Summary of continuity theorems

1. The **basic continuous functions** are:

$$f(x) = c \text{ } (c \text{ is constant}), \quad f(x) = x, \quad f(x) = e^x,$$
$$f(x) = \ln x \text{ } (\text{for } x > 0), \quad f(x) = |x|, \quad f(x) = \sin x, \quad f(x) = \cos x$$

2. **Combining continuous functions**. Assume that the functions $f(x)$ and $g(x)$ are continuous at a. Then:
   - The sum, $f(x) + g(x)$, and the difference, $f(x) - g(x)$, are continuous at a.
   - The products $c\,f(x)$ (where c is a real number) and $f(x)g(x)$ are continuous at a.
   - The quotient $f(x)/g(x)$ is continuous at a if $g(a) \neq 0$.

3. **Continuity of composition of functions**.
   - If $g(x)$ is continuous at a and $f(x)$ is continuous at $g(a)$, then the composition $(f \circ g)(x) = f(g(x))$ is continuous at a.

Summary of continuity theorems

1. **Continuous functions**. The following functions are continuous at all numbers in their domain:
   - polynomials, rational functions, roots, algebraic functions, absolute value function, exponential and logarithmic functions, trigonometric and inverse trigonometric functions.

2. **Interchanging a limit and a continuous function**.
   - Assume that a function g satisfies

$$\lim_{x \to a} g(x) = b$$

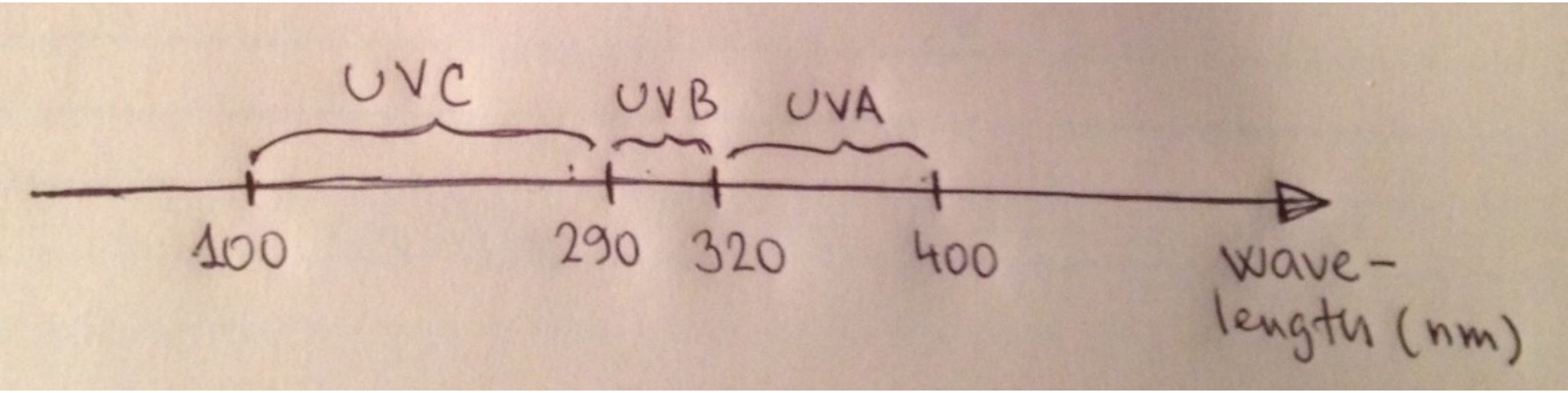
   and that a function f is continuous at b. Then,

$$\lim_{x \to a} f(g(x)) = f(b)\,.$$

UV Index

Ultraviolet (UV) light is an electromagnetic radiation (coming from the Sun, or from a tanning bed) with a wavelengths in the range 100-400 nm (nanometres; 1 nanometre = 10^{-9} metre)

UV light comes in three variants, called UVA, UVB and UVC, defined by the wavelength



