STATS 3Y03/3J04 Midterm #2
Greg Cousins, November 4, 2019

Name: ______________________________________ **ID #:** ________________

- The test is 75 minutes long.
- The exam has questions on page 2 through 11; there are 18 multiple-choice questions printed on BOTH sides of the paper.
- Scrap paper will be provided.
- You are responsible for ensuring that your copy of test is complete. Bring any discrepancies to the attention of the invigilator.
- Select the one correct answer for each question and enter that answer onto the answer sheet provided using an HB pencil.
- There are 18 multiple choice questions each worth 1 mark, and 1 question on correct computer card filling worth 1 mark (for a total of 19 marks)
- There is no penalty for a wrong answer.
- No marks will be given for the work in this booklet. Only the answers on the computer card (the scantron sheet) count for credit. You must submit this test booklet and any scrap paper along with your answer sheet.
- You may use a Casio FX-991 MS or MS Plus calculator and there is a **formula sheet at the end of the test**; no other aids are permitted.
- **Good luck!!**

1. Suppose that the random variable X has the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{for } x < 5 \\ \frac{x^2-25}{56} & \text{for } 5 \leq x < 9 \\ 1 & \text{for } x \geq 9. \end{cases}$$

What is $P(X > 6.33)$?

(a) 0.5613 **(b)** 0.1264 **(c)** **0.7309** **(d)** 0.2342

2. Using the cumulative distribution function from question 1, find $P(4.34 < X < 5.21)$.

(a) **0.0383** **(b)** 0.0243 **(c)** 0.5132 **(d)** 0.0813

Continued on page 3

3. Find the mean of the random variable X, whose cumulative distribution function is given in question 1.

(a) 3.4151 **(b)** 6.4251 **(c)** 7.9843 **(d)** **7.1905**

4. Find the variance of the random variable X, whose cumulative distribution function is given in question 1.

(a) **1.2971** **(b)** 3.156 **(c)** 1.247 **(d)** 2.901

Continued on page 4

5. The weight of a sophisticated running shoe is normally distributed with a mean of 15 ounces. Suppose that the standard deviation is 0.83. If we sample 6 such running shoes, find the probability that exactly 4 of those shoes weigh more than 16 ounces.

(a) 0.2378 **(b)** 0.0734 **(c)** 0.0047 **(d)** **0.0021**

6. Suppose that the random variables X and Y have the following joint probability density function:

$$f(x, y) = ce^{-4x-9y}, 0 < y < x.$$

Find c.

(a) **52** **(b)** 42 **(c)** 82 **(d)** 22

Continued on page 5

7. Suppose that X and Y are jointly distributed with joint probability density function given in question 6. Find $P(X < 1, Y < 1/9)$.

(a) 0.1508 **(b)** 0.7905 **(c)** 0.6393 **(d) 0.7474**

8. Suppose that X and Y have the following joint probability density function:

$$f(x, y) = \frac{3y}{464}, \text{ where } 0 < x < 6,\ 0 < y,\ \text{and } x - 4 < y < x + 4.$$

Find the covariance $\sigma_{XY} = \text{cov}(X, Y)$.

(a) 12.798 **(b)** 19.8811 **(c) 1.7327** **(d)** 21.6138

Continued on page 6

9. Consider the following data:

$$\{40, 67, 47, 52, 68, 54, 46, 42, 58, 45\}$$

.

Calculate the sample mean.
(a) 64.76 **(b)**55.22 **(c)** 25.65 **(d) 51.90**

10. Consider the following data (the same as the previous question):

$$\{40, 67, 47, 52, 68, 54, 46, 42, 58, 45\}$$

.

Calculate the sample variance.
(a) 82.34 **(b) 97.21** **(c)** 10.30 **(d)** 101.42

Continued on page 7

11. Consider the data set that is summarized in the Minitab Output below:

Stem-and-Leaf Display: C1

```
Stem-and-leaf of C1 N = 13
Leaf Unit = 1.0

  1    1  5
  2    2  7
  2    3
  3    4  5
 (7)   5  3367779
  3    6  456
```

Find Q_1, the median, and Q_3
(a) 49, 57, 61.5 **(b)** 45, 57, 61.5 **(c)** 49,59, 61.5 **(d)** 49, 57, 64

12. An 1868 paper by German physician Carl Wunderlich reported, based on over a million body temperature readings, that healthy adult body temperatures are approximately normally distributed with mean 98.6 degrees Fahrenheit and standard deviation 0.6. In a random sample of 55 healthy adults, find the probability that the average body temperature is between 98.43 and 98.74.
(a) 0.9403 **(b)** 0.9502 **(c)** 0.7503 **(d)** 0.5426

Continued on page 8

13. Let X denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. Suppose that X has the following Rayleigh pdf:

$$f(x) = \begin{cases} \frac{xe^{\frac{-x^2}{2\theta^2}}}{\theta^2} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

If $\theta = 95$, then 73% of the time the vibratory stress is greater than what value?
(a) 32.51 **(b)** 64.23 **(c)** **75.37** **(d)** 65.78

14. IQs are known to be normally distributed with mean 100 and standard deviation 15. What percentage of people have an IQ lower than 85?
(a) 42.01 **(b)** 19.73 **(c)** 54.83 **(d)** **15.87**

Continued on page 9

15. IQs are known to be normally distributed with mean 100 and standard deviation 15. Find the value for which 70% of the population has an IQ greater than that value.

