Extreme values of a function (III)

Extreme Value Theorem:

- Assume that $f(x)$ is a continuous function defined on a closed interval $[a, b]$. Then, $f(x)$ has an **absolute maximum** and an **absolute minimum** in $[a, b]$. That is, there exist numbers c_1 and c_2 in $[a, b]$ such that $f(c_1)$ is the absolute maximum and and $f(c_2)$ is the absolute minimum.

Finding Absolute Extreme Values: Assume that $f(x)$ is a continuous function defined on a closed interval $[a, b]$.

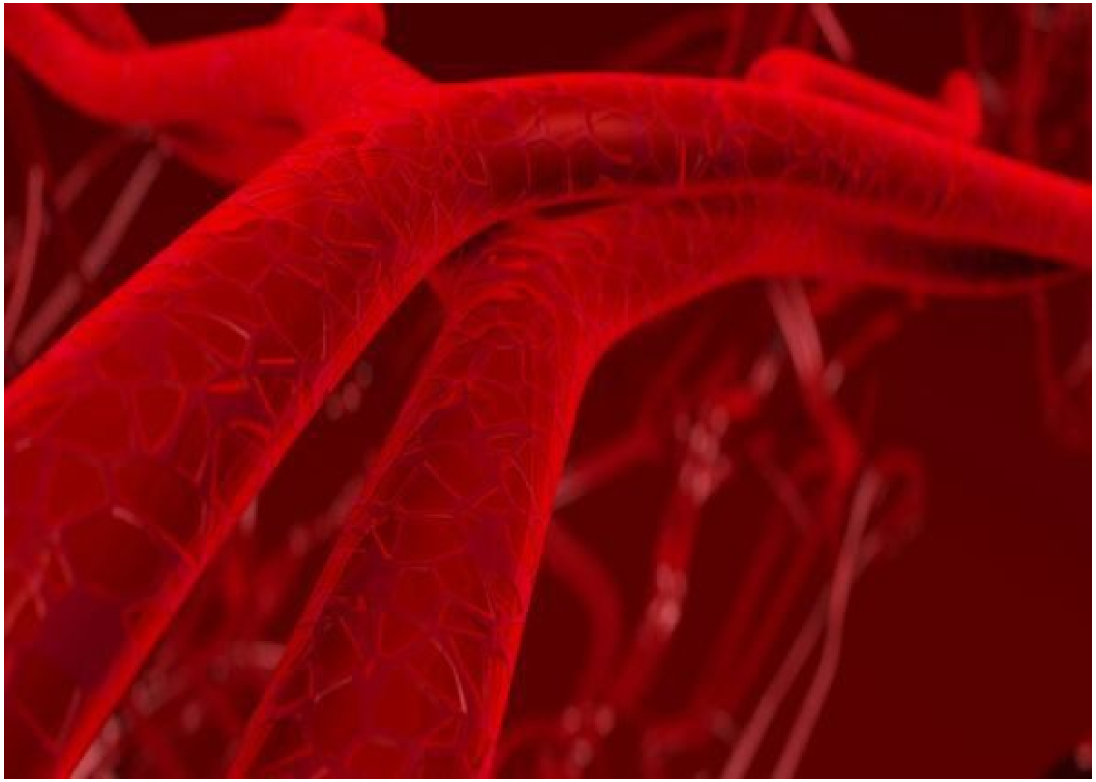
(1) Find all critical points ($f'(x) = 0$ and $f'(x)$ dne)

(2) Compute the values of $f(x)$ at all critical points.

(3) Compute the values $f(a)$ and $f(b)$.

(4) The largest value found in (2) and (3) is the absolute maximum, and the smallest the absolute minimum.

Practical application of critical points and extreme values: Vascular Branching



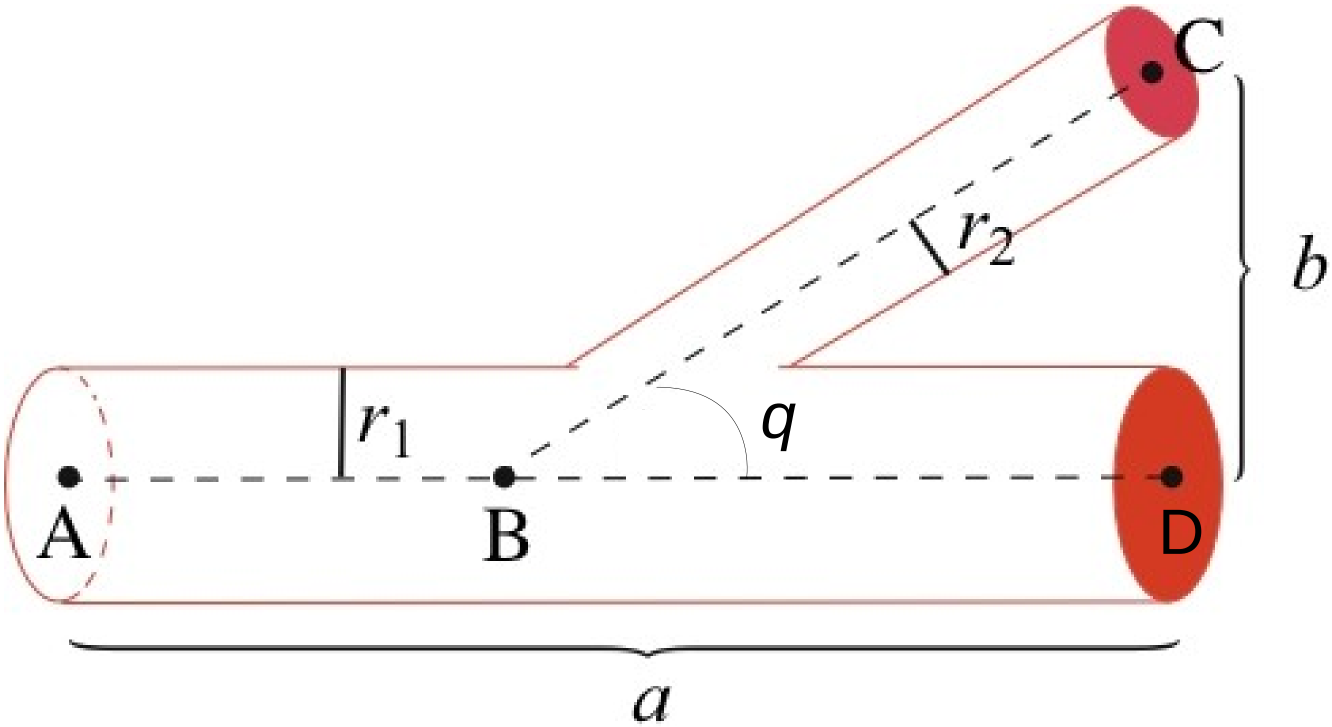
Poiseuille’s Law:

resistance *R* is proportional to length of the tube *L* and inversely proportional to the fourth power of its radius *r*

$$R = C\frac{L}{r^4}$$

vascular branching:

objective: minimize resistance, thus minimizing energy (i.e. minimizing work the heart needs to produce)



$$R(q) = C\left(\frac{a - b\cot q}{r_1^4} + \frac{b\csc q}{r_2^4}\right)$$

critical point:

$$\cos q = \frac{r_2^4}{r_1^4}$$

ratio of sizes of vessels (ratio of their radii)	optimal branching angle $q = \arccos \frac{r_2^4}{r_1^4}$
0.9	0.85 rad = 49.0 degrees
0.8	1.15 rad = 65.8 degrees
0.67=2/3	1.37 rad = 78.6 degrees
0.5	1.51 rad = 86.4 degrees