

Stability of DTDS

Theorem (Stability of DTDS). Suppose that the discrete-time dynamical system $x_{x+1} = f(x_t)$ has an equilibrium at x^* . The equilibrium at x^* is **stable** if

$$|f'(x^*)| < 1 \quad (1)$$

and **unstable** if

$$|f'(x^*)| > 1. \quad (2)$$

Comment: If $f'(x^*) = 1$, then x^* is neither stable nor unstable.

Example: Consider the DTDS

$$x_{t+1} = \frac{1}{2}x_t + 1 \quad (3)$$

which has an equilibrium at $x^* = 2$. The updating function and the derivative are

$$f(x) = \frac{1}{2}x + 1, \quad f'(x) = \frac{1}{2}. \quad (4)$$

Then

$$|f'(x^*)| = \left| \frac{1}{2} \right| < 1 \implies x^* \text{ is stable.} \quad (5)$$