Lecture 22

One Sided Confidence Bounds.

- The confidence intervals of the last section gave upper and lower bounds for μ, i.e. a two-sided CI.
- We can obtain one-sided confidence bounds by setting either $L = -\infty$ or $U = +\infty$, and replacing $z_{\alpha/2}$ by z_α:

- A $100(1-\alpha)\%$ upper-confidence bound for μ is
$$\mu \leq \bar{x} + z_\alpha \sigma/\sqrt{n}.$$

- A $100(1-\alpha)\%$ lower confidence bound for μ is
$$\bar{x} - z_\alpha \sigma/\sqrt{n} \leq \mu.$$

Ex: Find the 95% upper confidence bound for the mean of a normal distribution with $\sigma = 60$ and sample size 36.

Solution: $\alpha = 0.05$, so we want z_α s.t.
$$\Phi(z_\alpha) = 1 - \alpha = 0.95$$

tables $\rightsquigarrow z_\alpha = 1.645$.

Thus, $$\mu \leq \bar{x} + (1.645)\frac{60}{\sqrt{36}} = \bar{x} + 16.45.$$

Notice that this gives a tighter bound on the right than the simultaneous confidence interval, which had an error of 19.6 (compared to 16.45).

Confidence interval for large samples.

- In the last section we were finding confidence intervals for approximating μ of a normal distribution.
- If our distribution is <u>not</u> normal, we can still apply the above techniques because of the central limit theorem.

- If n is sufficiently large, say $n \geq 40$, then we still have

$$P\left(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha.$$

- i.e. the same confidence interval.

- Suppose σ is unknown.
  ↳ for <u>large enough</u> sample, $n \geq 40$,

$$\frac{\overline{X} - \mu}{\boxed{S}/\sqrt{n}} \sim N(0,1)$$

not σ → S

where S is the sample standard deviation.

- Thus, if we are sampling a (any) distribution with large enough sample size, the $100(1-\alpha)\%$

... large enough sample size, the $100(1-\alpha)\%$ CI for the mean is given by

$$\left[\bar{x} - z_{\alpha/2} S/\sqrt{n},\ \bar{x} + z_{\alpha/2} \frac{S}{\sqrt{n}}\right].$$ small s. (s is the sample std. dev.)

(8.2) Confidence Interval on the mean of a normal distribution, variance unknown

- We assume that we are sampling a population whose underlying distribution is Normal $N(\mu, \sigma^2)$, where now we know neither μ nor σ^2

- If n, the sample size, is large, then

$$\frac{\bar{X} - \mu}{S/\sqrt{n}},$$ where S is the sample standard deviation is approximately normal.

- what do we do when our sample size is small?

  ↳ - still use $\dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ } cannot assume that this is normal!

- Instead, we describe a new distribution:

- If our sample size is n, we say that

$$T := \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

has a "Student's t distribution with $n-1$ degrees of freedom"

- *Historical note*: t distribution was developed by William Gosset (AKA "Student") during his career as "Head Experimental Brewer" at the Guinness Brewery in Dublin, Ireland!

- The t distribution depends only on n, the sample size.
  - ↳ there is a different distribution for each n.
- As $n \to \infty$ $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}} \to N(0,1)$, the standard normal.
- to find t-values, we look them up in a chart as we do standard values.
- To find a $100(1-\alpha)\%$ confidence interval on the mean of a normal population of unknown variance with sample size n (n small), we find $t_{\alpha/2, n-1}$ in the tables such that

$$P(-t_{\alpha/2, n-1} < T < t_{\alpha/2, n-1}) = 1-\alpha$$

- then, using the same technique as with known variance, giving

$$\left[\bar{x} - t_{\frac{\alpha}{2}, n-1}\frac{s}{\sqrt{n}}\ ,\ \bar{x} + t_{\alpha/2, n-1}\frac{s}{\sqrt{n}}\right].$$

- here $\bar{x}$ is the sample mean,
- $t_{\alpha/2, n-1}$ is found in row $(n-1)$ column $\alpha/2$ of the t-distribution chart.

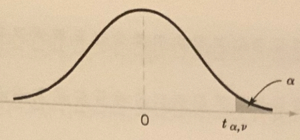

TABLE V **Percentage Points $t_{\alpha,\nu}$ of the t Distribution**

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.