1LS3: Week 1 (lectures 2 & 3)

1. What do we understand for *mathematical model*?
2. Example of mathematical model: Zika virus.
3. Continuos-time and discrete-time dynamical systems.
4. Linear models. Examples.
5. Relations (observation) and functions (prediction). Examples.

Suggested reading: sections 1.2, 1.3 and 2.1 from textbook.

1. What do we understand for *mathematical model*?

A mathematical model is not a very good looking mathematician…

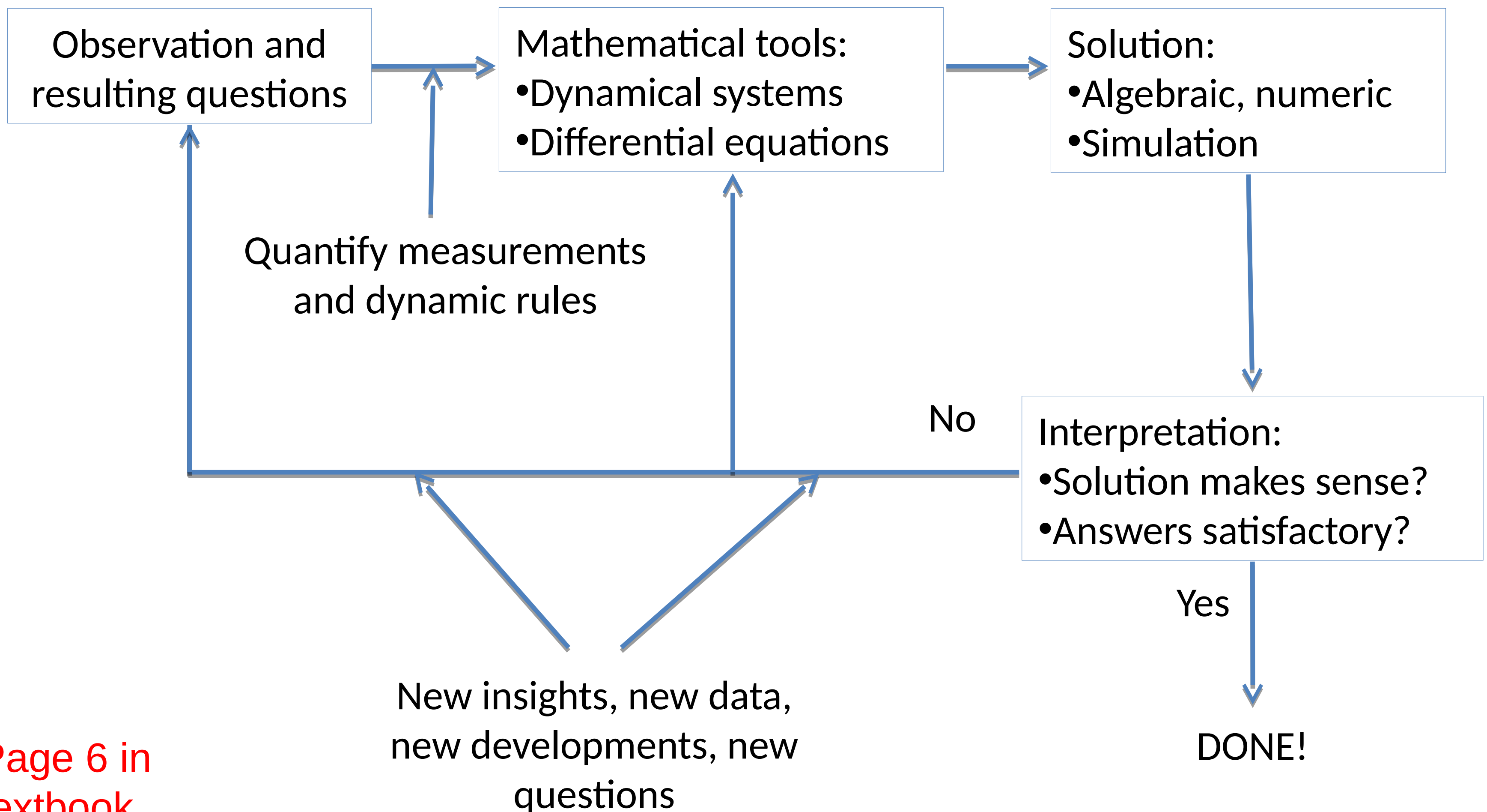
Definition by the Collins English Dictionary:

Mathematical model: a representation of a system, process, etc, in mathematical terms.

Definition by Wikipedia:

A mathematical model is a description of a system using mathematical concepts and language. (we could find many others…)

The cycle of mathematical models:



Page 6 in textbook.