(a)54.45 **(b) 92.20** **(c)**33.33 **(d)** 21.00

16. Suppose that we sample a normal distribution with standard deviation $\sigma = 100$. What sample size is required to have a 99% confidence interval for the mean with precision 20?

(a) 60 **(b)** 51 **(c)** 42 **(d)** 33

No Solution

Continued on page 10

17. Suppose that X is the number of observed "successes" in a sample of n observations where p is the probability of success on each observation and the observations are independent. Suppose that we use $\hat{p}^2 = \frac{X^2}{n^2}$ as an estimator of p^2. Find the amount of bias in the estimator $\hat{p}^2$.

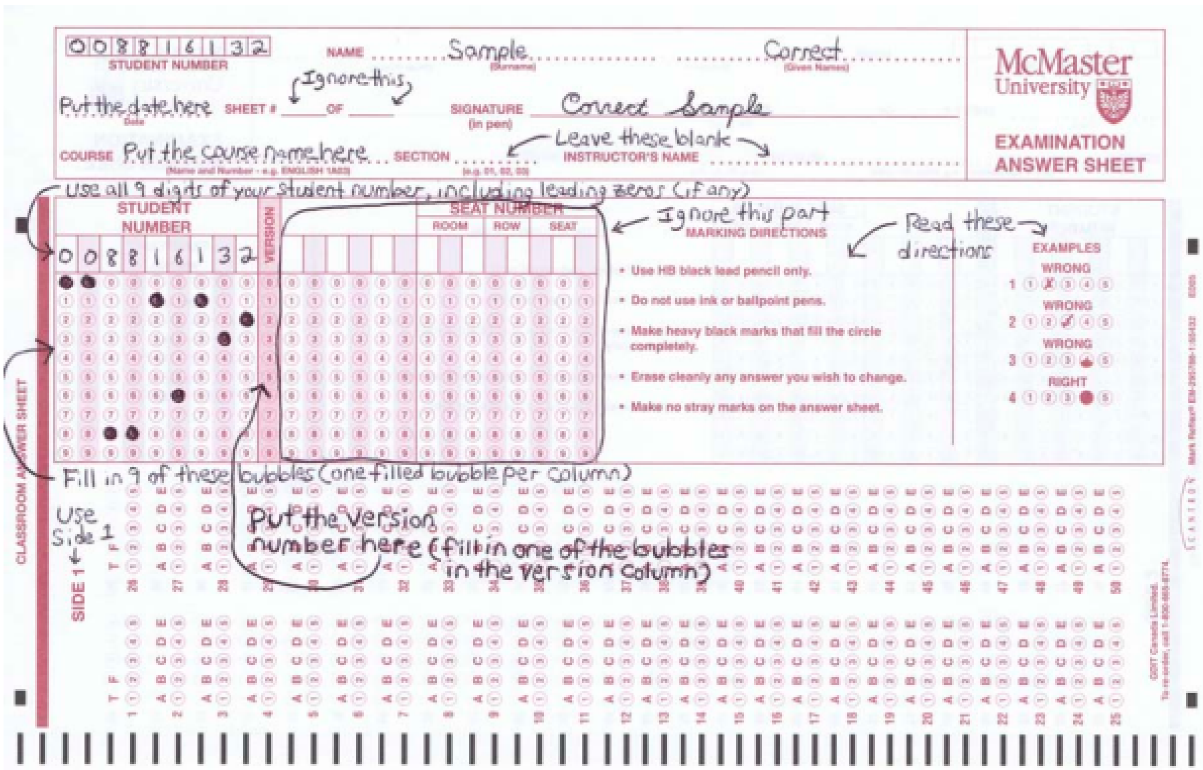
(a) $\frac{p(1-p)}{n^2}$ **(b)** $\frac{p}{n}$ **(c)** $\underbrace{\frac{p(1-p)}{n}}_{\text{this one}}$ **(d)** 0

18. The sample mean for the fill weights of 100 boxes is $\bar{x} = 12.050$. The population variance of the fill weights is known to be $\sigma^2 = (0.100)^2$. Find a 95% confidence interval for the population mean μ fill weight of the boxes.

