How to read and understand math in journal articles, books about science, and other sources?

Math in math textbooks is not exactly the same as math found elsewhere (we will see examples soon). Although the differences are mathematically insignificant (such as different notation), it takes some time to get used to math written in a non-textbook format

What do we do?

We learn concepts, formulas and algorithms using math textbooks, because it is easier that way

Then we use our knowledge to apply to contexts takes from various disciplines

Example – Journal Article

Forensic Science, Medicine, and Pathology

ISSN 1547-769X/07/3:33–40/$30.00 (Online) 1556-2891
DOI: 10.1385/Forensic Sci. Med. Pathol.:3:1:33

ORIGINAL ARTICLE

The Estimation of Blood Alcohol Concentration

Widmark Revisited

Douglas Posey[1] and Ashraf Mozayani[2]

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[2]Harris Country Medical Examiner's Office, Houston, TX

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Accepted for publication:
July 21, 2006

Abstract

Expert witnesses and others involved in toxicology are frequently asked to perform retrograde extrapolation of blood alcohol concentration (BAC) or to estimate BAC based on a proposed drinking scenario. Although many individuals are reluctant to perform these calculations and some jurisdictions expressly prohibit them, a significant number of practitioners routinely estimate BAC based on this type of calculation, using as a basis the fundamental work of Widmark. Although improvements to the Widmark formula and other data pertaining to the pharmacology of alcohol have been published, these improvements are frequently ignored when estimating BAC. This article summarizes five published models for the estimation of BAC and proposes a sixth model that incorporates recent data on the rate of absorption of alcohol from the GI tract into the existing five models. The five improved models can be computerized and used to construct comparative snapshots of the BACs calculated by the different algorithms. This will allow practitioners to provide a more balanced picture of the variability in BAC calculations.

Key Words: Forensic toxicology; Widmark; extrapolation; alcohol; ethanol; blood.

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INTRODUCTION

Driving under the influence of alcohol (DUI) is a persistent worldwide problem. Violators are subject to monetary fines, loss of driving privileges, arrest, and imprisonment. Owing to the potential serious consequences of DUI, many of these cases require appearances in court. A significant percentage of DUI trials involve the testimony of expert witnesses who in many instances are asked to extrapolate blood alcohol concentration (BAC) at a previous time based on laboratory BAC results (retrograde extrapolation), or to predict a BAC based on a particular drinking scenario.

Although some experts in the field discourage the practice of predicting BAC levels, the fact is that virtually every expert witness who testifies in DUI cases has been asked to do just that, and most of those that agree to perform the calculation rely on the Widmark equation *(1)*. This equation is based on elimination rates and a factor known as the "Widmark factor," ***r***, that were established by E. M. P. Widmark in 1932 by examining a number of men and women and determining average values for the elimination rate and ***r***.

The purpose of this article is to discuss the use of the Widmark equation in BAC calculations and to suggest improvements to the use of that equation.

ALCOHOL PHARMACOKINETICS

Absorption

The absorption of alcohol from the stomach has been recently studied *(2)*. The first order rate constant for the absorption of alcohol from an empty stomach was found to average 6.5 ± 1.5 hours^{-1} which translates to a half-life of 0.1066 hours and would indicate that 99% of the alcohol is absorbed in about 0.75 hours (45 minutes). At the other extreme, assuming that the absorption of 99% of the alcohol from a full stomach takes 2 hours, the rate constant is about 2.3 hours^{-1} and the half-life about 0.3009 hours. Many calculations are based on a general rule of thumb that says that alcohol is completely absorbed somewhere between 30 minutes and 2 hours after the ingestion of the alcohol. This assumption is valid if the time since the last drink is more than 2 hours, but is of little help for recent ingestions of alcohol (i.e., within the past hour).

What is in it?