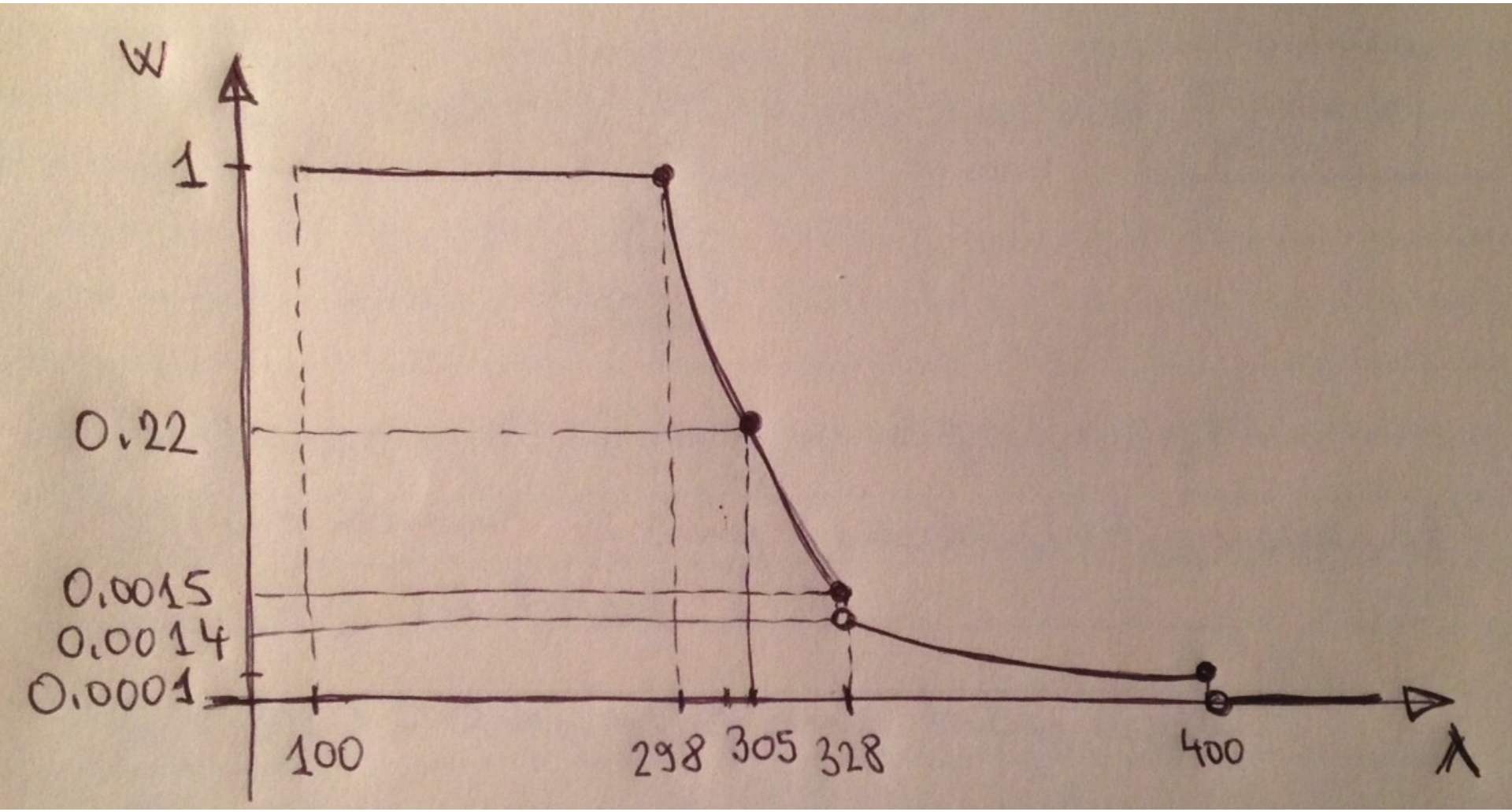
How dangerous is UV light?

