b) [1] What is the probability that the population will have increased after 3 years?

(c) [2] Suppose that conditions changed and now within any given year, there is a 75% chance that the population will increase by 50 and a 25% it will decrease by 120. What is more likely to happen to the number of frogs over time? A net increase or a decrease? Explain.

5. Consider the random experiment of rolling two, fair six-sided dice.

(a) [2] Find the probability that the sum is 7.

(b) [2] Find the probability that the sum is 7, given that at least one die shows a number larger than 3.

6. Suppose that within a given season, the incidence of influenza (flu) in Canada is about 43 cases per 1000 people. A certain rapid influenza diagnostic test (RIDT) returns a positive result in 89% of people who have it and in 12% of people who do not have it.

(a) [2] If a randomly selected person tests positive for influenza using a RIDT, what is the probability that they have it? Round your answer to two decimal places.

(c) [1] What is the probability that in a group of 20 healthy people, the RIDT test returns a false positive for all 20 people? Round your answer to two decimal places.

Name:_____
Student No.: _____

ROUGH WORK

THE END