Review: derivatives of basic functions (I)

1. Constant function:

$$f(x) = c \quad \text{(where } c \text{ is a constant)} \qquad \rightarrow \qquad f'(x) = 0$$

2. Power function:

$$f(x) = x^p \quad \text{(where } p \text{ is a real number)} \qquad \rightarrow \qquad f'(x) = px^{p-1}$$

3. Natural exponential/logarithm function:

$$f(x) = e^x \qquad \rightarrow \qquad f'(x) = e^x$$

$$f(x) = \ln x \qquad \rightarrow \qquad f'(x) = \frac{1}{x}$$

4. General exponential/logarithm function:

$$f(x) = a^x \qquad \rightarrow \qquad f'(x) = a^x \ln a$$

$$f(x) = \log_a x \qquad \rightarrow \qquad f'(x) = \frac{1}{x \ln a}$$

Review: derivatives of basic functions (II)

1. Basic trigonometric functions:

$$f(x) = \sin x \quad \rightarrow \quad f'(x) = \cos x$$
$$f(x) = \cos x \quad \rightarrow \quad f'(x) = -\sin x$$
$$f(x) = \tan x \quad \rightarrow \quad f'(x) = \sec^2 x$$

2. Other trigonometric functions:

$$f(x) = \sec x \quad \rightarrow \quad f'(x) = \sec x \tan x$$
$$f(x) = \cot x \quad \rightarrow \quad f'(x) = -\csc^2 x$$
$$f(x) = \csc x \quad \rightarrow \quad f'(x) = -\csc x \cot x$$

3. Inverse trigonometric functions:

$$f(x) = \arcsin x \quad \rightarrow \quad f'(x) = \frac{1}{\sqrt{1-x^2}}$$
$$f(x) = \arctan x \quad \rightarrow \quad f'(x) = \frac{1}{1+x^2}$$

Review: derivatives of algebraic operations

Consider two differentiable functions $f(x)$ and $g(x)$. Then,

1. Multiplication by a constant:

$$(cf(x))' = cf'(x)$$

2. Sum/difference rule:

$$(f(x) + g(x))' = f'(x) + g'(x)$$

$$(f(x) - g(x))' = f'(x) - g'(x)$$

3. Product rule:

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

4. Quotient rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Chain rule & Implicit differentiation

1. **Chain rule**. Let

$$F(x) = (f \circ g)(x) = f(g(x))$$

and assume that g is differentiable at x and f differentiable at $g(x)$. Then F is differentiable at x and

$$F'(x) = f'(g(x))\, g'(x)$$

2. **Implicit differentiation**. Sometimes the relation between the dependent and independent variable might be implicit. Example:

$$x^3 + y^4 = y \sin\, x - 7$$

How do we differentiate functions that are given implicitely? We combine usual differentiation and the chain rule (BB):

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^4) = \frac{d}{dx}(y\, \sin x - 7) \quad \to \quad \frac{dy}{dx} = \frac{y\, \cos x - 3x^2}{4y^3 - \sin x}$$