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Noise Trader Risk in Financial Markets

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We present a simple overlapping generations model of an asset market in which irrational noise traders with erroneous stochastic beliefs both affect prices and earn higher expected returns. The unpredictability of noise traders' beliefs creates a risk in the price of the asset that deters rational arbitrageurs from aggressively betting against them. As a result, prices can diverge significantly from fundamental values even in the absence of fundamental risk. Moreover, bearing a disproportionate amount of risk that they themselves create enables noise traders to earn a higher expected return than rational investors do. The model sheds light on a number of financial anomalies, including the excess volatility of asset prices, the mean reversion of stock returns, the underpricing of closed-end mutual funds, and the Mehra-Prescott equity premium puzzle.

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> If the reader interjects that there must surely be large profits to be gained . . . in the long run by a skilled individual who . . . purchase[s] investments on the best genuine long-term expectation he can frame, he must be answered . . . that there are such serious-minded individuals and that it makes a vast difference to an investment market whether or not they predominate. . . . But we must also add that there are several factors which jeopardise the predominance of such individuals in modern investment markets. Investment based on genuine long-term expectation is so difficult . . . as to be scarcely practicable. He who attempts it must surely . . . run greater risks than he who tries to guess better than the crowd how the crowd will behave. [Keynes 1936, p. 157]

There is considerable evidence that many investors do not follow economists' advice to buy and hold the market portfolio. Individual investors typically fail to diversify, holding instead a single stock or a small number of stocks (Lewellen, Schlarbaum, and Lease 1974). They often pick stocks through their own research or on the advice of the likes of Joe Granville or "Wall Street Week." When investors do diversify, they entrust their money to stock-picking mutual funds that charge them high fees while failing to beat the market (Jensen 1968). Black (1986) believes that such investors, with no access to inside information, irrationally act on noise as if it were information that would give them an edge. Following Kyle (1985), Black calls such investors "noise traders."

Despite the recognition of the abundance of noise traders in the market, economists feel safe ignoring them in most discussions of asset price formation. The argument against the importance of noise traders for price formation has been forcefully made by Friedman (1953) and Fama (1965). Both authors point out that irrational investors are met in the market by rational arbitrageurs who trade against them and in the process drive prices close to fundamental values. Moreover, in the course of such trading, those whose judgments of asset values are sufficiently mistaken to affect prices lose money to arbitrageurs and so eventually disappear from the market. The argument "that speculation is . . . destabilizing . . . is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on . . . average sell . . . low . . . and buy . . . high" (Friedman 1953, p. 175). Noise traders thus cannot affect prices too much and, even if they can, will not do so for long.

In this paper we examine these arguments by focusing explicitly on the limits of arbitrage dedicated to exploiting noise traders' misperceptions. We recognize that arbitrageurs are likely to be risk averse and to have reasonably short horizons. As a result, their willingness to take positions against noise traders is limited. One source of risk that limits the power of arbitrage—fundamental risk—is well understood. Figlewski (1979) shows that it might take a very long time for noise traders to lose most of their money if arbitrageurs must bear fundamental risk in betting against them and so take limited positions. Shiller (1984) and Campbell and Kyle (1987) focus on arbitrageurs' aversion to fundamental risk in discussing the effect of noise traders on stock market prices. Their results show that aversion to fundamental risk can by itself severely limit arbitrage, even when arbitrageurs have infinite horizons.

But there is another important source of risk borne by short-horizon investors engaged in arbitrage against noise traders: the risk that noise traders' beliefs will not revert to their mean for a long time and might in the meantime become even more extreme. If noise traders today are pessimistic about an asset and have driven down its price, an arbitrageur buying this asset must recognize that in the near future noise traders might become even more pessimistic and drive the price down even further. If the arbitrageur has to liquidate before the price recovers, he suffers a loss. Fear of this loss should limit his original arbitrage position.

Conversely, an arbitrageur selling an asset short when bullish noise traders have driven its price up must remember that noise traders might become even more bullish tomorrow, and so must take a position that accounts for the risk of a further price rise when he has to buy back the stock. This risk of a further change of noise traders' opinion away from its mean—which we refer to as "noise trader risk"—must be borne by any arbitrageur with a short time horizon and must limit his willingness to bet against noise traders.