Source, title, author and contact information

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Introduction and body of the article

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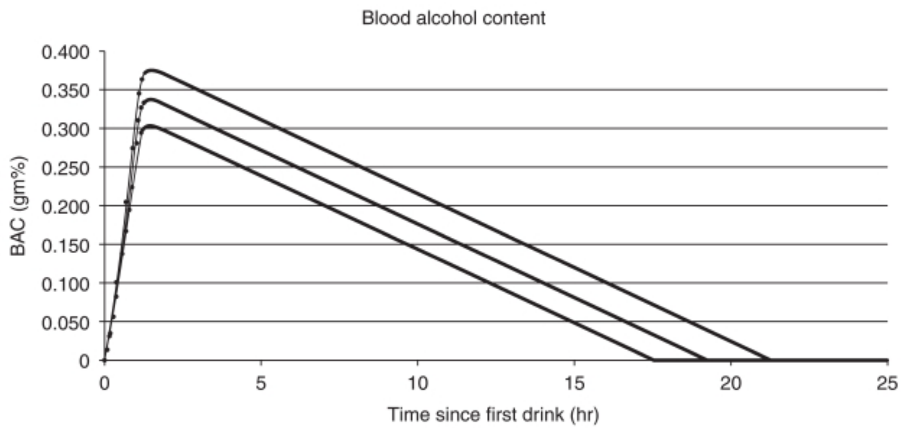
Math parts – diagrams (graphs), tables …



Legend: Upper curve = BAC calculated from r_{avg} using Widmark parameters
Middle curve = BAC calculated from r_{avg} using Watson parameters
Lower curve = BAC calculated from r_{avg} using Forrest and Seidl parameters. The Forrest and Seidl curves are nearly coincident.

Fig. 1. BAC curves for example 2. Legend: Upper curve = BAC calculated from $\boldsymbol{r}_{avg}$ using Widmark parameters Middle curve = BAC calculated from $\boldsymbol{r}_{avg}$ using Watson parameters Lower curve = BAC calculated from $\boldsymbol{r}_{avg}$ using Forrest and Seidl parameters. The Forrest and Seidl curves are nearly coincident.

EXAMPLE 2

A 23-year-old female, height 5′4″ (163 cm) and weight 126 pounds (57.3 kg) consumed one shot of 80 proof liquor every 6 minutes for 1 hour, beginning at 9 PM, for a total of 11 shots. Each shot was assumed to contain 1.25 ounces of liquor. Figure 1 shows the result of calculating the BAC using an elimination rate of 0.018 g% hour^{-1}, an absorption rate constant of 6.5 hours^{-1} and Widmark's average $\boldsymbol{r}$ value for females of 0.55. Table 2 shows the magnitude and time of the peak BAC. The lower curve on Fig. 1 is composed of both the Seidl

Table 2
Comparison of Time of Example 2 Peak BAC by Five Models

Peak BAC (g%)	*Time of peak*	*Method*
0.376	10:30 PM	Widmark *(1)*
0.338	10:30 PM	Watson et al. *(3)*
0.306	10:30 PM	Forrest *(4)*
0.305	10:30 PM	Seidl et al. *(5)*
N/A for females		Ulrich et al. *(6)*

Math parts – formulas

Distribution

Most BAC calculations are based on the Widmark formula,

$$C = \frac{A}{rW} - (\beta t) \qquad (1)$$

where C is the blood alcohol concentration; A is the mass of alcohol consumed; r is the Widmark factor; W is body weight; t is elapsed time since the start of drinking; and β is the elimination rate.

The Widmark factor r is variable and depends on body mass, percentage body fat, age, and sex. Widmark tested a large number of individuals and found that the average value of r for males is 0.68 ± 0.17 and for females is 0.55 ± 0.11. Researchers after Widmark *(3–6)* attempted to improve the Widmark formula, primarily by developing methods to more

Seidl et al. *(8)* gathered height (in centimeters), weight (in kilograms), blood water content, and TBW data for 256 women and 273 men and used the data to fine tune the Widmark equation according to the formulae

$$r_{Seidl}\text{ (female)} = 0.31223 - 0.006446W + 0.4466H$$

$$r_{Seidl}\text{ (male)} = 0.31608 - 0.004821W + 0.4632H$$

Ulrich, Cramer, and Zink *(9)* developed an equation to determine the Widmark r value based on experiments with 386 males (weight in kilograms and height in meters), viz.

$$r_{Ulrich}\text{(male)} = 0.715 - 0.00462W + 0.22H$$

Ulrich used only male subjects, so the resulting BAC parameters are not applicable to calculations of BAC in females.

