L'Hôpital's Rule (I)

Definition of indeterminate forms:

- The limit of the ratio

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

(where a can be $\pm\infty$) is called an **indeterminate form of type** $0/0$ if

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = 0\,,$$

and an **indeterminate form of type** ∞/∞ if

$$\lim_{x \to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty\,.$$

L'Hôpital's Rule (II)

Theorem. L'Hôpital's Rule:

- Suppose that $f(x)$ and $g(x)$ are differentiable functions such that

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is an indeterminate form of type $0/0$ or ∞/∞. If $g'(x) \neq 0$ near a (could be 0 at a) then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

provided that the limit on the right side exists (i.e., is equal to a real number), or is equal to $-\infty$ or $+\infty$.

Differential Equations

Table : Finding the rate of change by differentiating a value or measurement.

Value (measured)	Differentiate	Rate of Change (comptued)
position	$\rightarrow$	speed
mass	$\rightarrow$	rate of change of mass (growth rate)
amount of sodium	$\rightarrow$	rate sodium enters a cell
population size	$\rightarrow$	rate of change of population size

Table : Find a value or measurement from the rate of change (by solving the differential equation).

Rate of Change (measured)	Solve Differential Equation	Value (comptued)
speed	$\rightarrow$	position
rate of change of mass (growth rate)	$\rightarrow$	mass
rate sodium enters a cell	$\rightarrow$	amount of sodium
rate of change of population size	$\rightarrow$	population size