Area Under the Curve

Definition (Area under the curve). Let $f(x)$ be a continuous function defined on $[a, b]$ such that $f(x) \geq 0$ for all x in $[a, b]$. The **area, A, of the region under $f(x)$ on $[a, b]$** is

$$A = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \, f(x_i^*) \tag{1}$$

where S_n is a Riemann sum (Right Sum, Left sum, ...), provided that the limit exists. The **definite integral of $f(x)$ on $[a, b]$**, denoted by

$$\int_a^b f(x) dx \tag{2}$$

is defined by

$$\int_a^b f(x) dx = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \, f(x_i^*) \tag{3}$$

if the limit exists.