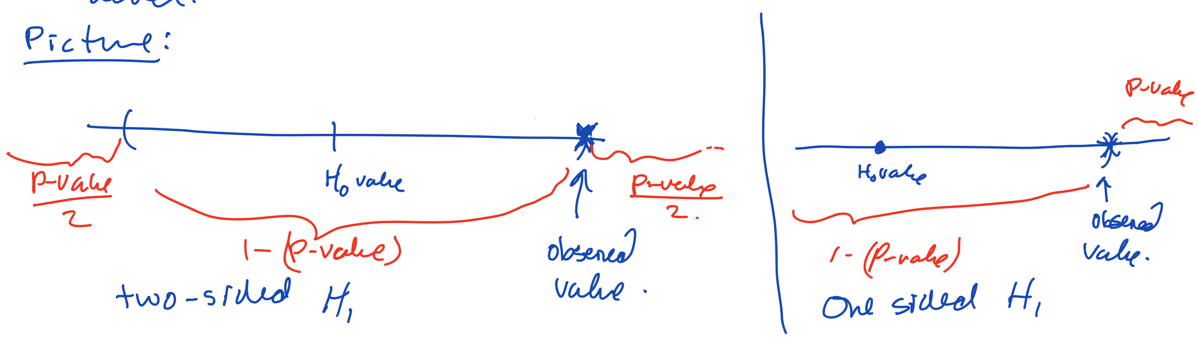
# Lecture 26

* Given an experiment with null hypothesis $H_0$, the <u>P-value</u> of the experiment is the smallest significance level that would lead to a rejection of $H_0$ with the given data. *

- the P-value is sometimes called the "observed significance level."

<u>Picture:</u>

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<u>Motto:</u> The p-value is a measure of the risk that we make an incorrect decision if we reject $H_0$.

- So for example, if we are given a value of our test statistic and it has a very high p-value, then we are at high risk of a Type I error if we reject $H_0$.

- Notice that p-values give us a refinement of the "fixed significance level" testing.

↳ - fix a significance level $\alpha$.

- if the observed value is in the critical region then the p-value is <u>smaller</u> than $\alpha$

↳ if p-value $\leq \alpha$, Reject $H_0$.

- otherwise, accept $H_0$, report the result with p-value.

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## Relationship between Hypothesis Test and Confidence Interval

- close relationship between a hypothesis test for a parameter $\theta$ and the confidence interval for $\theta$.

- suppose that $[L, U]$ is a $100(1-\alpha)\%$ confidence interval for $\theta$

- <u>Then</u> a hypothesis test of significance level $\alpha$ for the hypothesis

$$H_0: \theta = \theta_0 \quad , \quad H_1: \theta \neq \theta_0$$ ($\theta_0$ some number)

will lead to a rejection of $H_0$ if and only if $\theta_0 \notin [L, U]$.

- This gives an equivalent way of performing fixed sig. level testing:

↳ - Given a hypothesis $H_0: \theta = \theta_0$ $H_1: \theta = \theta_0$, choose a test statistic $\hat{\Theta}$ for $\theta$ and a sig. level $\alpha$.

- for a given point estimate $\hat{\Theta} = \hat{\theta}$, construct the CI around $\hat{\theta}$.

- If $\theta$ is in the interval, accept $H_0$.

- If $\theta_0$ is not on the interval, reject $H_0$.

## 9.2 Tests on the mean of a Normal distribution, Variance known.

- We study in more detail the case of sampling the normal distribution with known variance, $\sigma^2$.
- We fix a null hypothesis

$$H_0 : \mu = \mu_0 \leftarrow \text{some } \#.$$

- We break $H_1$ into three cases:

(I) $H_1 : \mu \neq \mu_0$ (II) $H_1 : \mu > \mu_0$ (III) $H_1 : \mu < \mu_0$.

- As in all of the previous examples, we standardize everything we take as our test stat

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}.$$

- Thus, if $H_0$ is true, then $Z_0 = N(0,1)$.

Any statistical test of this form is called a Z-test.

Z-tests are particularly important, since even if we are not sampling a normal distribution, having a large sample means the central limit theorem will make the test still apply.

Z-test structure (for significance level $\alpha$):

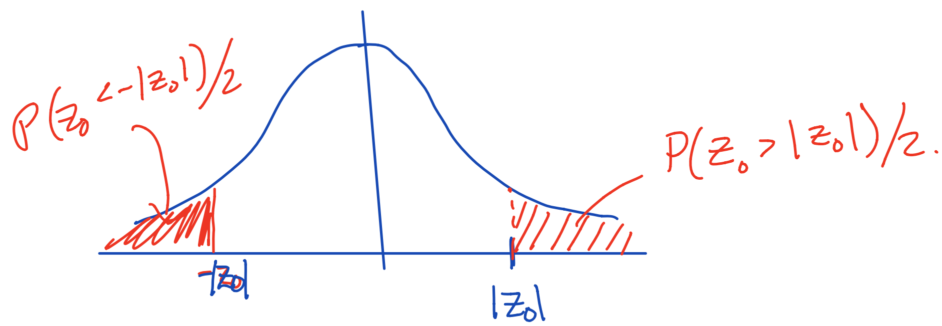
i) From data, compute $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$. } some #.

ii) Find p-value for the test with observation $z_0$.

iii) Reject $H_0$ if $\underbrace{\text{p-value of } z_0 \text{ is } \leq \alpha}_{\text{three cases of this.}}$

Case I, $H_1: \mu \neq \mu_0$.