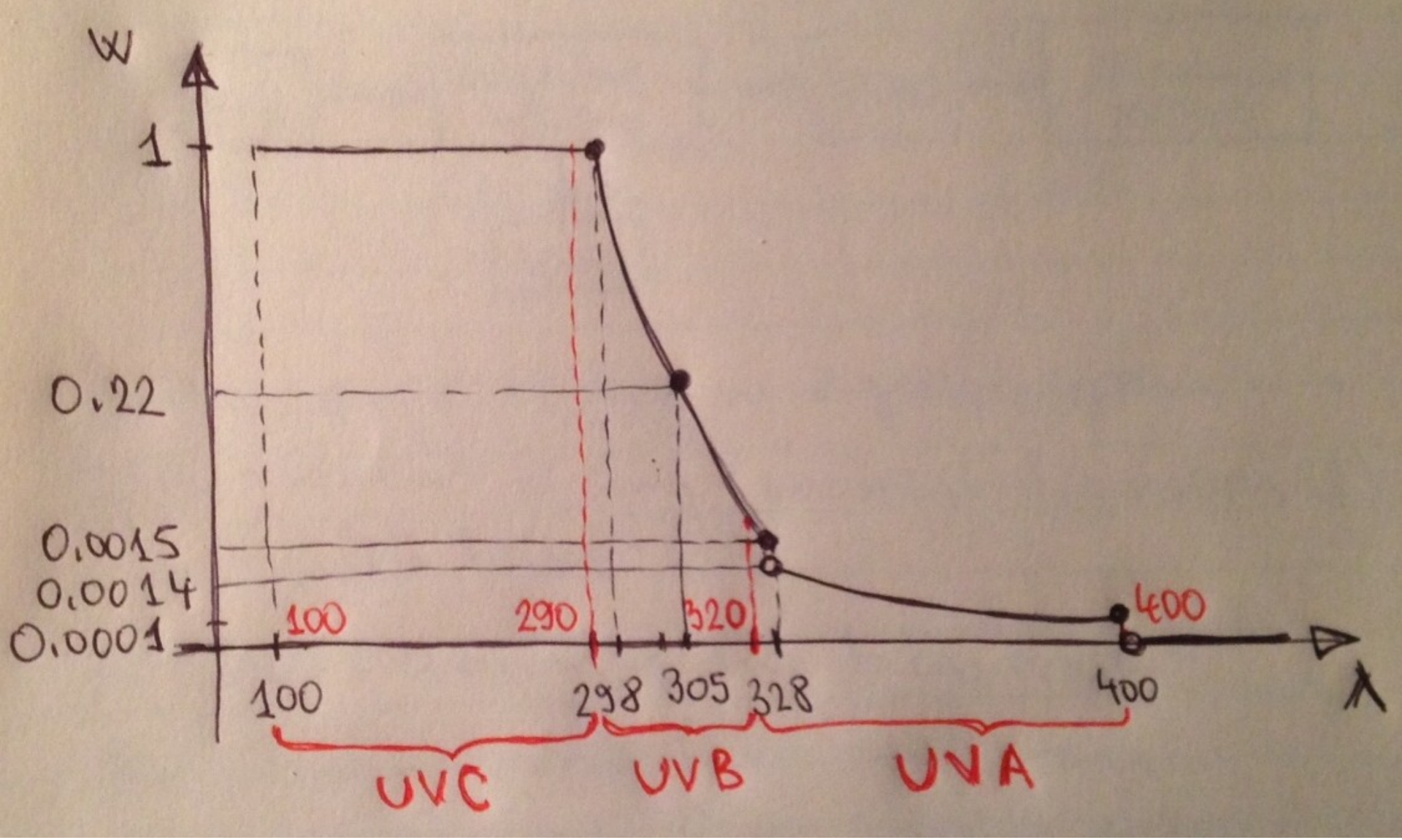
Common way to quantify the danger to humans is to use *erythemal action spectrum function* (*erythema* for short)

Erythema is a piecewise function of the wavelength:

