Q1 : $n=144$

$P(\text{face 4 comes up once}) = \frac{1}{12}$

Poisson Approximation:

$$\binom{n}{x} p^{x} (1-p)^{n-x} \longrightarrow \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\mathcal{E}(X) = np = 144 \cdot \frac{1}{12} = 12$$

$$\mathcal{E}(X) = \mu = \lambda = 12$$

$$P(A) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-12} 12^{0}}{0!} + \frac{e^{-12} 12^{1}}{1!} + \frac{e^{-12} 12^{2}}{2!}$$

$$= 85e^{-12} \approx 0.000516$$

Q2 : let A = event that an individual buys an item in the store

$$P(A) = 0.45 + 0.1 = 0.55 = p$$

If $n = 33$, then $\mathcal{E}(x) = np = 33(0.55) = 18.15$

Q3 $X \sim N(\mu = 13, \sigma = 0.82)$, $n=8$

$P(\text{one shoe} > 14 \text{ ounces}) = 1 - P(\text{one shoe} < 14 \text{ ounce})$

$$= 1 - P\left(z \leq \frac{14-13}{0.82}\right) = 1 - \Phi(1.219) \approx 0.111422 = p$$

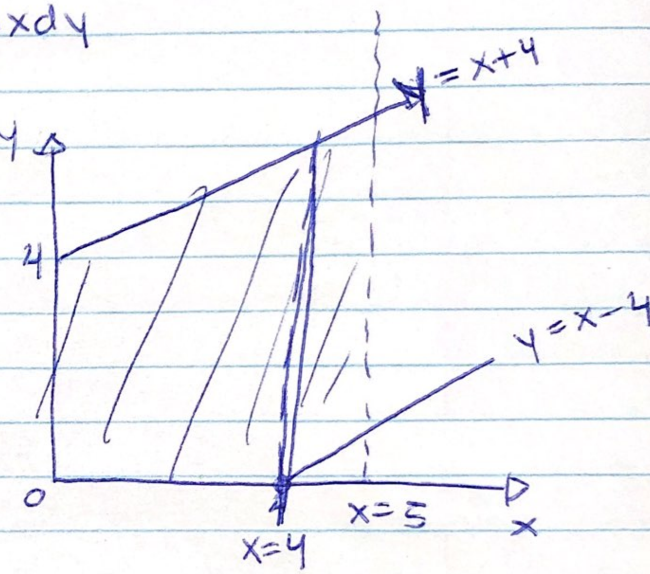
Q3 continued

Then, using the binomial formula,

$$P(X=3) = \binom{8}{3} (0.11142)^3 (0.88858)^5$$

$$\simeq 0.0427$$

Q4 $\quad E(XY) = \iint_A f(x,y)\,dx\,dy$



Need to separate into two integrals:

$$\int_{x=0}^{4}\int_{y=0}^{x+4} xy\, f(x,y)\,dx\,dy$$

$$+ \int_{x=4}^{5}\int_{y=x-4}^{x+4} xy\, f(x,y)\,dx\,dy$$

$$= \int_{x=0}^{4}\int_{y=0}^{x+4} xy\left(\frac{3x}{268}\right)dy\,dx + \int_{x=4}^{5}\int_{y=x-4}^{x+4} xy\left(\frac{3x}{268}\right)dy\,dx$$

$$= 14.1836$$

Q5 a) $P(0.1 < X < 0.6)$

$$= P(0.1 < X < 0.2) + P(0.2 < X < 0.6)$$

$$= \int_{0.1}^{0.2} 0.2\,dx + \int_{0.2}^{0.6} 2x\,dx$$

$$= 0.2(0.2 - 0.1) + \frac{2}{2}\left[0.6^2 - 0.2^2\right]$$

$$= \underline{0.3400}$$

(b) $$E(X) = \int_{0}^{0.2} 0.2x\,dx + \int_{0.2}^{1} 2x^2\,dx$$

$$= 0.1x^2 \Big|_{0}^{0.2} + \frac{2x^3}{3} \Big|_{0.2}^{1}$$

$$= 0.6653$$

Q6 n(females) = 25 n(males) = 23
n(total) = 48

$$\frac{n(\text{all values up to } 140)}{n(\text{total})} = \frac{19}{48} = P(X \leq 140)$$

