

## Stats 3403/3J04 - Lecture #1.

- This course will be an introduction to statistics and probability.
- Statistics is the science of collecting and analyzing data, and of inferring information from that data, esp. when the information is incomplete.
- Probability is the mathematics of random events.

### Probability theory (2.1)

- An experiment is/can be anything that produces data.
- A random experiment is an experiment is one which can produce different outcomes even though it is performed in the same way.

Ex:1: The experiment is "flip a coin". The outcome is either heads or tails.

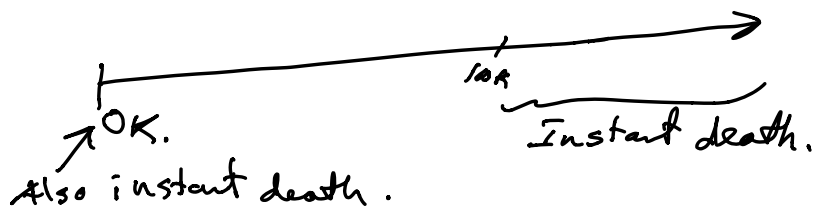
Ex: The experiment is "Take the temperature": you stick your head out the window and read the thermometer.

Def: A sample space,  $S$ , is the set of all outcomes of an experiment.

A sample space can be discrete (i.e. finite, countably infinite) or continuous (contains an interval of  $\mathbb{R}$ ).

Ex: - For the coin-tossing experiment, the sample space is  $S = \{H, T\}$ , so it is discrete.

- For the temperature taking experiment, the sample space is  $S = \{x \geq 0 \text{ (Kelvin)}\} \in \mathbb{R}$ , so it is a continuous random variable.

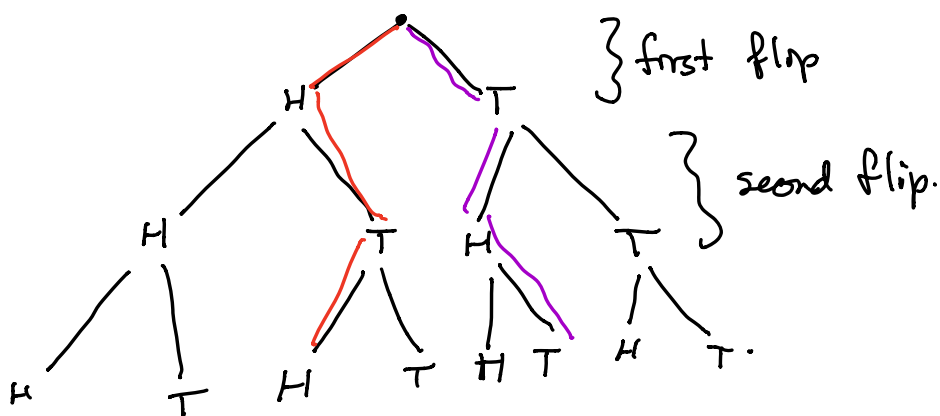


- Rolling a six-sided die:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

- For diabetics, blood glucose level,  
 $S = \{x \geq 0\}$ . mg/—.

Tree diagrams Sometimes we can represent sample spaces as trees! Suppose our experiment B "flip a coin three times in succession". Then the sample space B given by paths through a tree



Each outcome is determined by the last node, so there are 8 possible outcomes.

Sometimes we are interested in a collection of related outcomes. For example, in the last situation we may be only interested in the outcomes that have two or more heads.

Such a subset is called an event.

Defn: An event is a subset of a sample space of a random experiment.

New events can be constructed from other events as follows:

① Unions: If  $E_1$  and  $E_2$  are events, then  $E_1 \cup E_2 = \{x : x \in E_1 \text{ or } x \in E_2\}$  is an event.

② Intersections: If  $E_1, E_2$  are events then  $E_1 \cap E_2 := \{x : x \in E_1 \text{ and } x \in E_2\}$  is an event.

③ Complement: If  $E \subseteq S$  is an event, then the complement  $E' = \{x \in S : x \not\in E\}$  is an event.  
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NB: Sometimes people write  $E^c$  instead of  $E'$ .

Note:  $E \cup E' = S$  and  $S' = \emptyset \leftarrow$  empty set.  
 $\nwarrow$  the whole sample space.