Differential Equations

There are two popular types of differential equations:

- **Pure-time Differential Equations**: Only the independent variable appears on the right hand side.

  Examples:

  $$\frac{df(t)}{dt} = sin(t), \quad \frac{df(t)}{dt} = t + e^t, ... \tag{1}$$

- **Autonomous Differential Equations**: The independent variable does not appear explicitly on the right hand side.

  Examples:

  $$\frac{df(t)}{dt} = f(t), \quad \frac{df}{dt} = f(t)^3 + cos(f(t)), ... \tag{2}$$

Note: Even if the independent variable does not represent time, a differential equation can still be calissified as pure-time differential equation. For instance,

$$\frac{dg(x)}{dx} = x^3. \tag{3}$$

Euler's method to solve DEs

Algorithm for Euler's method to solve pure-time DE: Assume that we need to solve the differential equation

$$f'(t) = G(t)$$

with the initial condition $f(t_0) = y_0$. Then,

1. Choose a time step Δt (length of time between approximations).
2. Use the formulas

$$t_{n+1} = t_n + \Delta t \tag{4}$$
$$y_{n+1} = y_n + G(t_n)\Delta t \tag{5}$$

   to find approximations $y_1, y_2, y_3, \ldots$ of the solution $f(t)$ at $t_1, t_2, t_3, \ldots$.