Extreme values of a function (I)

Definition of absolut maximum/minimum:

- A function $f(x)$ has an **absolute maximum** at $x = c$ if $f(x) \leq f(c)$ for all x in the domain of $f(x)$. The value $f(c)$ is called the **absolute maximum value** of $f(x)$.
- A function $f(x)$ has an **absolute minimum** at $x = c$ if $f(x) \geq f(c)$ for all x in the domain of $f(x)$. The value $f(c)$ is called the **absolute minimum value** of $f(x)$.

Definition of relative maximum/minimum:

- A function $f(x)$ has a **relative maximum** at $x = c$ if $f(x) \leq f(c)$ for all x near c. The value $f(c)$ is called the **relative maximum value** of $f(x)$.
- A function $f(x)$ has a **absolute minimum** at $x = c$ if $f(x) \geq f(c)$ for all x near c. The value $f(c)$ is called the **relative minimum value** of $f(x)$.

Definition of critical point: (reminder)

- The point $x = c$ in the domain of a function $f(x)$ is called a **critical point** if $f'(c) = 0$ or if $f'(c)$ is not defined.

Extreme values of a function (II)

Fermat's Theorem:

- If $f(x)$ has a relative minimum or a relative maximum at $x = c$, and if $f'(c)$ exists, then $f'(c) = 0$.

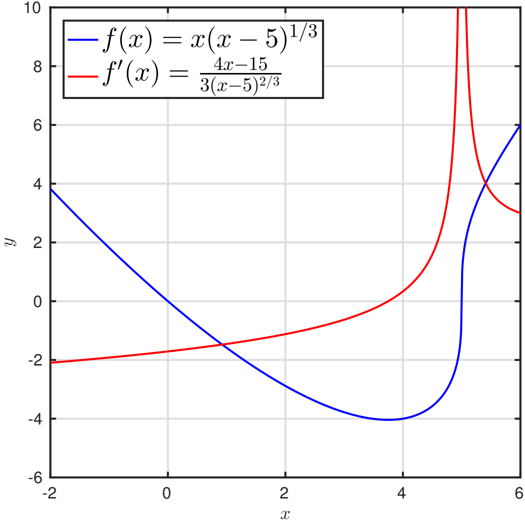
  In other words, if $f'(c) = 0$ (so c is a critical point) we may have a relative maximum/minimum.

Frist Derivative Test: Assume that $f(x)$ is continuous at $x = c$, where $x = c$ is a critical point of $f(x)$. Then,

- To ensure that $x = c$ is a relative maximum (or minimum) we need to verify that $f'(x)$ changes from positive to negative (or negative to positive) at $x = c$. If $f'(x)$ does not change sign at $x = c$ then f has neither a relative minimum nor a relative maximum at $x = c$.

Second Derivative Test: Assume that $f''(x)$ is continous near $x = c$ and $f'(c) = 0$. Then,

- If $f''(x) > 0$, then $f(x)$ has a relative minimum at $x = c$. If $f''(x) < 0$, then $f(x)$ has a relative maximum at $x = c$. If $f''(x) = 0$ the test provides no answer.



$f(x) = x(x-5)^{1/3}$
$f'(x) = \frac{4x-15}{3(x-5)^{2/3}}$
10
8
6
4
2
0
-2
-4
-6
y
-2
0
2
4
6
x