MATHEMATICS 1LT3E FINAL EXAMINATION

Evening Class
Duration of Examination: 3 hours
McMaster University
24 April 2015

E. Clements

FIRST NAME (PRINT CLEARLY): SOL'NS +

FAMILY NAME (PRINT CLEARLY): GRADING

Student No.: GUIDE.

THIS EXAMINATION PAPER HAS 18 PAGES AND 12 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF THE INVIGILATOR.

Note: Page 18 contains a partial table of values for $F(z)$.

Total number of points is 80. Marks are indicated next to the problem number. You may use the McMaster standard calculator, Casio fx991 MS+. Write your answers in the space provided. EXCEPT ON QUESTION 1, YOU MUST SHOW WORK TO OBTAIN FULL CREDIT. **Good luck!**

Problem	Points	Mark
1	20	
2	6	
3	5	
4	4	
5	3	
6	4	
7	5	
8	7	
9	7	
10	6	
11	7	
12	6	
TOTAL	80	

MC ... (0) or (2) ... no part marks!

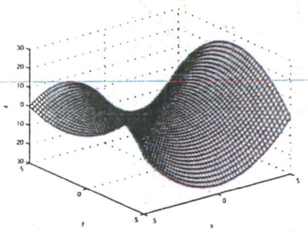
Name:________________

Student No.: ________________

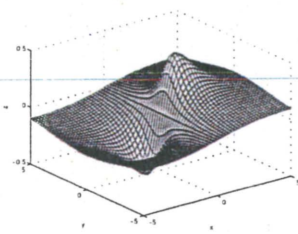
Question 1: For part (a), write the letter corresponding to the graph of the function next to the equation in the space provided. For parts (b)-(j), clearly circle the one correct answer.

1. (a) [2] Match the equation of each function with its graph below.

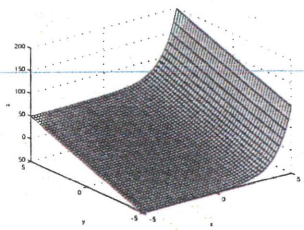
$f(x, y) = e^x + 10y$ C $\qquad g(x, y) = x^2 - y^2$ A $\qquad h(x, y) = \dfrac{x}{x^2 + y^2 + 1}$ B



(A)



(B)



(C)

MC: CAB C | HD | EH | AH | AH

(b) [2] In the basic model for the spread of a disease, $\dfrac{dI}{dt} = \alpha I(1 - I) - \mu I$ where $\alpha, \mu > 0$, which of the following statements are true?

(I) $I^* = 0$ is a stable equilibrium. ✗
(II) If $\mu > \alpha$, then the disease will eventually die out. ✓
(III) If $\mu < \alpha$ and $I(0) > 0$, then $I(t) \to 1$ as $t \to \infty$. ✗

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(c) [2] A partial table of values for a function $f(x, y)$ is given below. Which of the following are positive?

(I) $f(4,1)$ (II) $f_x(4,1)$ (III) $f_{xx}(4,1)$

+ + +

	$x = 3$	$x = 4$	$x = 5$	$x = 6$
$y = 0$	2.3	2.2	2.0	1.7
$y = 1$	2.4	2.5 $\rightarrow$	2.7	3.0
$y = 2$	2.5	2.7	2.9	3.2
$y = 3$	2.6	3.0	3.0	3.3

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(d) [2] The linearization of $f(x, y) = xe^{xy}$ at $(1,0)$ is

(A) x (B) $-x$ (C) $y - x$ (D) $x + y$

(E) $x - y$ (F) y (G) $2x + y$ (H) 0

$f_x = 1e^{xy} + x \cdot e^{xy} \cdot y \;\ldots\; f_x(1,0) = 1$

$f_y = x^2 e^{xy} \;\ldots\; f_y(1,0) = 1$

$L_{(1,0)} = 1 + 1(x-1) + 1(y-0) = x + y$

(e) [2] Various surveys have found that about 95% of claims that certain products are "green" (or "ecofriendly" or "organic") are either misleading or not true at all. Suppose that you buy 20 products that claim to be "green". Which of the following statements are true?