(a) [12.030, 12.070] **(b)** [19.231, 21.170] **(c)** [11.050,13.050] **(d)** [12.020,12.080]

Continued on page 11

19. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. Note: You are writing **VERSION 1**. Hint:



END OF TEST QUESTIONS

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Formula Sheet (Page 1)

1. Addition Rule (events not mutually exclusive): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. Addition Rule (events not mutually exclusive):

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

3. Addition Rule (mutually exclusive events): $P(A \cup B) = P(A) + P(B)$

4. Multiplication Rule (dependent events): $P(A \cap B) = P(A)P(B|A)$

5. Multiplication Rule (independent events): $P(A \cap B) = P(A)P(B)$

6. Conditional Probability: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

7. Binomial: $f(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n;\quad \binom{n}{x} = \frac{n!}{x!(n-x)!};\quad \mu = np,\ \sigma^2 = np(1-p)$

8. Negative Binomial: $f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r,\ x = r, r+1, r+2, \ldots;\quad \mu = \frac{r}{p}, \sigma^2 = \frac{r(1-p)}{p^2}$

9. Hypergeometric Distribution: $f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}},\quad \mu = np,\ \sigma^2 = np(1-p)\left(\frac{N-n}{N-1}\right)$

10. Poisson Distribution: $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots;\qquad \mu = \sigma^2 = \lambda$

11. Normal Distribution: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2},\ -\infty < x < \infty$

12. Exponential Distribution: $f(x) = \lambda e^{-\lambda x},\ 0 < x < \infty;\ \mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$

13. Marginal Distributions: $f_X(x) = \int f_{XY}(x, y)\, dy,\ \ f_Y(y) = \int f_{XY}(x, y)\, dx$

14. Correlation: $\rho_{XY} = \frac{\text{cov}(X,Y)}{\sqrt{V(X)V(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

15. Transformation to standard normal: $z = \frac{X-\mu}{\sigma}$

16. Sample variance: $s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2}{n-1}$

17. Outliers: $Q_1 - 1.5\,\text{IQR}$, $Q_3 + 1.5\,\text{IQR}$

18. Normal probability plot: $\Phi(z_j) = \frac{j-0.5}{n},\ \ j = 1, 2, \ldots, n$

19. Central Limit Theorem formula: $z = \frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$

20. z confidence interval for the mean: $\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

Formula Sheet (Page 2)

21. t confidence interval for the mean: $\overline{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

22. Confidence interval for a proportion: $\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$

23. z test for a mean: $Z_0 = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$; $\quad \beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$

24. t test for a mean: $T_0 = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$ **25.** z test for proportions: $Z_0 = \frac{X-np_0}{\sqrt{np_0(1-p_0)}} = \frac{\widehat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$;

$$\beta = \Phi\left(\frac{p_0-p+z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0-p-z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

Confidence interval for a difference in means:

26. Variances equal: $\overline{x}_1 - \overline{x}_2 \pm t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$, $\quad s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

27. Variances unequal: $\overline{x}_1 - \overline{x}_2 \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$, $\quad \nu = \left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2 \Big/ \left(\frac{(s_1^2/n_1)^2}{n_1-1}+\frac{(s_2^2/n_2)^2}{n_2-1}\right)$

28. t test for comparing two means (variances equal): $t_{n_1+n_2-2} = \frac{\overline{x}_1-\overline{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

29. t test for comparing two means (variances unequal): $t_\nu = \frac{\overline{x}_1-\overline{x}_2}{\sqrt{(s_1^2/n_1)+(s_2^2/n_2)}}$,

30. Single variable Least Squares Regression line: $\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1\overline{x}$, $\widehat{\beta}_1 = \frac{s_{xy}}{s_{xx}}$, where

$s_{xx} = \sum_{i=1}^{n} x_i^2 - n\overline{x}^2$ and $s_{xy} = \sum_{i=1}^{n} x_i y_i - n\overline{x}\,\overline{y}$

31. t-test for single variable regression: $t_{n-2} = \frac{\widehat{\beta}_1}{\sqrt{\widehat{\sigma}^2/s_{xx}}}$, where $\widehat{\sigma}^2 = \frac{SS_E}{n-2}$

32. Residual sum of squares: $SS_E = \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$ **33.** Regression sum of squares: $SS_R = \widehat{\beta}_1 s_{xy}$

34. Total sum of squares: $SS_T = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$

35. Prediction Interval: $\widehat{y}_0 \pm t_{\alpha/2,n-2}\sqrt{\widehat{\sigma}^2\left(1+\frac{1}{n}+\frac{(x_0-\overline{x})^2}{s_{xx}}\right)}$

36. Sample correlation coefficient: $r = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}}$ **37.** Coefficient of determination: $R^2 = \frac{SS_R}{SS_T}$

38. Total sum of squares: $SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij} - \overline{y}_{\bullet\bullet})^2 = \sum_{i=1}^{a}\sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{\bullet\bullet}^2}{N}$ (d.f. $= N-1$)

39. Error sum of squares: $SS_E = \sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij} - \overline{y}_{i\bullet})^2 = \sum_{i=1}^{a}(n_i - 1)s_i^2$ (d.f. $= N-a$)

40. Treatment sum of squares: $SS_{\text{Treatments}} = \sum_{i=1}^{a} n_i(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2 = \sum_{i=1}^{a}\frac{y_{i\bullet}^2}{n_i} - \frac{y_{\bullet\bullet}^2}{N}$ (d.f. $= a-1$)

41. Fisher's LSD Test: $\text{LSD} = t_{\alpha/2,N-a}\sqrt{MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$, **42.** Fisher's CI: $\overline{y}_{i\bullet} - \overline{y}_{j\bullet} \pm \text{LSD}$

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

END OF TEST PAPER