Two sided/tailed test.




So the p-value for $z_0$ is

$$P(z_0 \geq |z_0|) + P(z_0 \leq -|z_0|) = 2P(z_0 \geq |z_0|)$$
$$= 2(1 - P(z_0 \leq |z_0|)).$$

- Therefore, we reject $H_0$ if $\alpha \geq 2(1 - P(z_0 \leq |z_0|))$

- equivalently, reject $H_0$ when $\frac{\alpha}{2} \geq 1 - P(z_0 \leq |z_0|)$

- equivalently, reject $H_0$ when $P(z_0 \leq |z_0|) \geq 1 - \frac{\alpha}{2}$.

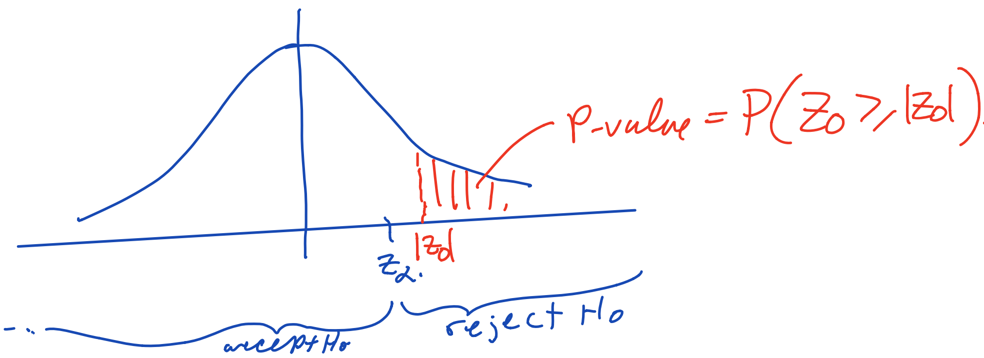
$$\Updownarrow$$

$$z_{\alpha/2} \leq |z_0|$$

(Recall, $z_{\alpha/2}$ is the # s.t. $P(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2}$).

So the critical region is $Z_0 \leq -z_{\alpha/2}$ or $Z_0 \geq z_{\alpha/2}$.

Case II: $H_1: \mu > \mu_0$ (one sided/tailed).




Then reject $H_0$ if $P(Z_0 \geq |z_0|) \leq \alpha$.

$$\Updownarrow$$

$$z_0 \geq z_\alpha.$$

where $z_\alpha$ is such that

$$\Phi(z_\alpha) = 1 - \alpha.$$

Similarly:

Case III: $H_1: \mu < \mu_0$. (one sided/tailed)

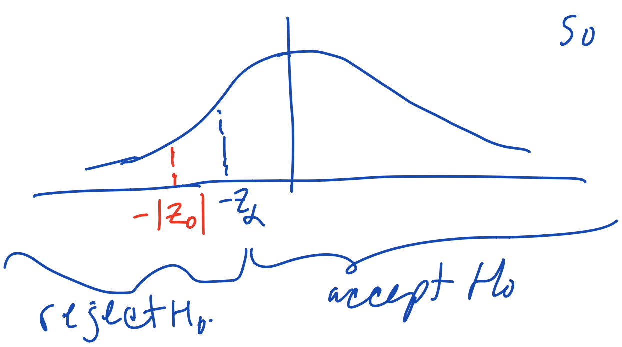
p-value is $P(Z_0 \leq -|z_0|)$

So Reject $H_0$ if.

$$P(Z_0 \leq -|z_0|) < \alpha.$$

$$\Updownarrow$$

$$-|z_0| \leq -z_\alpha.$$



Example: Sample of 25 adults have a mean body temperature of 36.8°C. Assume body temp is normally distributed with $\sigma = 0.34°C$.. Suppose our test is

$H_0 : \mu = 37°C$.

$H_1 : \mu \neq 37°C$. ← (two-sided!) case (I)

$\alpha = 0.01$.

Accept or Reject $H_0$?

solution:

$$z_0 = \frac{\bar{x} - \mu_0}{(\sigma/\sqrt{n})} = \frac{36.8 - 37}{0.34/\sqrt{25}} = -2.94$$

So the p-value (2-sided) of the test is

$$2P(Z \leq -2.94) = 0.003282 \underbrace{\leq 0.01 = \alpha}_{\text{So reject } H_0!}$$