(I) The expected number of truly "green" products is 1. ✓

(II) The probability that none of the products are truly "green" is 0.3585. ✓

(III) The probability that all of the products are truly "green" is 0.00001935. ✗

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

Let X = # of truly "green" products. $X \sim B(20, 0.05)$

$E(X) = 20 \times 0.05 = 1$

$P(X=0) = \binom{20}{0}(0.95)^{20} \approx 0.3585$

$P(X=20) = \binom{20}{20}(0.05)^{20} \approx$

(f) [2] Let X count the number of heads obtained after three tosses of a fair coin. Which of the following statements are true?

(I) $E(X) = 1.5$ ✓

(II) $P(X \geq 1) = 0.875$ ✓

(III) $F(2) = 0.875$, where $F(x)$ is the cumulative distribution function of X. ✓

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8

$E(X) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} \cdot 3 \cdot \frac{1}{8} = \frac{12}{8} = 1.5$

$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - \frac{1}{8} = \frac{7}{8} = 0.875$

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \frac{7}{8} = 0.875$

(g) [2] Consider a population of school aged children comprised of 160 girls and 145 boys. Suppose that 3% of girls and 5% of boys within this population are estimated to be affected by ADHD. What is the probability that a randomly chosen child will be affected by ADHD?

(A) 0.03951	(B) 0.3248	(C) 0.3333	(D) 0.4051
(E) 0.1883	(F) 0.1286	(G) 0.1205	(H) 0.0421

$$P(ADHD) = 0.03 \times \frac{160}{305} + 0.05 \times \frac{145}{305} \approx 0.03951$$

(h) [2] Amoung women aged 40-50, the prevalence of breast cancer is 0.8%. A test for the presence of breast cancer (a mammogram, for example) shows a positive result in 90% of women who have breast cancer and in 5% of women who do not have breast cancer. Suppose that a woman in this age group tests positive for breast cancer. What is the probability (approximately) that she actually has it?

(A) 0.233	(B) 0.768	(C) 0.685	(D) 0.562
(E) 0.148	(F) 0.865	(G) 0.921	(H) 0.127

$$P(C|+) = \frac{P(+|C)\cdot P(C)}{P(+|C)\cdot P(C) + P(+|C^c)\cdot P(C)} = \frac{(0.9)(0.008)}{(0.9)(0.008) + (0.05)(1-0.008)}$$

$$\approx 0.127$$

(i) [2] Certain types of a rare strain of respiratory infection occur in about 3 out of 2,000 people. During a particularly bad flu season, 12 out of 5,000 people were diagnosed with the infection. What is the probability of this event occurring?

(A) 0.036575	(B) 0.048574	(C) 0.037425	(D) 0.133589
(E) 0.046471	(F) 0.865751	(G) 0.0016575	(H) 0.0055238

Let X = # of people w/ infection

$\lambda = 3 \times 2.5 = 7.5$

$X \sim P_o(7.5)$

$P(X=12) = \frac{e^{-7.5}(7.5)^{12}}{12!} \approx 0.036575$

(j) [2] Suppose that $X \sim N(5, 2^2)$. Which of the following statements is/are true?

(I) $P(1 \leq X \leq 9) \approx 0.955$ ✓

(II) $P(X > 4) \approx 0.691$ ✓

(III) $P(X \leq x) = 0.8$ when $x \approx 6.7$ ✓

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

$\mu = 5$
$\sigma = 2$

$P(1 \leq X \leq 9) = P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0.955$

$P(X > 4) = 1 - P(X \leq 4)$

$= 1 - P(Z \leq \frac{4-5}{2})$

$= 1 - [1 - F(0.5)]$

≈ 0.691

$P(X \leq 6.7) = P(Z \leq \frac{6.7-5}{2})$

$= F(0.85)$

≈ 0.80

CABC | HD | EH | AH | AH

Questions 2-12: You must show work to obtain full credit.

2. State whether each statement is **true or false** and then **explain** your reasoning.

(a) [2] $x^* = 0$ is a stable equilibrium of the autonomous differential equation $\frac{dx}{dt} = 1 - e^x$.