Because the unpredictability of noise traders' future opinions deters arbitrage, prices can diverge significantly from fundamental values even when there is no fundamental risk. Noise traders thus create their own space. All the main results of our paper come from the observation that arbitrage does not eliminate the effects of noise because noise itself creates risk.[1]

The risk resulting from stochastic changes in noise traders' opinions raises the possibility that noise traders who are on average bullish

[1] Our paper is related to other examinations of Friedman's arguments, including Hart and Kreps (1986), Ingram (1987), and Stein (1987). Also relevant are Haltiwanger and Waldman (1985) and Russell and Thaler (1985). We discuss these papers after presenting our model.

earn a higher expected return than rational, sophisticated investors engaged in arbitrage against noise trading. This result obtains because noise trader risk makes assets less attractive to risk-averse arbitrageurs and so drives down prices. If noise traders on average overestimate returns or underestimate risk, they invest more in the risky asset on average than sophisticated investors and may earn higher average returns. This result is more interesting than the point that if noise traders bear more fundamental risk they earn higher returns: our point is that noise traders can earn higher expected returns solely by bearing more of the risk that they themselves create. Noise traders can earn higher expected returns from their own destabilizing influence, not because they perform the useful social function of bearing fundamental risk.

Our model also has several implications for asset price behavior. Because noise trader risk limits the effectiveness of arbitrage, prices in our model are excessively volatile. If noise traders' opinions follow a stationary process, there is a mean-reverting component in stock returns. Our model also shows how assets subject to noise trader risk can be underpriced relative to fundamental values. We apply this idea to explain the underpricing of closed-end mutual funds, as well as the long-run underpricing of stocks known as the Mehra-Prescott (1985) puzzle. Finally, our model has several implications for the optimal investment strategy of sophisticated investors and for the possible role of long-term investors in stabilizing asset prices.

We develop our two main arguments—that bearing noise trader risk raises noise traders' returns and that noise trader risk can explain several financial anomalies—in five sections. Section I presents a model of noise trader risk and shows how prices can diverge significantly from fundamental values. Section II calculates the relative expected returns of noise traders and of sophisticated investors. Section III analyzes the persistence of noise traders in an extended model in which successful investors are imitated (as in Denton [1985]). Section IV presents qualitative implications of the model for the behavior of asset prices and market participants. Section V presents conclusions.

I. Noise Trading as a Source of Risk

The model contains noise traders and sophisticated investors. Noise traders falsely believe that they have special information about the future price of the risky asset. They may get their pseudosignals from technical analysts, stockbrokers, or economic consultants and irrationally believe that these signals carry information. Or in formulating their investment strategies, they may exhibit the fallacy of excessive subjective certainty that has been repeatedly demonstrated in experimental contexts since Alpert and Raiffa (1982). Noise traders select their portfolios on the basis of such incorrect beliefs. In response to noise traders' actions, it is optimal for sophisticated investors to exploit noise traders' irrational misperceptions. Sophisticated traders buy when noise traders depress prices and sell when noise traders push prices up. Such active contrarian investment strategies push prices toward fundamentals, but not all the way.

A. *The Model*

Our basic model is a stripped-down overlapping generations model with two-period-lived agents (Samuelson 1958). For simplicity, there is no first-period consumption, no labor supply decision, and no bequest. As a result, the resources agents have to invest are exogenous. The only decision agents make is to choose a portfolio when young.

The economy contains two assets that pay identical dividends. One of the assets, the safe asset s, pays a fixed real dividend r. Asset s is in perfectly elastic supply: a unit of it can be created out of, and a unit of it turned back into, a unit of the consumption good in any period. With consumption each period taken as numeraire, the price of the safe asset is always fixed at one. The dividend r paid on asset s is thus the riskless rate. The other asset, the unsafe asset u, always pays the same fixed real dividend r as asset s. But u is not in elastic supply: it is in fixed and unchangeable quantity, normalized at one unit. The price of u in period t is denoted p_t. If the price of each asset were equal to the net present value of its future dividends, then assets u and s would be perfect substitutes and would sell for the same price of one in all periods. But this is not how the price of u is determined in the presence of noise traders.

We usually interpret s as a riskless short-term bond and u as aggregate equities. It is important for the analysis below that noise trader risk be marketwide rather than idiosyncratic. If noise traders' misperceptions of the returns to individual assets are uncorrelated and if each asset is small relative to the market, arbitrageurs would eliminate any possible mispricing for the same reasons that idiosyncratic risk is not priced in the standard capital asset pricing model.