2. Example of mathematical model: Zika virus.

Zika virus (ZIKV) outbreak

Research needed to understand the spread of infection, so that it can be controlled effectively

Zika virus (ZIKV) outbreak – math model

Prevention and Control of Zika as a Mosquito-Borne and Sexually Transmitted Disease: A Mathematical Modeling Analysis

Daozhou Gao, Yijun Lou, Daihai He, Travis C. Porco, Yang Kuang, Gerardo Chowell & Shigui Ruan

Scientific Reports **6**, Article number: 28070 (2016)
doi:10.1038/srep28070

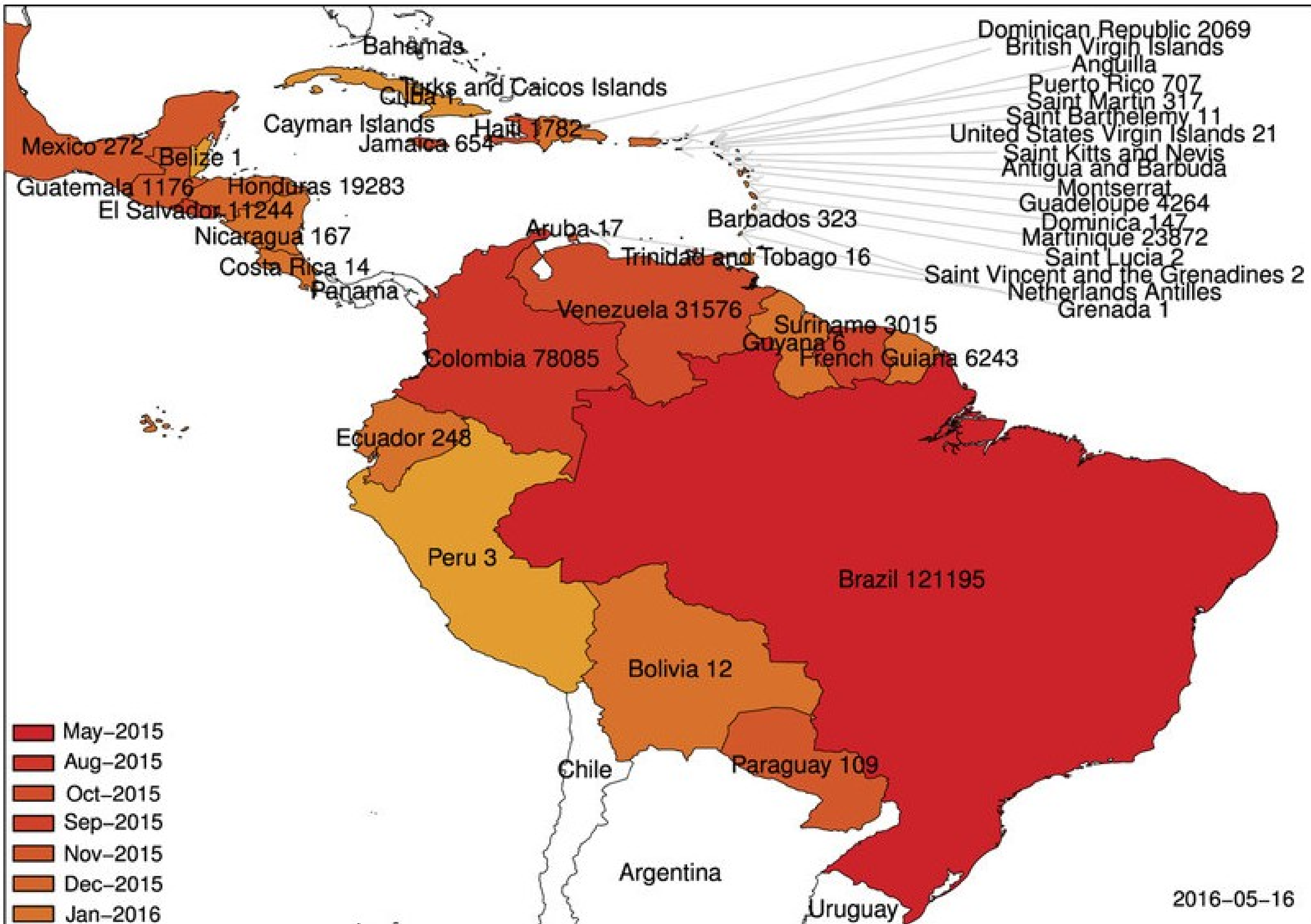
Received: 13 April 2016
Accepted: 31 May 2016
Published online: 17 June 2016

new research since previous models are not adequate

Next slide: number of cases, as of January 2016

Figure 2

From: Prevention and Control of Zika as a Mosquito-Borne and Sexually Transmitted Disease: A Mathematical Modeling Analysis



Research problem:

The ongoing Zika virus (ZIKV) epidemic in the Americas poses a major global public health emergency. While ZIKV is transmitted from human to human by bites of *Aedes* mosquitoes, recent evidence indicates that ZIKV can also be transmitted via sexual contact with cases of sexually transmitted ZIKV reported in Argentina, Canada, Chile, France, Italy, New Zealand, Peru, Portugal, and the USA.

Authors introduce a mathematical model in order to investigate the impact of mosquito-borne and sexual transmission on the spread and control of ZIKV and calibrate the model to ZIKV epidemic data from Brazil, Colombia, and El Salvador.

to *calibrate* means to assign numeric values to parameters (details follow)