$f(x) = 1 - e^x$

$f(0) = 1 - e^0 = 0 \Rightarrow x^* = 0$ is an eq^m of ⊛

STABILITY THM $\rightarrow$ $f'(x) = -e^x$

$f'(0) = -e^0 = -1 \Rightarrow x^* = 0$ is a STABLE eq^m

∴ TRUE (1)

(1) for using some stability test (could also use phase line $\rightarrow \bullet \leftarrow$ 0)

(b) [2] The range of $g(x, y) = e^{x^2+y^2}$ is $(0, \infty)$.

$x^2 + y^2 \geq 0 \Rightarrow e^{x^2+y^2} \geq e^0 \Rightarrow \underbrace{g(x,y) \geq 1}$

∴ FALSE. (1)

(1) for determining correct range

(c) [2] Suppose that $\nabla f(2, 3) = 4\mathbf{i} - \mathbf{j}$. Then $D_{\mathbf{u}} f(2, 3) = 5$ for some direction $\mathbf{u}$.

max. directional derivative $= \|\nabla f(2,3)\|$

$= \sqrt{4^2 + (-1)^2}$

$= \sqrt{17}$

So $D_u f(2,3) \leq \sqrt{17} < 5$ for all directions $\vec{u}$

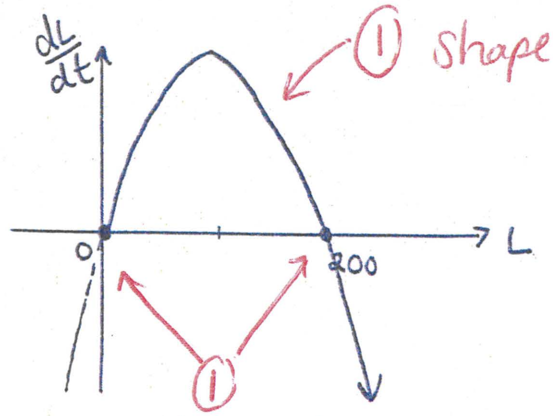
∴ FALSE (1)

(1) for explaining that the max directional derivative is less than 5

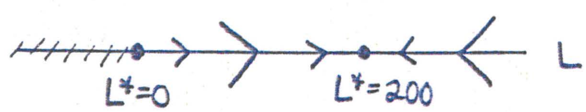
3. A population of ladybugs changes according to the logistic differential equation

$$\frac{dL}{dt} = 0.05L\left(1 - \frac{L}{200}\right)$$

(a) [2] Graph the rate of change, $\frac{dL}{dt}$, as a function of L. Clearly label intercepts.



(b) [2] Draw a phase-line diagram for this differential equation.



(1) correct direction of all arrows
(1) correct size of arrows.

(c) [1] Suppose that the population will die out if the number of ladybugs drops below 30. Write a new differential equation (i.e., modify the one above) to reflect this observation.

$$\frac{dL}{dt} = 0.05L\left(1 - \frac{L}{200}\right)\underbrace{\left(1 - \frac{30}{L}\right)}_{1}$$

4. The following pair of equations represent the population growth of two different species where one is the predator, the other is the prey and t is measured in months.

$$\frac{dx}{dt} = 0.4x - 0.001xy \qquad \frac{dy}{dt} = -0.01y + 0.0002xy$$

(a) [2] State which variable represents the prey population and explain why.

0.5

1 $\boxed{x \text{ represents the prey}}$ since in the absence of predators (ie, $y=0$), x will increase exponentially but interactions between x and y will decrease the growth rate of x 0.5 which is reflected by the negative coefficient of the interaction term "xy".

(b) [2] Suppose that $x_0 = 500$ and $y_0 = 80$. Using Euler's Method with a step size of one month, estimate the size of population x two months from now.

$x_1 = 500 + (0.4(500) - 0.001(500)(80)) = 660$
$y_1 = 80 + (-0.01(80) + 0.0002(500)(80)) = 87.2$

1 good start (ie using correct formula)

$x_2 = 660 + (0.4(660) - 0.001(660)(87.2)) \approx 866$

$\therefore$ The size of pop^{N} x 2 months from now is about $\boxed{866}$

1

(accept 866 ± 1 due to different rounding errors)

5. [3] Solve the separable equation $\dfrac{dP}{dt} = \dfrac{2tP}{1+t^2}$, with initial condition $P(0) = 1$.