Elimination

End … conclusion and references

BAC curves by calculating BAC at frequent small time intervals. The computer can also easily incorporate the first-order absorption of ethanol from the gut.

CONCLUSION

From these examples, two significant phenomena are obvious. The first is that the sex (and thus the Widmark value) contribute to a significant difference in the calculated BAC value and thus the magnitude of the Widmark value must be accurately estimated. The second is that the simple Widmark calculation used by many experts results in a higher BAC than any of the other methods, whereas the other methods tend to agree with each other and not with Widmark. The primary reason for the elevated numbers obtained when using the Widmark ***r*** value is that the other methods calculate ***r*** based on measured parameters rather than relying on the average value obtained from a small group of individuals in 1932.

Many alcohol experts are reluctant to perform retrograde extrapolations or calculate estimated BAC values. For those that do perform these calculations, however, it seems apparent that a "one equation fits all" approach is inadequate in an adversarial legal proceeding. Expert witnesses and any other individual who calculates BAC values should look at several different methods for calculating BAC. Any necessary assumptions should be based as much as possible on scientific data and any calculations should include all three factors affecting the final BAC (i.e., absorption, distribution, and elimination). The utilization of an average ***r*** value provides a more balanced prediction of the BAC at a given time, whereas the utilization of a computer program that calculates the BAC at intervals of six minutes provides a simple, accurate way to predict the magnitude of the estimated BAC at any particular time, based on easily available physical measurements. Although the approach may be somewhat more mathematically challenging, it is easily computerized and will result in a more uniform, and perhaps more accurate, reflection of the true BAC at the time in question.

Educational Message

1. Improved algorithm for the estimation of BAC at relatively high concentrations.
2. Expression of the need to include several established methods when estimating BAC.

ACKNOWLEDGMENTS

The authors would like to acknowledge Farrokh Tabeee for his mathematical and computer assistance.

The authors have stated that they do not have a significant financial interest or other relationship with any product manufacturer or provider of services discussed in this article.

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Our focus is on math parts

Distribution

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Elimination

Compare to formulas in our textbook - what's different?

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Elimination

Lots of symbols! Some uppercase, some lowercase, Greek alphabet

No f(x) notation … we need to read the text to figure out what symbols represent dependent and independent variables, and what symbols represent parameters

As well, what is a variable and what is a parameter might change within the article (as in the textbook example on the body mass index)

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Elimination

Many parameters are not given numeric values

Numeric values are approximate, written in the form 0.68 plus/minus 0.17

Not all text is mathematically relevant (big difference from textbook exercises and examples!)

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Elimination

Parentheses around the second term on the right side are not necessary (and in math we would not use them)

Another example

The following excerpt is taken from

The emergence of environmental homeostasis in complex ecosystems. James G. Dyke and Iain S. Weaver. PLoS Computational Biology. 9.5 (May 2013)

This analysis assumes that values of K are very large. Numerical simulations allow us to explore the behaviour of the model as K is increased from 1. We find that beyond a threshold value of K, the expected number of times F changes sign and so the expected number of stable points remains constant. The threshold value of K is approximately

$$K \approx \left(\frac{5R}{\sigma_E} \right)^N . \qquad (7)$$

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Parts of above text are not relevant for this formula
Some information is missing (we need to read more text to figure things out)

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We do not need to know much about the quantities involved to figure out math (for instance, K is an exponential function of N; no need to know what a threshold value actually is)

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$$K \approx \left(\frac{5R}{\sigma_E}\right)^N. \tag{7}$$

Symbols: Greek letter, subscript
Approximate sign (nothing in real world has an exact value – big difference between math and reality!)

Back to

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Elimination

How do we make ourselves understand this formula? I.e., what do we have to know?

$$C = \frac{A}{rW} - \beta t$$

If *C* is a function and *t* is independent variable:

(1) Which quantities are parameters
(2) Replace all parameters by numbers – do you recognize the equation?
(3) Keep the parameters (do not give them numeric values); what is the graph of *C* (you will need to make assumptions, for instance β could be positive, negative, or zero)

(4) If *C* is a function of *A*, what is its graph?
(5) If *C* is a function of *r*, what is its graph?
(6) If *C* is a function of *W*, what is its graph?