r.v. d.r.v.

Lecture 5

Independence

In some situations, the occurence of one event has no affect on the probability of some other event. Such events are called independent.

Ex: Consider a box of 50 car parts. Six parts are selected randomly from a box and each part is replaced before the next is chosen. Suppose there are 3 defective parts and 47 non-defective. What is the probability that the second is defective if the first is?

Let A_i = the ith part is defective.

We want $P(A_2 | A_1)$. Since the first part is replaced, there are still 3 defective in the box of 50 when we choose the second.

So $P(A_2 | A_1) = 3/50$.

Furthermore $P(A_1 \cap A_2) = \frac{3}{50} \cdot \frac{3}{50}$.

Defn we say that A_1, A_2 are *independent* if one of the following equivalent conditions *holds:*

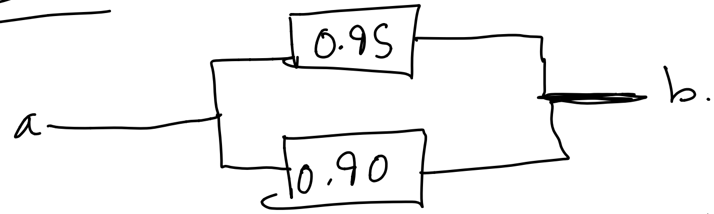
1) $P(A_1 | A_2) = P(A_1)$
2) $P(A_2 | A_1) = P(A_2)$
3) $P(A_1 \cap A_2) = P(A_1) P(A_2)$.
4) $P(A_1' \cap A_2') = P(A_1') P(A_2')$

Warning Mutually exclusive events are automatically *dependent*: if $P(A_i) \neq 0$ and $A_1 \cap A_2 = \emptyset$, then $P(A_1 \cap A_2) = 0$ but $P(A_1)P(A_2) \neq 0$ so A_1 and A_2 are *not* independent.

In general, we say $A_1, \ldots, A_n$ are independent if

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)P(A_2)\ldots P(A_n).$$

Ex: Consider the parallel circuit.



The numbers indicate the probability that the device is functional. Assuming the devices fail independently, what is the probability that there is a path from a to b?

Let T = "the top device is functional"

B = "the bottom device is functional"

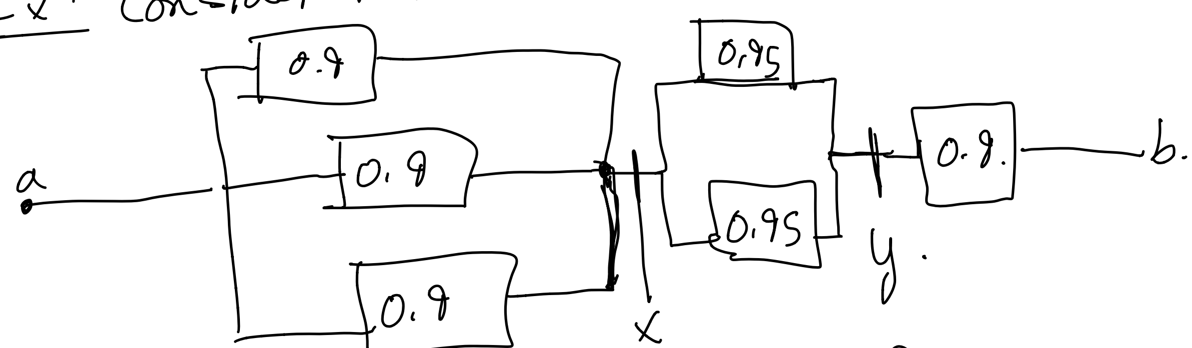
Then $P(B \cup T) = 1 - P((B \cup T)')$

$= 1 - P(B' \cap T')$

$= 1 - P(B')P(T')$ (by independence)

$= 1 - (1-P(B))(1-P(T)) = 0.995.$

Ex: Consider the circuit



What is the probability of a path from a to b?

Divide into three circuits. There is a path from a to b iff there is a path from a to x and a path from x to y and a path from y to b.

So $P(a \text{ to } x) = 1-(1-0.9)^3$

$= 1-(0.1)^3 = 0.999.$

$P(x \text{ to } y) = 1-(1-0.95)^2$

$= 0.999875.$

$P(y \text{ to } b) = 0.99.$

Now, since all of the devices fail independently, we have

$P(\text{"a to x"} \cap \text{"x to y"} \cap \text{"y to b"}) = P(a \text{ to } x)P(x \text{ to } y)P(y \text{ to } b)$

$= (0.999)(0.999875)(0.99) = 0.987.$

Random Variables

Defn Given a sample space of a random experiment, a random variable is a function that assigns a real value to each outcome

Remk: We usually denote a random variable by a capital letter X (or Y or Z, R, or whatever) After the experiment takes place, the outcome is usually assigned to a lowercase letter ~~ex~~ i.e. $x = 70$ mg (resp y, z, r, etc...).

Note that, like sample spaces, random variables can either discrete or continuous.

X is discrete if the range is finite or countably infinite.

X is continuous if the range contains a real interval (i.e. $[0,1]$, or $(0,5)$ or $(0,\infty)$ or all of $\mathbb{R}$, etc...).

Ex:

Discrete random vars:

- \# of heads in 1000 coin flips.
- \# of defective parts per 10000 on a production line.
- \# of errors per binary string of a given length.

Continuous Random Vars

- electrical current
- temperature
- pressure
- time
- volume

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