

## Stability of DTDS

**Theorem (Stability of DTDS).** Suppose that the discrete-time dynamical system  $x_{t+1} = f(x_t)$  has an equilibrium at  $x^*$ . The equilibrium at  $x^*$  is **stable** if

$$|f'(x^*)| < 1 \quad (1)$$

and **unstable** if

$$|f'(x^*)| > 1. \quad (2)$$

**Comment:** If  $f'(x^*) = 1$ , then  $x^*$  is neither stable nor unstable.

**Example:** Consider the DTDS

$$x_{t+1} = \frac{1}{2}x_t + 1 \quad (3)$$

which has an equilibrium at  $x^* = 2$ . The updating function and the derivative are

$$f(x) = \frac{1}{2}x + 1, \quad f'(x) = \frac{1}{2}. \quad (4)$$

Then

$$|f'(x^*)| = \left| \frac{1}{2} \right| < 1 \implies x^* \text{ is stable.} \quad (5)$$