Let $u = 1+t^2$. Then $du = 2t\,dt$.
$\Rightarrow \int \frac{2t}{1+t^2}dt = \int \frac{1}{u}du = \ln|u| + C$

$\int \frac{1}{P}dP = \int \frac{2t}{1+t^2}dt$

$\ln|P| = \ln(1+t^2) + C \;\Big|_{e}$

$|P| = e^{c}\cdot(1+t^2)$

$P = \underbrace{\pm e^{c}}_{=A}\cdot(1+t^2)$

1 correct integratn by substitution

1 applying "e" to both sides

$P(0)=1 \Rightarrow 1 = A(1+0^2) \Rightarrow A = 1$

$\therefore \boxed{P(t) = t^2+1}$ 1 final answer.

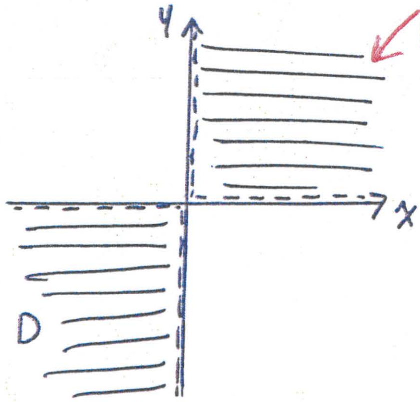
6. Consider the function $g(x, y) = \ln(xy)$.

(a) [2] Find and sketch the domain of g.

$xy > 0 \Rightarrow x > 0$ and $y > 0$ OR $x < 0$ and $y < 0$

① shading 1ST quadrant

* if no picture but domain is stated algebraically, give 1 mark



① shading 3rd quadrant

(b) [2] Create a contour map for g including level curves for $k = -1$, $k = 0$, and $k = 1$.

$\ln(xy) = k \,\Big|_e$

$xy = e^k$

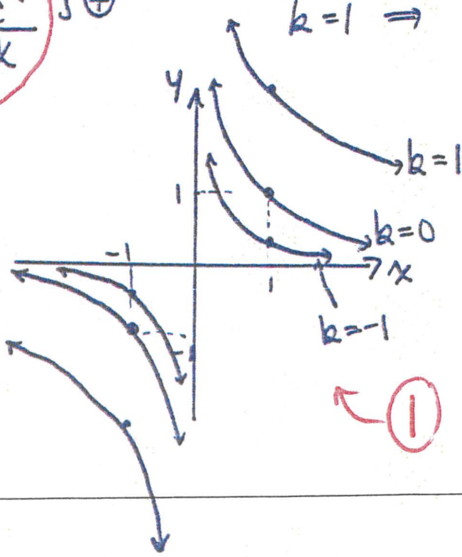
① $y = \dfrac{e^k}{x}$ } ⊕

formula for level curves

$k = -1 \Rightarrow y = \dfrac{e^{-1}}{x}$

$k = 0 \Rightarrow y = \dfrac{1}{x}$

$k = 1 \Rightarrow y = \dfrac{e}{x}$



① similar picture

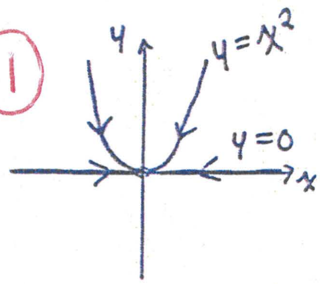
7. (a) [2] Describe what is meant by $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

$f(x,y)$ approaches L as (x,y) approaches (a,b) along all paths to (a,b) in the domain of f.

(2) more or less this exact statement should be here

(b) [3] Show that $\lim_{(x,y)\to(0,0)} \overbrace{\frac{x^2y}{5x^4+y^2}}^{f}$ does not exist. Sketch the domain of the function and the paths you've chosen to approach $(0,0)$ along.