Research question: which of the two routes

Mosquito-bourne or sexual transmission

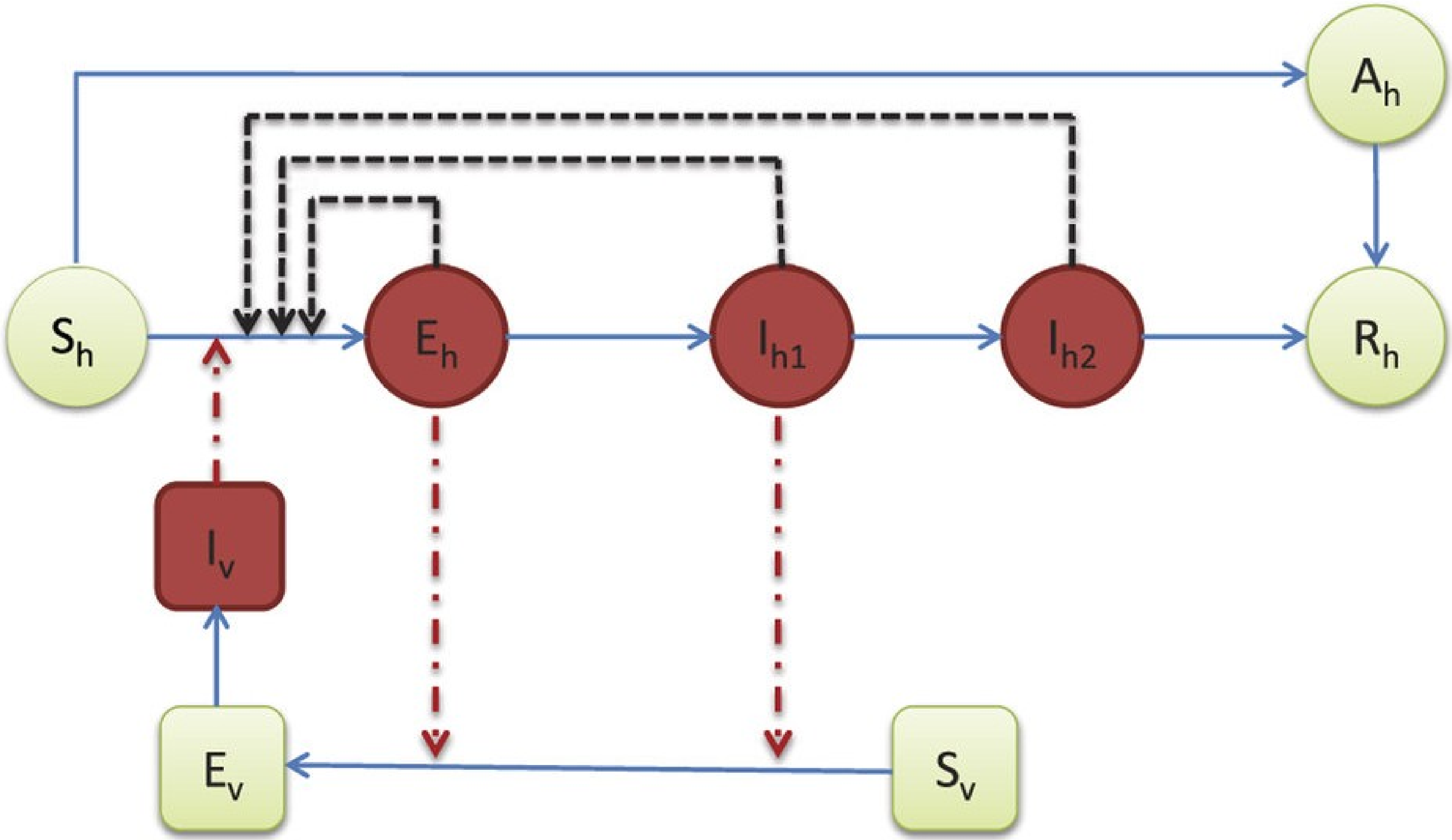
to target in order to control and prevent the spread of ZIKV?

Building a math model

Divide the human and mosquito populations into groups

Susceptible
Exposed
Infected
Convalescent
Recovered

Investigate and develop mathematical relationships between these groups …



Green nodes are non-infectious and red nodes are infectious. Blue solid arrows show the progression of infection. Black dashed arrows show direction of human-to-human transmission and red dash-dotted lines show direction of transmission between humans and mosquitoes. An individual may progress from susceptible (S_h) to asymptomatically infected (A_h) to recovered (R_h), or exposed (E_h) to symptomatically infected (I_{h1}) to convalescent (I_{h2}) to recovered (R_h), while a mosquito may progress from susceptible (S_v) to exposed (E_v) to infectious (I_v).

$$
\begin{aligned}
\frac{dS_h}{dt} &= -ab\frac{I_v}{N_h}S_h - \beta\frac{\kappa E_h + I_{h1} + \tau I_{h2}}{N_h}S_h,\\
\frac{dE_h}{dt} &= \theta\left(ab\frac{I_v}{N_h}S_h + \beta\frac{\kappa E_h + I_{h1} + \tau I_{h2}}{N_h}S_h\right) - \nu_h E_h,\\
\frac{dI_{h1}}{dt} &= \nu_h E_h - \gamma_{h1} I_{h1},\\
\frac{dI_{h2}}{dt} &= \gamma_{h1} I_{h1} - \gamma_{h2} I_{h2},\\
\frac{dA_h}{dt} &= (1-\theta)\left(ab\frac{I_v}{N_h}S_h + \beta\frac{\kappa E_h + I_{h1} + \tau I_{h2}}{N_h}S_h\right) - \gamma_h A_h,\\
\frac{dR_h}{dt} &= \gamma_{h2} I_{h2} + \gamma_h A_h,\\
\frac{dS_v}{dt} &= \mu_v N_v - ac\frac{\eta E_h + I_{h1}}{N_h}S_v - \mu_v S_v,\\
\frac{dE_v}{dt} &= ac\frac{\eta E_h + I_{h1}}{N_h}S_v - (\nu_v + \mu_v)E_v,\\
\frac{dI_v}{dt} &= \nu_v E_v - \mu_v I_v.
\end{aligned}
$$

We will learn math which will help us understand all this!

Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

derivative=rate of change, so this equation describes how

$$I_{h1} = \text{number of symptomatically infected people}$$

changes over time (*t* is time in days)

$$E_h = \text{number of exposed people}$$

Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

The numbers v_h and γ_{h1}

are constants, called parameters

In the article, we find numeric values of parameters (figuring out these values is called calibration)

reduction in mosquito population and 50% reduction in unprotected sex behaviors). In all scenarios, $b = 0.4$, $c = 0.5$, $\eta = 0.1$, $\varkappa = 0.6$, $\tau = 0.3$, $\theta = 0.18$, $\nu_h = 1/5$, $\nu_v = 1/10$, $\gamma_{h1} = 1/5$, $\gamma_{h2} = 1/20$, $\gamma_h = 1/7$, $\mu_v = 1/14$. The three columns refer to controllable parameters, basic reproduction numbers,

Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

This term is positive, i.e., the number of symptomatically infected people will increase due to the exposed people

[positive derivative means increase!]

Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

The rate of change is a multiple of

$$E_h = \text{number of exposed people}$$

thus, it is *proportional* to the number of exposed people

Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

This term is negative, i.e., in the absence of exposed people (ignore the first term), the number of symptomatically infected people will decrease

[negative derivative means decrease!]

Look at the third equation

$$\frac{dI_{h1}}{dt} = -\gamma_{h1} I_{h1}$$

This is an example of exponential decay (more about it in lectures).

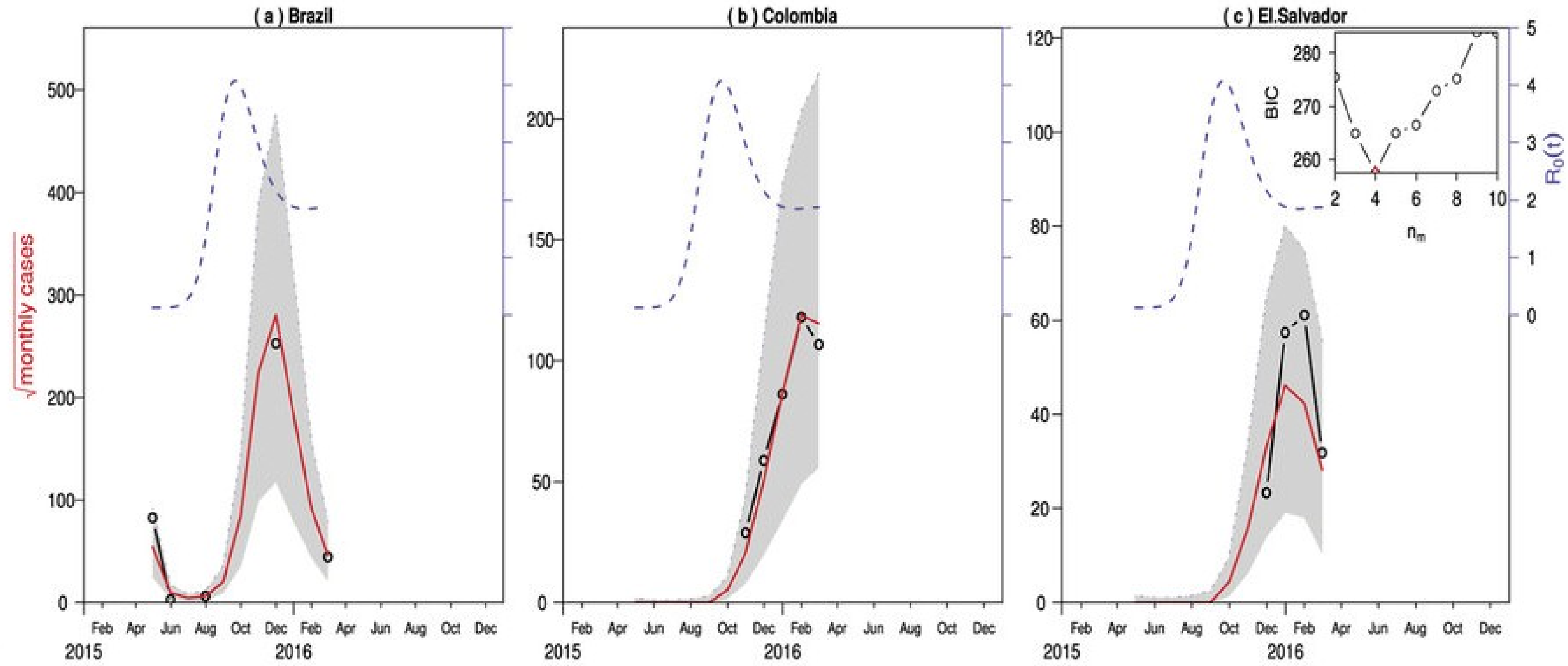
Look at the third equation

$$\frac{dI_{h1}}{dt} = v_h E_h - \gamma_{h1} I_{h1}$$

this is called a differential equation

to solve, we have to un-apply the derivative, i.e., we have to use integration

So, is the model realistic?



