Mathematics 2R3 Practice Test 2

Dr. Hart Fall, 2019

Last Name:_____ Initials:_____

Student No.:_____

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a McMaster standard Casio fx-991 calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

Good Luck!

Score

Question	1	2	3	4	5	Total
Points	5	5	5	5	5	25
Score						

continued ...

1. (5 marks) Put your answer in the space provided for each part.

   (a) The range of a linear transformation is a vector space. True or False.

   _____

   (b) If V is an n-dimensional vector space and $T : V \to V$ is a linear operator with range V then T is one-to-one. True or False.

   _____

   (c) The real vector spaces of 2×2 real matrices and polynomials of degree at most 3 with real coefficients are isomorphic. True or False.

   _____

   (d) Suppose that A is an $n \times n$ matrix and for every $x \in \mathbf{R}^n$, $T(x) = x^T Ax$ then T is a linear transformation. True or False.

   _____

   (e) Suppose that $T : V \to W$ is a surjective linear transformation, the dimension of W is 4 and the dimension of V is 7. What is the dimension of the kernel of T?

   _____

continued . . .

2. (5 marks) Consider the inner product space $\mathrm{C}[-1,1]$ with the inner product given by

$$\langle f, g\rangle = \int_{-1}^{1} fg\,dx.$$

(a) (3 marks) Apply the Gram-Schmidt process to the linearly independent set $\{1, x, x^2\}$ in $\mathrm{C}[-1,1]$ to obtain an orthogonal set.

(b) (2 marks) If W is the subspace generated by $\{1, x, x^2\}$, determine the projection of x^3 onto W.

continued . . .

3. P_2 is the real vector space of polynomials of degree less than or equal to 2. Define $T : P_2 \to P_2$ by $T(p(x)) = p(x-1)$.

   (a) (3 marks) Show that T is a linear transformation.

   (b) (2 marks) Determine the range and kernel of T

continued ...

4. Suppose that P_2 is the vector space of real polynomials of degree ≤ 2 and $T : P_2 \to \mathbf{R}^3$ is a linear transformation satisfying

$$T(1) = (1, 0, 3), T(x + 1) = (2, -3, 0) \text{ and } T(x^2 + x + 1) = (1, 0, 1).$$

(a) (2 marks) Compute $T(1 - x + x^2)$.

(b) (3 marks) Write out a formula for $T(a_0 + a_1x + a_2x^2)$.

continued . . .

5. Prove that if $T : V \to W$ is a one-to-one linear transformation then T^{-1} is a linear transformation from $R(T)$ to V.

THE END