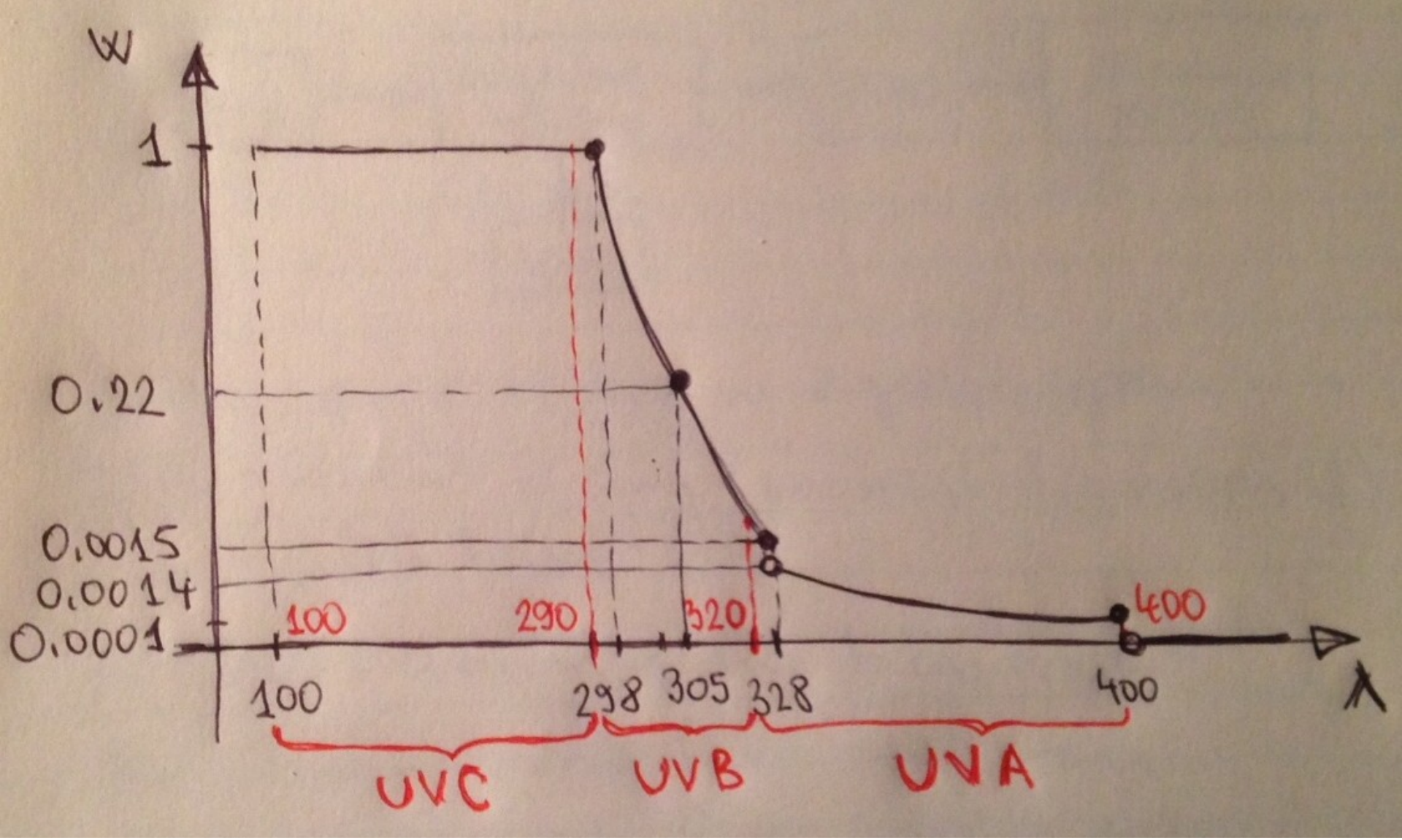
$$w(\lambda) = \begin{cases} 1 & 100 \le \lambda \le 298 \\ 10^{0.094(298-\lambda)} & 298 < \lambda \le 328 \\ 10^{0.015(139-\lambda)} & 328 < \lambda \le 400 \\ 0 & \lambda > 400 \end{cases}$$