(B)

(A) ZIKV outbreaks in South and Central Americas. The map indicates the month of first reported cases and the cumulative cases by May 16, 2016, in each country. The map was made with the free software "R: A Language and Environment for Statistical Computing, R Core Team, R Foundation for Statistical Computing, Vienna, Austria (2016) https://www.R-project.org." accessed on February 1, 2016. (B) Fitting model to data in Brazil, Colombia, and El Salvador up to February 27, 2016. Each panel shows the simulation (red solid curve) versus the observed (black circle), with the best fitting parameters. The red solid curves show median values of 1000 simulations and shaded region show the 95% range. The blue dash curves show the estimated mosquito-human population ratio $m(t)$. The inset panel shows Bayesian Information Criterion (BIC) as a

Model = red solid curve, observed = black circles

Research question: which of the two routes mosquito-bourne or sexual transmission to target in order to control and prevent the spread of ZIKV?

Conclusion:

Sexual activity transmission is about 4.5% of all transmissions, BUT culling large fraction of mosquito population might not suffice in the absence of sexual risk-reduction.

Important message:

We need to learn math in order to understand a vastly increasing number of publications in biology and health sciences which use mathematics and statistics.

Even if you do not plan to become a researcher, you will need to read and understand all kinds of documents, manuals, and reports which use quantitative information.

Is this going to be on the test?

Yes, but only after we cover all math that's necessary

We will be explicit about what each test covers (as well, there are sample tests in your coursepack)

3. Continuos-time and discrete-time dynamical systems

- Dynamical system: system that evolves with time. (we will use “model” or “dynamical system” indistinctively, but the word “model” is somewhat more general…)

- Time and variables can be discrete or continuos. (Example: Infected people with flu → BB)