cumulative
$$\Rightarrow \% = \frac{19}{48}(100) = \boxed{\frac{475}{12}}$$

Q7 $534 = n = 145 + 74 + 304$
$534 \times 0.2727 = 145.62$

* The ~~145~~ 146th value is 65
∴ 27.27% of data are below 65.

Hilroy

(Q8) $X \sim N(\mu = 164, \sigma = 90)$

$P(X > 180) = 1 - P(X \leq 180)$

$= 1 - P\left(Z \leq \frac{180 - 164}{90}\right)$

$= 1 - \Phi\left(\frac{16}{90}\right) = 1 - 0.5704$

$= \underline{\underline{0.4286}}$

(Q9) $X \sim N(\mu, \sigma = 21)$

$*P(X < 73) = 0.86$

$\Phi\left(\frac{73 - \mu}{21}\right) = 0.86$

$\hookrightarrow P(Z \leq z) = 0.86,$
$z = 1.080319$

$\Rightarrow \frac{73 - \mu}{21} = 1.08$

$\Rightarrow \mu = 73 - 22.68$

$\underline{\underline{\mu = 50.32}}$

(Q10) $X \sim N(\mu = 123.1, \sigma = 3.1)$

$P(122.2 \leq \bar{X} \leq 125.3) = P(\bar{X} \leq 125.3) - P(\bar{X} \leq 122.2)$

$= \Phi\left(\frac{125.3 - 123.1}{\underbrace{3.1/\sqrt{10}}}\right) - \Phi\left(\frac{122.2 - 123.1}{3.1/\sqrt{10}}\right) \doteq 0.8087$

$\hookrightarrow \sigma_{\bar{x}} = \frac{3.1}{\sqrt{10}} = \frac{\sigma}{\sqrt{n}}$

Q11 (a) $P(X>4.11) = 1 - P(X \le 4.11)$

$= 1 - F_X(4.11) = 0.9009$

(b) $P(2.9 < X < 4.05)$

$= P(X < 4.05) - P(X < 2.9)$

$= F_X(4.05) - F_X(2.9)$

$= 0.0447$

(c) $E(X) = \int_{x=4}^{5} x f(x)\,dx$, $\quad f(x) = \frac{d}{dx} F(x)$

$= \frac{2x}{9}$

$\Rightarrow E(X) = \int_4^5 x\left(\frac{2x}{9}\right) dx$

$= \int_4^5 \frac{2x^2}{9}\,dx = \frac{2x^3}{9}\Big|_{x=4}^{5} = \underline{\frac{122}{27}}$

(d) $Var(X) = E(X^2) - [E(X)]^2$

$\Rightarrow E(X^2) = \int_{x=4}^{5} x^2 f(x)\,dx = \int_4^5 \frac{2x^3}{9}\,dx$

$= \frac{2x^4}{9 \cdot 4}\Big|_4^5 = \frac{738}{36}$

$Var(X) = \frac{738}{36} - \left[\frac{122}{27}\right]^2$

$= \frac{738}{36} - \frac{14884}{729} = \boxed{121/1458}$

Q12 $SD(X) = 8$

$Var(X) = 8^2 = 64$

$$Var(9X+4) = 9^2 \cdot Var(X)$$
$$= 81 \cdot Var(X)$$
$$= 81\ (64)$$
$$= 5184$$

$$\Rightarrow SD(9X+4) = \sqrt{5184}$$
$$= \boxed{72}$$

Q13 (a) First, need to find c.