graph of the erythema $w(\lambda)$

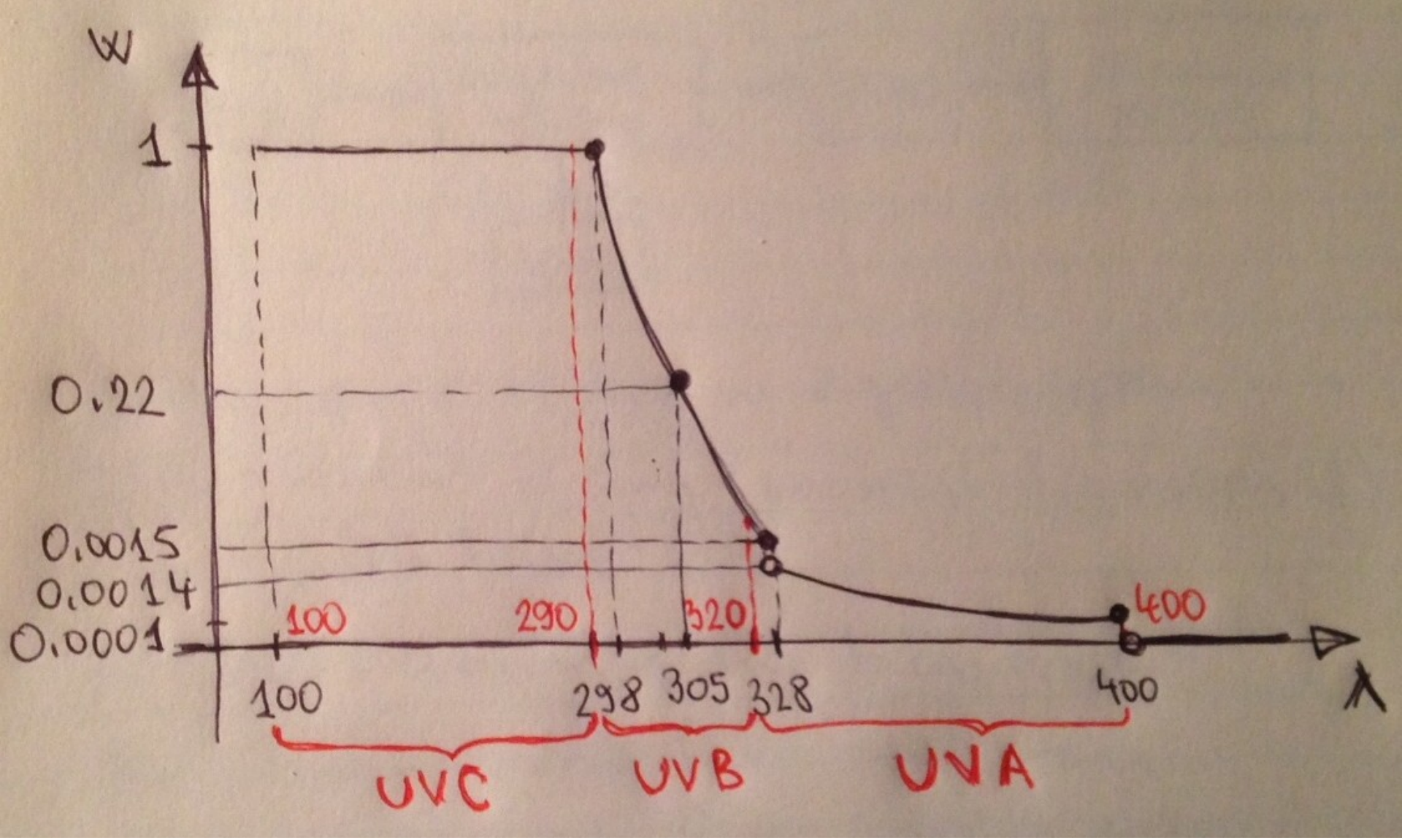




values close to 1 … extremely dangerous (severe burns on exposure, death); includes all of UVC and some of UVB



mid-range UVB: w(305) =0.22 (could cause mild to severe sunburn, eye damage)



within UVA range : w(340) =0.001 (longer exposure: suntan, sunburn, freckles, immunosupression, risk of skin cancer)

Severity of UV is affected by other factors such as:

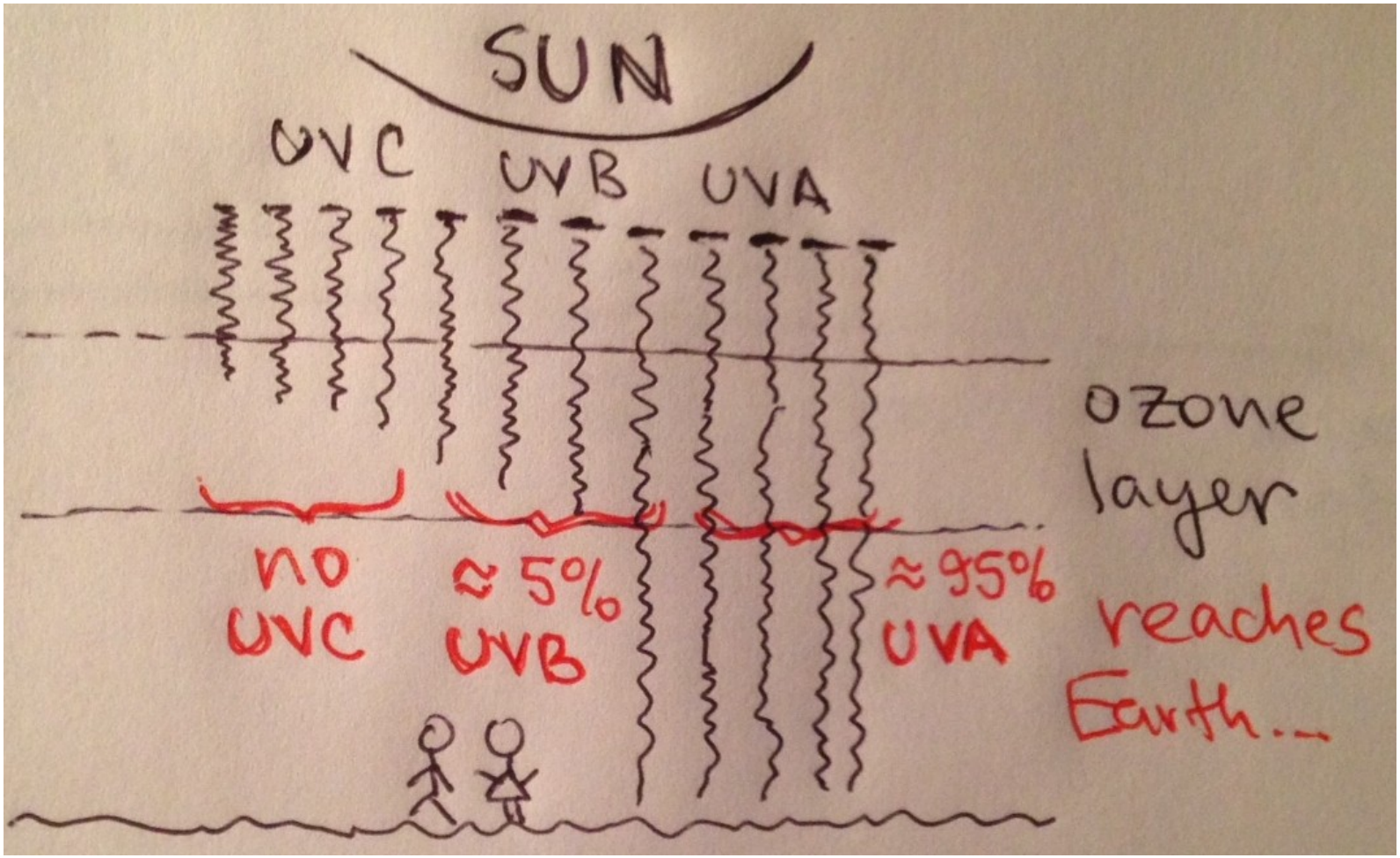
* time of day (strongest at solar noon, i.e., when the angle with respect to horizontal is the largest)

