

Substitution and Integration by Parts

Substitution rule for indefinite integrals (Theorem): Assume that $f(x)$ and $g(x)$ are such that the composition $f(g(x))$ is defined, and that $f(x)$ and $g'(x)$ are continuous. If $u = g(x)$, then

$$\int f(g(x))g'(x)dx = \int f(u)du. \quad (1)$$

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$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du. \quad (2)$$

Integration by parts:

$$\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx \quad (3)$$

$$\int_a^b u(x)v'(x)dx = u(x)v(x)|_a^b - \int_a^b v(x)u'(x)dx \quad (4)$$