

Discrete-Time Dynamical Systems

A **discrete-time dynamical system** (DTDS) gives a relation between the **present** (m_t) and the **future** (m_{t+1}) value of a quantity or measurement (m). The relation between m_t and m_{t+1} is given by the **updating function**, $f(m_t)$. So, a DTDS takes the form

$$m_{t+1} = f(m_t). \quad (1)$$

Comment: As in Initial Value Problems we need to specify an initial condition, m_0 , to solve the problem.

Definition (Solution of DTDS). The sequence of values of m_t for $t = 0, 1, 2, \dots$ is the solution of the discrete-time dynamical system $m_{t+1} = f(m_t)$ starting from the initial condition m_0 .

Basic Exponential DTDS. If we have the DTDS $b_{t+1} = rb_t$ where the initial condition is given by b_0 , then the solution is $b_t = b_0 r^t$.

Basic Additive DTDS. If we have the DTDS $h_{t+1} = h_t + a$ where the initial condition is given by h_0 , then the solution is $h_t = h_0 + at$.