(1)



domain : $\mathbb{R}^2 \setminus \{(0,0)\}$

$f(x,0) = \frac{0}{5x^4} = 0$

so $f \to 0$ as $(x,y)\to(0,0)$ along $y=0$ (1)

$f(x,x^2) = \frac{x^2 \cdot x^2}{5x^4+(x^2)^2} = \frac{1}{6}$

so $f \to \frac{1}{6}$ as $(x,y)\to(0,0)$ along $y=x^2$ (1)

$\therefore \lim_{(x,y)\to(0,0)} f(x,y)$ D.N.E.

* students may have chosen different paths of approach please give 2 marks if 2 paths to $(0,0)$ were used where f approaches different values along each path

8. Consider the function $f(x,y) = (2x - y + 5)^{\frac{3}{2}}$.

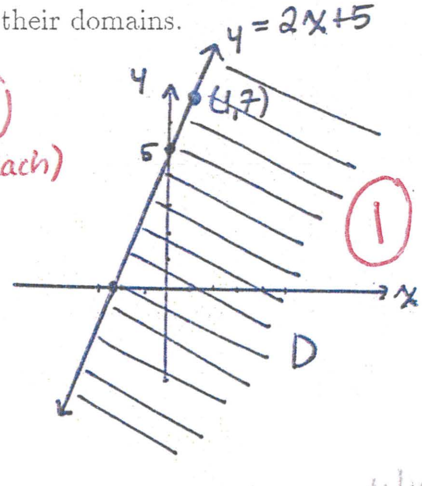
(a) [2] Compute f_x and f_y. Find and sketch their domains.

$$\left.\begin{aligned} f_x &= \tfrac{3}{2}(2x-y+5)^{\frac{1}{2}}(2) \\ f_y &= \tfrac{3}{2}(2x-y+5)^{\frac{1}{2}}(-1) \end{aligned}\right\} \text{1 (.5 each)}$$

demain of f_x and f_y:

$2x - y + 5 \geq 0$

$y \leq 2x + 5$



(b) [2] Is f differentiable at $(1,7)$? Explain why or why not.

1 No! We cannot use this theorem since every disk $B_r(1,7)$ will include points where f_x and f_y are not continuous.

1 similar explanation

OR

No. We cannot find an open disk centred at $(1,7)$ on which f_x and f_y are continuous.

(c) [3] Compute the directional derivative of $f(x,y) = (2x - y + 5)^{\frac{3}{2}}$ at the point $(0,1)$ in the direction specified by $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$.

$\|\vec{v}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$ so $\vec{u} = \frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}$ is a unit vector in the same direction as $\vec{v}$

1 unit vector

$$D_u f(0,1) = f_x(0,1)\cdot\frac{2}{\sqrt{5}} + f_y(0,1)\cdot\frac{1}{\sqrt{5}}$$

1 correct formula.

$$= (6)\frac{2}{\sqrt{5}} + (-3)\frac{1}{\sqrt{5}}$$

$$= \frac{9}{\sqrt{5}} \quad (\approx 4.02)$$

1 final answer.

9. (a) [2] Show that $(1,1)$ is the only critical point of $f(x,y) = x + y + \frac{1}{xy}$. $x \neq 0, y \neq 0$

$f_x = 1 + \frac{0 - 1y}{x^2y^2} = 1 - \frac{y}{x^3y^2} = \frac{x^2y^2 - y}{x^2y^2}$, $f_y = 1 - \frac{x}{x^2y^2} = \frac{x^2y^2 - x}{x^2y^2}$

$f_x = 0$ when $x^2y^2 - y = 0 \Rightarrow y = \frac{1}{x^2}$ ①

$f_y = 0$ when $x^2y^2 - x = 0 \Rightarrow x = \frac{1}{y^2}$ ②

① getting to this point

sub ① into ②: $x = \frac{1}{(\frac{1}{x^2})^2} \Rightarrow x = x^4 \Rightarrow 1 = x^3 \Rightarrow x = 1$

sub $x=1$ into ①: $y = \frac{1}{1^2} = 1$

① solving to get $(1,1)$

$\therefore (1,1)$ is the only critical point.

(b) [3] Use the second derivatives test to determine whether f has a local maximum, a local minimum, or a saddle point at $(1,1)$.