$$1 = \int_{x=0}^{\infty} \int_{y=0}^{x} c e^{-8x-3y}\, dy\, dx$$

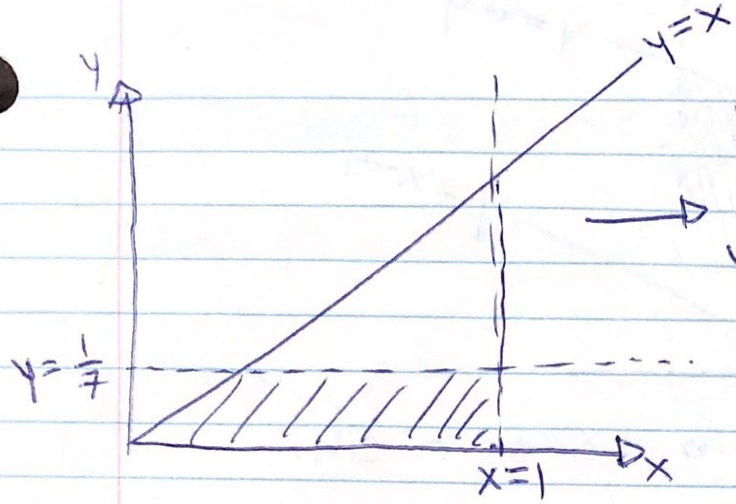
$$\frac{1}{c} = \int_{x=0}^{\infty} \frac{-c}{3} e^{-8x-3y} \Big|_{y=0}^{x} dx$$

$$\frac{3}{c} = \int_0^{\infty} (e^{-8x} - e^{-11x})\, dx$$

$$\frac{3}{c} = -\frac{e^{-8x}}{8} + \frac{e^{-11x}}{11} \Big|_{x=0}^{\infty}$$

$$\Rightarrow \frac{3}{c} = -\frac{1}{11} + \frac{1}{8} = \frac{11-8}{88} = \frac{3}{88}$$

$\therefore$ $c = 88$



$$\rightarrow \int_{y=0}^{1/7} \int_{x=y}^{1} f(x,y)\,dx\,dy$$

note: this is one way to organize the integral

$$\Rightarrow P(X<1,\ Y<1/7) = \int_{y=0}^{1/7} \int_{x=y}^{1} 88e^{-8x-3y}\,dx\,dy$$

$$= \int_{y=0}^{1/7} \left(11e^{-11y} - 11e^{-8-3y}\right) dy$$

$$= 1 - e^{-11/7} - \left(\frac{11}{3}e^{-8} - \frac{11}{3}e^{-59/7}\right)$$

$$= 0.7918$$

(b) $$f_X(x) = \int_{y=0}^{x} 88e^{-8x-3y}\,dy$$

$$= -\frac{88}{3}e^{-8x-3y}\Big|_{y=0}^{x}$$

$$= \frac{88}{3}e^{-8x}\left(1-e^{-3x}\right)$$

Q14 Same integral bounds as for Q4

$$\int_{x=0}^{4}\int_{y=0}^{x+4} x\, f(x,y)\,dy\,dx + \int_{x=4}^{5}\int_{y=x-4}^{x+4} x\, f(x,y)\,dy\,dx$$

$$\left(= \frac{234}{67}\right) = \int_{x=0}^{4}\int_{y=0}^{x+4} \frac{3x^2}{268}\,dy\,dx + \int_{x=4}^{5}\int_{y=x-4}^{x+4} \frac{3x^2}{268}\,dy\,dx$$

Q16

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + [Bias(\hat{\theta})]^2$$

$$= \frac{2(n-1)\sigma^4}{n^2} + \left[\frac{n-1}{n} E(S^2) - \sigma^2\right]^2$$

$$= \frac{2(n-1)}{n^2}(\sigma^2)^2 + \left(\frac{n-1}{n}\sigma^2 - \sigma^2\right)^2$$

$$= \frac{2n-1}{n^2}(\sigma^2)^2$$

$\Rightarrow$ sub in $n=21$, $\sigma^2=4$

$$MSE = \frac{2(21)-1}{(21)^2}(4)^2 = 1.4875 \simeq \underline{\underline{1.49}}$$