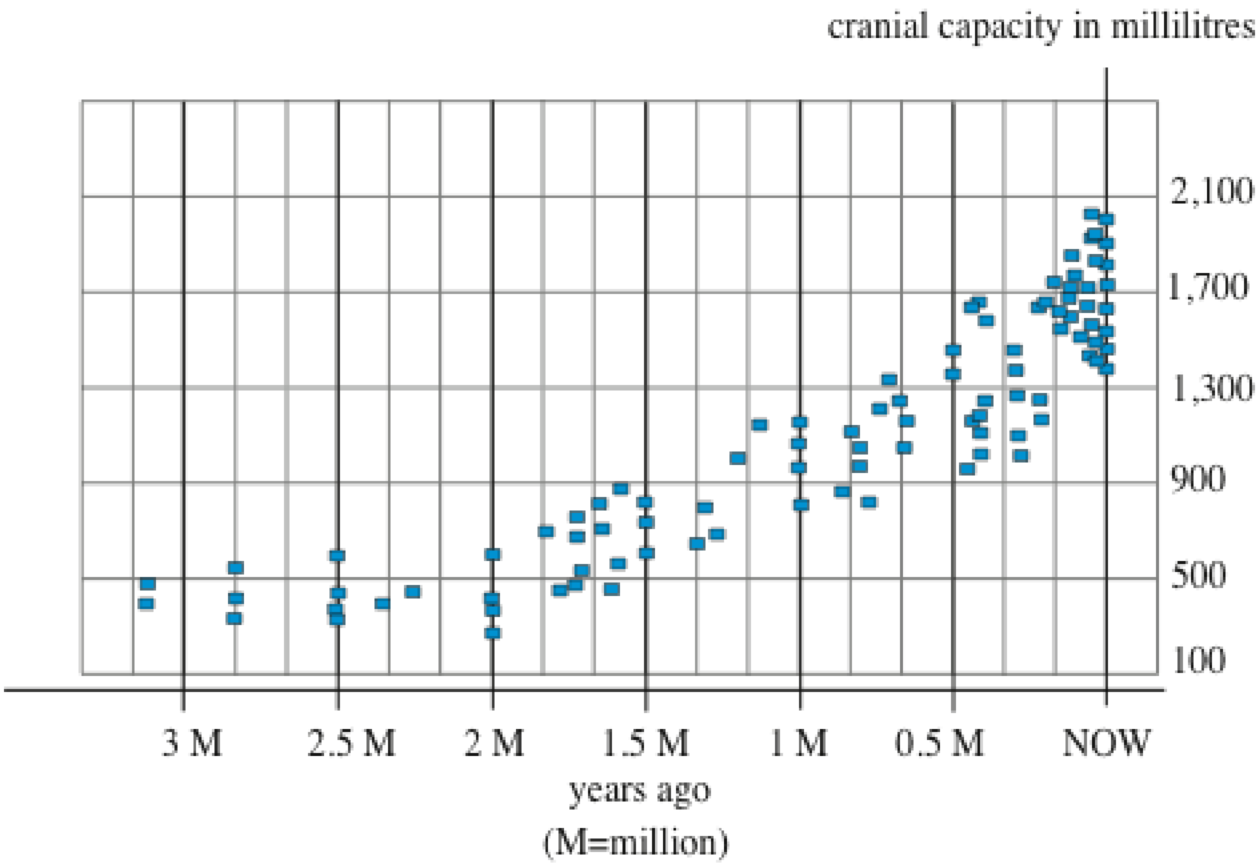
4. Linear Models

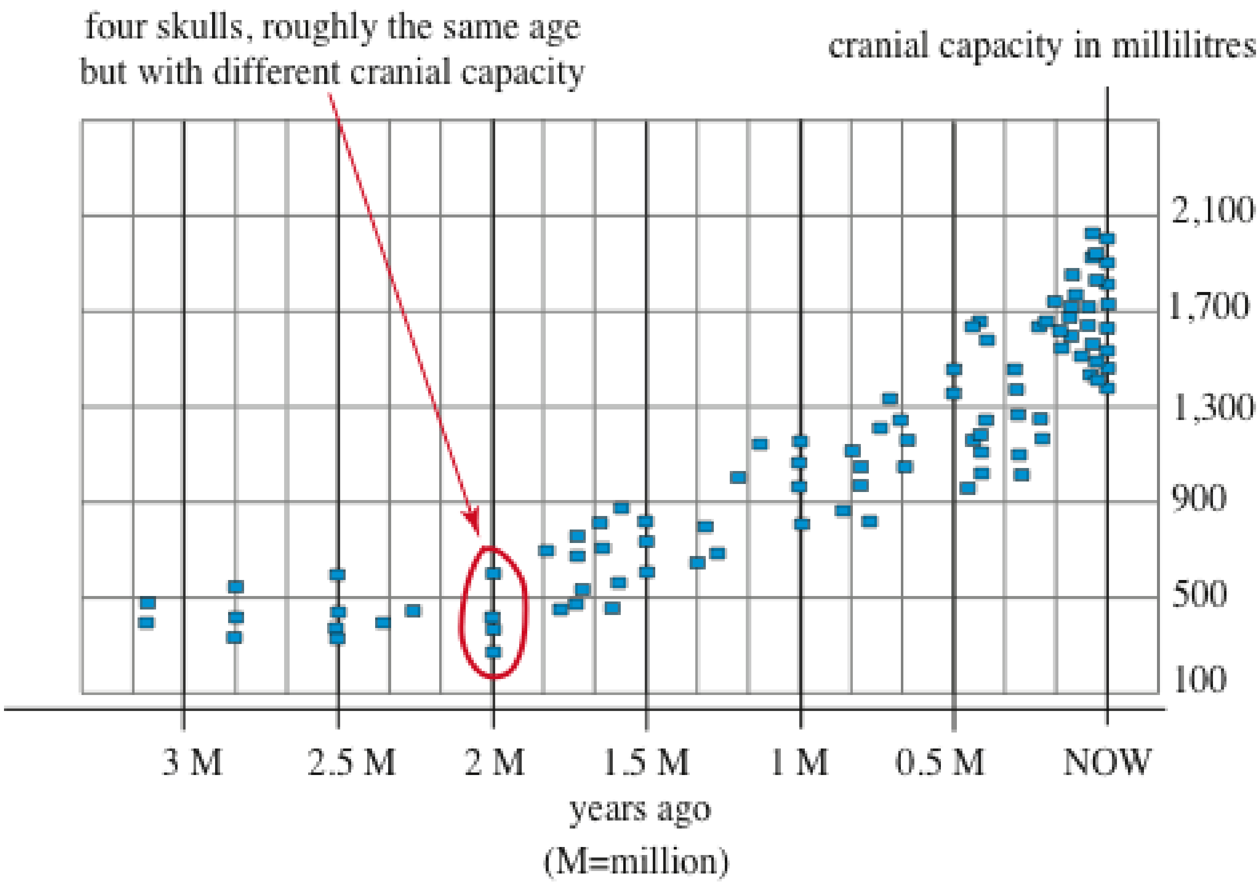
- Linear function. (in BB)
- Proportional relationship. (in BB)
- Example of linear model: Linear model for the population of Canada. (in BB)
- Example of proportional relationship: Proportional relation between mass and Volume. (in BB)

5. Relations (observation) and functions (prediction). Examples.

Most (almost all) DATA collected in life sciences reflect a RELATIONSHIP, but not necessarily a specific FUNCTION