① mixed derivative correct

$f_{xx} = \frac{2}{x^3y}$ $\boxed{f_{xy} = \frac{1}{x^2y^2}}$ $f_{yy} = \frac{2}{xy^3}$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = \frac{4}{(xy)^4} - \frac{1}{(xy)^4} = \boxed{\frac{3}{(xy)^4}}$ ① correct "D"

① conclusion

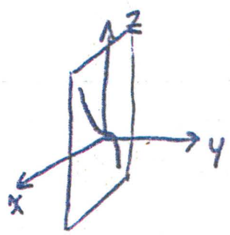
$\left.\begin{array}{l} D(1,1) = 3 \\ f_{xx}(1,1) = 2 \end{array}\right\} \Rightarrow f$ has a $\boxed{\text{local min}}$ at $(1,1)$

[local min value is $f(1,1) = 3$]

(c) [2] Without using the second derivatives test, determine whether $(0,0)$ corresponds to a local maximum, local minimum, or saddle point of the function $g(x,y) = x^3 - 2y^2 + 3xy + 1$.

$g(0,0) = 1$

$g(x,0) = x^3 + 1$



①

In any open disk centred at $(0,0)$, there will be values >1 and <1 $\therefore g$ has a $\boxed{\text{saddle point}}$ at $(0,0)$.

① →

[In particular, along $y=0$ $g(x,y) > 1$ when $x > 0$ and $g(x,y) < 1$ when $x < 0$ so $g(0,0) = 1$ is not an extreme value of $g(x,y)$.

similar explanation

10. A population of bears is modelled by $b_{t+1} = b_t + I_t$, where b_t represents the number of bears in year t. Suppose that the immigration term is $I_t = 8$ with a 40% chance and $I_t = 0$ with a 60% chance. Assume that $b_0 = 40$ and that immigration from year to year is independent.

(a) [3] Let X count the number of bears after 2 years. Find the expected value and standard deviation of X.

$S = \{40, 48, 56\}$

x	$p(x)$
40	$(.6)^2 = 0.36$
48	$2(.6)(.4) = 0.48$
56	$(.4)^2 = 0.16$

(1) correct probabilities

$\mu = E(x) = 40 \cdot p(40) + 48 \cdot p(48) + 56 \cdot p(56)$

$= \boxed{46.4}$ (1)

$Var(x) = (40 - 46.4)^2 \cdot p(40) + (48 - 46.4)^2 \cdot p(48) + \ldots$

$\ldots + (56 - 46.4)^2 \cdot p(56)$

$= 30.72$

$\sigma = \sqrt{Var(x)}$

$= \sqrt{30.72}$

$\approx \boxed{5.54}$ (1)

* if probabilities are wrong but calculations for μ and σ are correct, give 2/3

(b) [3] What is the probability that there will be more than 100 bears after 10 years? (Hint: Let N count the number of years immigration occurs and use the Binomial Distribution.)

MAX POP^N SIZE $= 40 + 10 \times 8 = 120$

POSSIBLE POP^N SIZES OVER 100 : 120, 112, 104

$P(b_{10} > 100) = P(b_{10} = 104) + P(b_{10} = 112) + P(b_{10} = 120)$

$= \boxed{P(N=8) + P(N=9) + P(N=10)}$ where $N \sim B(10, .4)$

$= \binom{10}{8}(.4)^8(.6)^2 + \binom{10}{9}(.4)^9(.6) + \binom{10}{10}(.4)^{10}$

(1) this line

$\approx \boxed{0.01229}$ (1) final answer.

(1) using correct binomial pmf somewhere

Name: ____________________

Student No.: ____________________

11. Suppose that the lifetime of a tree is given by the probability density function $f(t) = 0.01e^{-0.01t}$, where t is measured in years, $0 \le t < \infty$.

(a) [2] Determine the cumulative distribution function, $F(t)$.

$$F(t) = \int_0^t 0.01e^{-0.01x}\,dx$$

① dy ✓

$$= -e^{-0.01x}\Big|_0^t$$

$$= -e^{-0.01t} - (-e^0)$$

$$= \boxed{1 - e^{-0.01t}}$$ ①

(b) [2] What is the probability that the tree will live longer than 70 years?

$$P(T>70) = 1 - P(T \le 70)$$

$$= 1 - \int_0^{70} f(t)\,dt$$

$$= 1 - [F(70) - F(0)]$$

$$= 1 - [(1 - e^{-0.01(70)}) - (1 - e^0)]$$

$$\approx \boxed{0.4966}$$ ①

① some correct work.

