Antiderivatives (I)

Definition of Antiderivative:

- A function $F(x)$ defined on an interval (a, b) is called an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in (a, b).

Comments:

- The interval (a, b) allows infinity. For instance, $(-\infty, b)$, $(a, +\infty)$ or $(-\infty, +\infty)$.
- To say that $F(x)$ is an antiderivative of $f(x)$, we write

$$\int f(x)dx = F(x)\,. \tag{1}$$

- The integral sign $\int$ comes from a streched uppercase letter S, used to denote sum (this will become clear soon). The sympol dx is called the **differential** and is also part of the notation (and will also become clear soon).

Antiderivatives (II)

Theorem (Indefinite Integral):

- If $F(x)$ is an antiderivative of $f(x)$, then the most general antiderivative (also called the **indefinite integral**) of $f(x)$ is $F(x) + C$, i.e.,

$$\int f(x)dx = F(x) + C\,, \tag{2}$$

where C is a real number.

Comments:

- Geometrically, all antiderivatives $F(x) + C$ of a function $f(x)$ are vertical shifts of each other.
- Differentiation and antidifferentiation are inverse operations

$$\int F'(x)dx = F(x) \qquad \text{and} \qquad \left(\int f(x)dx\right)' = f(x) \tag{3}$$

similar to inverse functions or cancellation formulas ($e^{\ln(x)}$=x).