Definite Integral

Definition (Definite Integral). Let $f(x)$ be a continuous function defined on $[a, b]$. The **definite integral of** $f(x)$ **on** $[a, b]$ is the real number

$$\int_a^b f(x)dx = \lim_{n\to\infty} S_n\,. \tag{1}$$

Comments:

- Now, we are including the case $f(x) < 0$, so the definite integral does not represent the area under the curve any more. Now,

$$\int_a^b f(x)dx = \text{area above x-axis} - \text{area below the x-axis}\,. \tag{2}$$

- In the notation, a and b are called **limits of integration** (a is the **lower limit** and b the **upper limit**), and $f(x)$ is called the **integrand**.
- The Riemann sum has the same definition. The rectangles under the x-axis contribute negatively to the sum.

Properties of Integrals

Properties of Integrals:

$$\int_a^a f(x)dx = 0\,, \tag{3}$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx\,, \tag{4}$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx\,, \tag{5}$$

$$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx\,, \tag{6}$$

$$\int_a^b cdx = c(b-a)\,, \tag{7}$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\,, \tag{8}$$

If $m \leq f(x) \leq M$ for all x in $[a, b]$, then

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)\,. \tag{9}$$