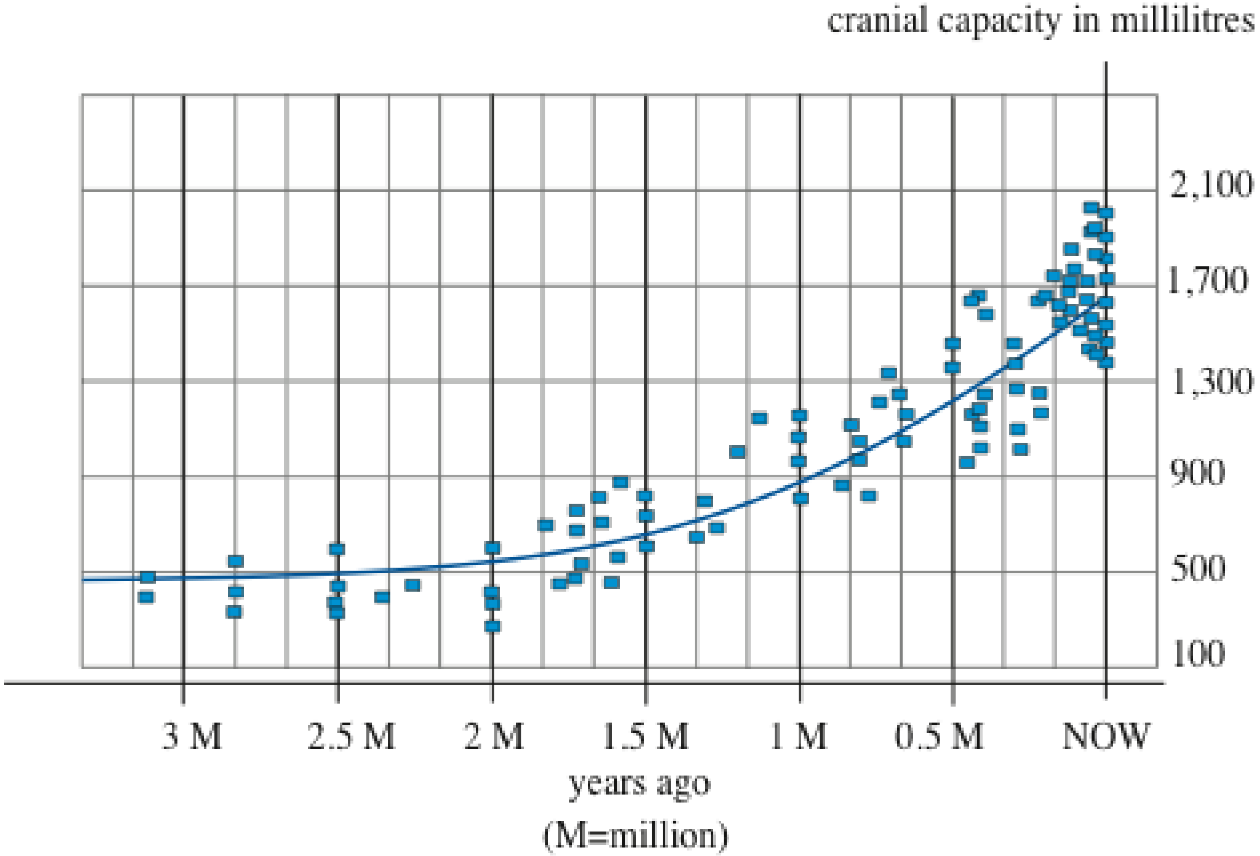
Example: the graph on the next slide shows the cranial capacity (i.e., the brain volume) calculated from the skulls of early humans and modern humans, between 3 million years in the past and today


cranial capacity in millilitres
2,100
1,700
1,300
900
500
100
3 M
2.5 M
2 M
1.5 M
1 M
0.5 M
NOW
years ago
(M=million)

