Functions approaching ∞ at ∞

Definition: Suppose that

$$\lim_{x\to\infty} f(x) = \infty\,, \quad \lim_{x\to\infty} g(x) = \infty$$

1. The function $f(x)$ approaches infinity **more quickly** than $g(x)$ as x approaches infinity if
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$$
2. The function $f(x)$ approaches infinity **more slowly** than $g(x)$ if
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$
3. $f(x)$ and $g(x)$ approach infinity at the **same rate** if
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$$
where L is any real number other than 0.

Functions approaching 0 at ∞

Definition: Suppose that

$$\lim_{x \to \infty} f(x) = 0\,, \quad \lim_{x \to \infty} g(x) = 0$$

1. The function $f(x)$ approaches zero **more quickly** than $g(x)$ as x approaches infinity if
$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$$
2. The function $f(x)$ approaches zero **more slowly** than $g(x)$ as x approaches infinity if
$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \pm\infty$$
3. $f(x)$ approaches zero at the **same rate** as $g(x)$ if
$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$$
where L is any real number other than 0.