(c) [3] Find the average lifetime of the tree. (Recall: $\int u\,dv = uv - \int v\,du$.)

$$E(T) = \int_0^\infty 0.01te^{-0.01t}\,dt$$

$$= \lim_{T\to\infty} \int_0^T 0.01te^{-0.01t}\,dt$$

$$= \lim_{T\to\infty}\left[-Te^{-0.01T} - 100e^{-0.01T} + 100\right]$$

$$= 0 - \underbrace{100e^{-\infty}}_{=0} + 100$$

$$= \boxed{100 \text{ years.}}$$ ① final answer

① $\int 0.01te^{-0.01t}\,dt$

$u = t \qquad dv = 0.01e^{-0.01t}\,dt$

$du = dt \qquad v = -e^{-0.01t}$

$$= -te^{-0.01t} + \int e^{-0.01t}\,dt$$ ①

$$= \boxed{-te^{-0.01t} - 100e^{-0.01t}} + C$$

int. by parts

② $\lim_{T\to\infty} \dfrac{-T}{e^{0.01T}}$

$$\overset{LH}{=} \lim_{T\to\infty} \frac{-1}{-0.01e^{0.01T}}$$

$$= \frac{-1}{\infty}$$

$$= 0$$

① showing this limit is 0

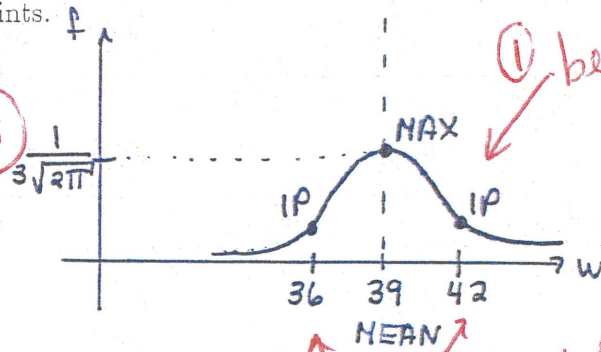
12. The wingspan, W, of a blue jay is normally distributed with a mean of 39 cm and a standard deviation of 3 cm.

(a) [2] Sketch the graph of the probability density function for W, labelling the mean, maximum, and location of inflection points.

$W \sim N(39, 3^2)$

$$f(w) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{w-39}{3}\right)^2}$$

0.5



1 bell curve shape

0.5 IPs labelled

(b) [2] What is the probability that a randomly chosen blue jay has a wingspan wider than 42 cm?

$$\begin{aligned} P(W > 42) &= 1 - P(W \leq 42) \\ &= 1 - P\left(Z \leq \frac{42-39}{3}\right) \\ &= 1 - F(1) \\ &\approx \boxed{0.158655} \end{aligned}$$

1 z-score of 1

1 (decimal places not impt.)

(c) [2] Using substitution and the fact that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$, show that $\int_{-\infty}^{\infty} f(w)\,dw = 1$ (this, along with your graph in part (a), verifies that $f(w)$ is a valid probability density function).

$$\int_{-\infty}^{\infty} \frac{1}{3\sqrt{2\pi}} e^{-\left(\frac{w-39}{\sqrt{2}\cdot 3}\right)^2} dw$$

Let $u = \boxed{\frac{w-39}{\sqrt{2}\cdot 3}}$ Then $du = \frac{1}{\sqrt{2}\cdot 3} dw$

1 correct choice of u.

$$= \int_{-\infty}^{\infty} \frac{1}{\cancel{3}\cancel{\sqrt{2}}\sqrt{\pi}} e^{-u^2} \cdot \cancel{\sqrt{2}}\cancel{3}\,du$$

$$= \frac{1}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-u^2}\,du}_{=\sqrt{\pi}}$$

1 correct pdf for w.

$= 1$