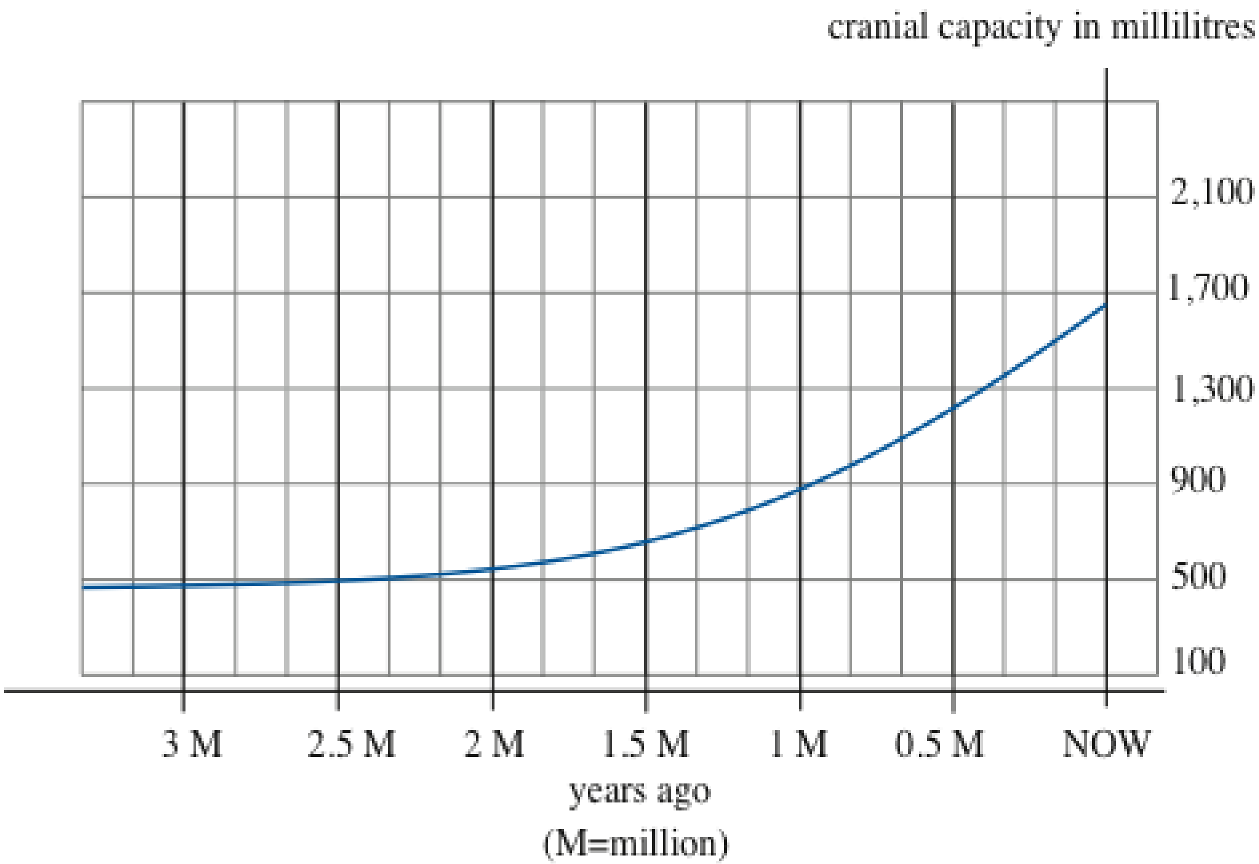


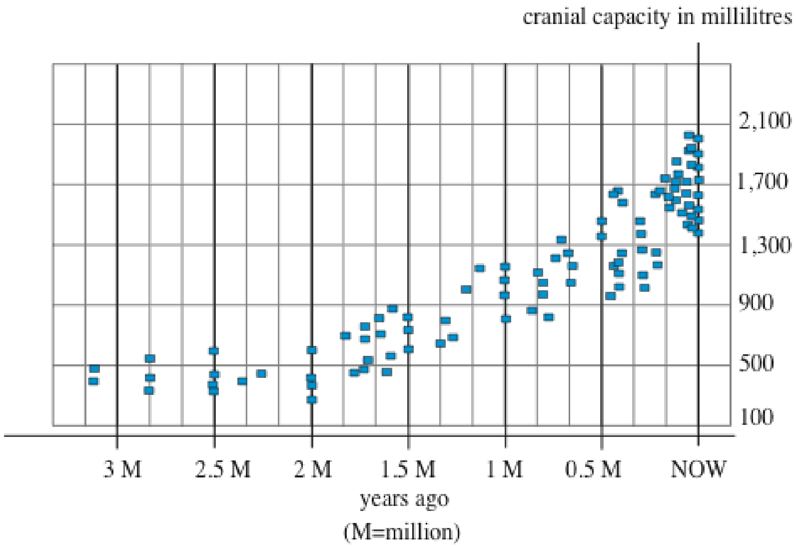
This diagram shows a relationship (sometimes called a *relation*), but it is not a function

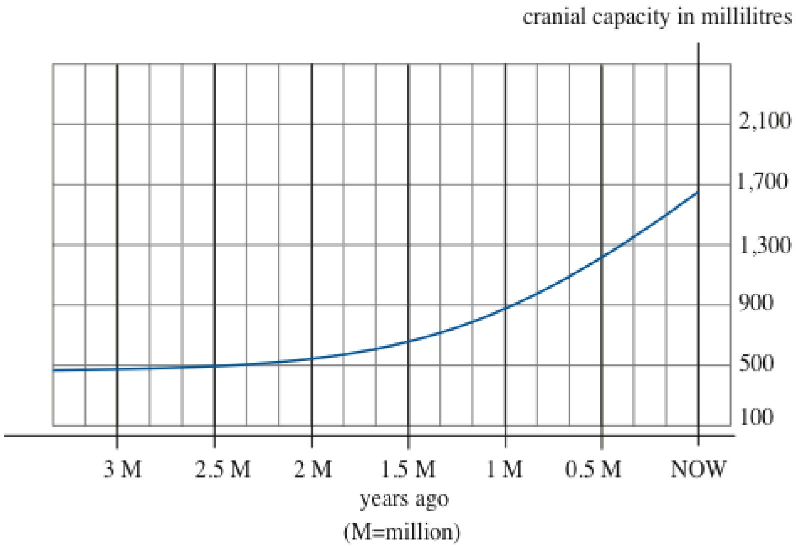
Using statistical methods such as regression (these methods are covered in statistics courses in levels 2 and above), we can identify a function which approximates the data



And then we work with the function we obtained.
Why? Because we have no choice. In order to obtain the quantitative results desired in our research in the life sciences, we need to model the relationship with a function.







Of course, we can say something … for example, the data on the left suggests some kind of exponential growth. But in order to quantify that growth, and further work with it, we need to have a function

Examples of functions that model relationships:

- Example 1: (in BB)
  Heartbeat frequency in mammals.

- Example 2: (in BB)
  Body mass index.

When BMI makes no sense …

9-year
old girl Ana

Source:
http://www.cnn.com/video/data/2.0/video/us/2014/05/26/mxp-wabc-bmi-school-program-girl-overweight.hln.html

According to her school administrators, Ana is overweight …

NORMAL GROWTH PATTERN FOR KIDS

height (m)	weight (kg)	BMI	
1.24	28		
1.25	30		
1.27	31		
1.30	31		
1.33	32		

Ana: height = 4 ft 1 in = 1.25 m

mass = 66 lb = 29.94 kg = 30kg

BMI=mass/height^2 = 19.2

height (m)	weight (kg)	BMI	
1.24	28		
1.25	30	19.2	
1.27	31		
1.30	31		
1.33	32		

height (m)	weight (kg)	BMI	
1.24	28	18.2	
1.25	30	19.2	
1.27	31	19.2	
1.30	31	18.3	
1.33	32	18.1	

height (m)	weight (kg)	BMI	
1.24	28	18.2	normal
1.25	30	19.2	overweight
1.27	31	19.2	overweight
1.30	31	18.3	normal
1.33	32	18.1	normal

PATTERNS OF CHILD GROWTH VARY SIGNIFICANTLY – NO STRAIGHTFORWARD RELATION BETWEEN HEIGHT AND WEIGHT

REASONING BASED ON BMI DOES NOT APPLY TO CHILDREN !!!

According to her school administrators,
Ana is overweight …

NONSENSE !!!!!!!!

This kind of reasoning only hurts children, yet the above school officials DID NOT CHANGE their opinion. They continue using BMI to “help students get healthy.”