* duration of exposure

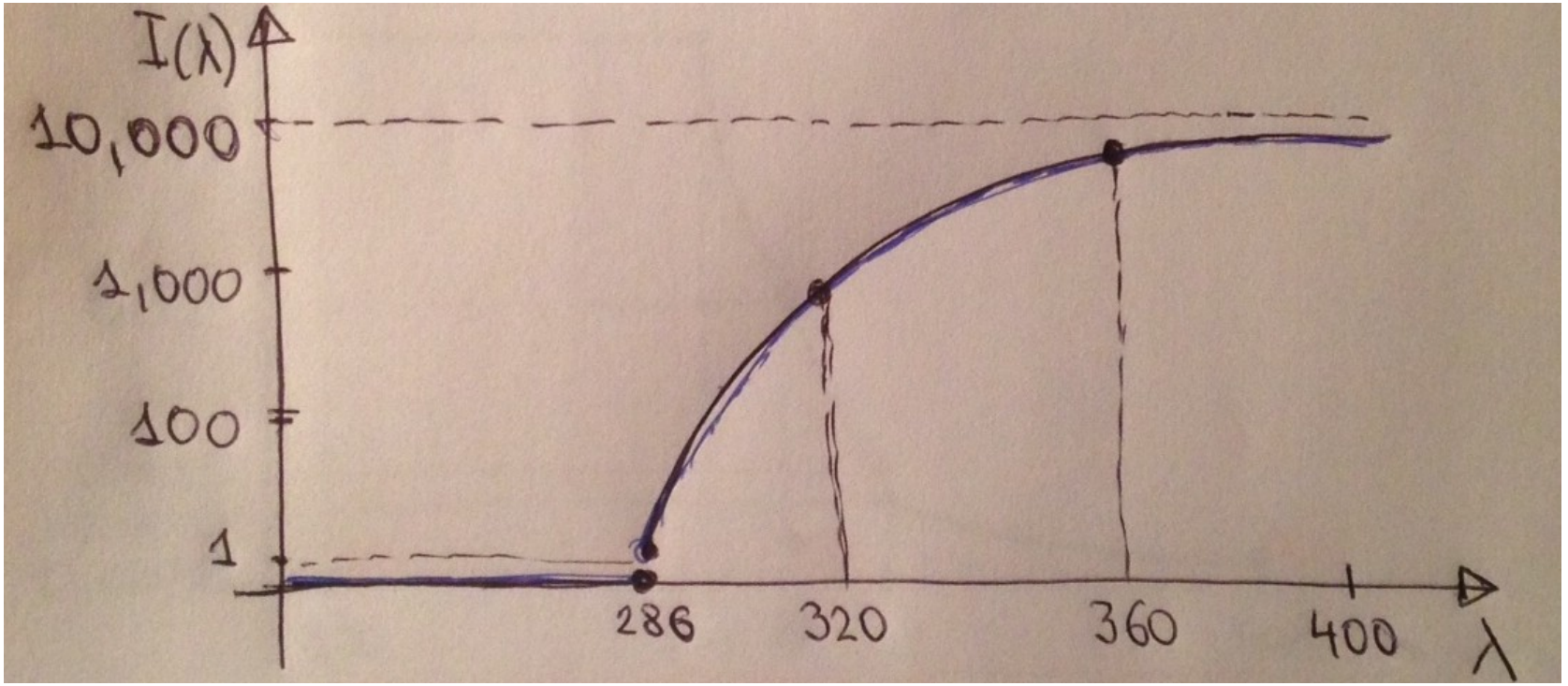
* reflection from water, snow, concrete, or sand

* skin tone

Good news: ozone blocks UV radiation!



The “quantity” of each frequency which reaches Earth is given by the irradiance or intensity function $I(\lambda)$



Definition of UV index:

$$\mathrm{UVI} = \frac{1}{25} \int_{100}^{400} w(\lambda)\, I(\lambda)\, d\lambda$$

$w(\lambda)\, I(\lambda)$ is the product of level of danger and "quantity" i.e. intensity for each frequency

we integrate over all frequencies (later: integral = area under curve)

why divide by 25: UV index was defined in 1992 by Canadian scientists (Canada is the first country in the world to include UV into weather forecasts); they obtained UV of 250 for a clear skies hot sunny summer day in downtown Toronto, and wanted to scale UV so that it goes from 0 to 10

Typical (not maximum!) summer UVI:

Toronto, Montreal, Calgary … 8
Vancouver, St John's, Winnipeg … 7
Yellowknife … 5

New Zealand (due to thin ozone) … 14
Grand Canyon, Arizona … 14
Mauna Loa, Hawaii (thin ozone, high altitude, reflection)… 18
Antarctica (thin ozone, reflection) … 16

Tanning beds … 11 or 12 !!!

Advice:

UVI > 3 … use some protection if longer exposure

use combined UVA/UVB protection (“broad spectrum sunscreen”; some protect only against UVB or only against UVA)

avoid sun around solar noon (to be safe: avoid sun as long as your shadow is smaller than your height)

Later, and in assignments, we will discuss details (and tell you what you need to know about this)