There are two types of agents: sophisticated investors (denoted i) who have rational expectations and noise traders (denoted n). We assume that noise traders are present in the model in measure μ, that sophisticated investors are present in measure $1 - \mu$, and that all agents of a given type are identical. Both types of agents choose their portfolios when young to maximize perceived expected utility given their own beliefs about the ex ante mean of the distribution of the price of u at $t + 1$. The representative sophisticated investor young in period t accurately perceives the distribution of returns from holding the risky asset, and so maximizes expected utility given that distribution. The representative noise trader young in period t misperceives the expected price of the risky asset by an independent and identically distributed normal random variable ρ_t:

$$\rho_t \sim N(\rho^*, \sigma_\rho^2). \tag{1}$$

The mean misperception ρ^* is a measure of the average "bullishness" of the noise traders, and σ_ρ^2 is the variance of noise traders' misperceptions of the expected return per unit of the risky asset.[2] Noise traders thus maximize their own expectation of utility given the next-period dividend, the one-period variance of p_{t+1}, and their false belief that the distribution of the price of u next period has mean ρ_t above its true value.

Each agent's utility is a constant absolute risk aversion function of wealth when old:

$$U = -e^{-(2\gamma)w}, \tag{2}$$

where γ is the coefficient of absolute risk aversion. With normally distributed returns to holding a unit of the risky asset, maximizing the expected value of (2) is equivalent to maximizing

$$\bar{w} - \gamma\sigma_w^2, \tag{3}$$

where w is the expected final wealth, and σ_w^2 is the one-period-ahead variance of wealth. The sophisticated investor chooses the amount λ_t^i of the risky asset u held to maximize

$$\begin{aligned} E(U) &= \bar{w} - \gamma\sigma_w^2 \\ &= c_0 + \lambda_t^i[r + {}_tp_{t+1} - p_t(1 + r)] - \gamma(\lambda_t^i)^2({}_t\sigma_{p_{t+1}}^2), \end{aligned} \tag{4}$$

where c_0 is a function of first-period labor income, an anterior subscript denotes the time at which an expectation is taken, and we define

$${}_t\sigma_{p_{t+1}}^2 = E_t\{[p_{t+1} - E_t(p_{t+1})]^2\} \tag{5}$$

to be the one-period variance of p_{t+1}. The representative noise trader maximizes

$$\begin{aligned} E(U) &= \bar{w} - \gamma\sigma_w^2 \\ &= c_0 + \lambda_t^n[r + {}_tp_{t+1} - p_t(1 + r)] - \gamma(\lambda_t^n)^2({}_t\sigma_{p_{t+1}}^2) + \lambda_t^n(\rho_t). \end{aligned} \tag{6}$$

[2] The assumption that noise traders misperceive the expected price hides the fact that the expected price is itself a function of the parameters ρ^* and σ_ρ^2. Thus we are implicitly assuming that noise traders know how to factor the effect of future price volatility into their calculations of values. This assumption is made for simplicity. We have also solved a more complicated model that parameterizes noise traders' beliefs by their expectations of future prices, not by their misperceptions of future returns. The thrust of the results is the same.

The only difference between (4) and (6) is the last term in (6), which captures the noise traders' misperception of the expected return from holding λ_t^n units of the risky asset.

Given their beliefs, all young agents divide their portfolios between u and s. The quantities λ_t^n and λ_t^i of the risky asset purchased are functions of its price p_t, of the one-period-ahead distribution of the price of u, and (in the case of noise traders) of their misperception ρ_t of the expected price of the risky asset. When old, agents convert their holdings of s to the consumption good, sell their holdings of u for price p_{t+1} to the new young, and consume all their wealth.

One can think of alternative ways of specifying noise trader demands.[3] There are well-defined mappings between misperceptions of returns ρ_t and (*a*) noise traders' fixing a price p_t at which they will buy and sell, (*b*) noise traders' purchasing a fixed quantity λ_t^n of the risky asset, or (*c*) noise traders' mistaking the variance of returns (taking them to be σ^{2*} instead of σ^2). The equilibrium in which noise traders matter found in our basic model exists regardless of which primitive specification of noise traders' behavior is assumed.

Solving (4) and (6) yields expressions for agents' holdings of u:

$$\lambda_t^i = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma({}_t\sigma^2_{p_{t+1}})}, \tag{7}$$

$$\lambda_t^n = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma({}_t\sigma^2_{p_{t+1}})} + \frac{\rho_t}{2\gamma({}_t\sigma^2_{p_{t+1}})}. \tag{8}$$

We allow noise traders' and sophisticated investors' demands to be negative; they can take short positions at will. Even if investors hold only positive amounts of both assets, the fact that returns are unbounded gives each investor a chance of having negative final wealth. We use a standard specification of returns at the cost of allowing consumption to be negative with positive probability.[4]

Under our assumptions on preferences and the distribution of re-

[3] Let noise traders set

$$p_t = 1 - \frac{2\gamma}{r}\sigma^2 + \frac{\mu\rho^*}{r} + \frac{\mu(\rho_t - \rho^*)}{1+r},$$

where σ^2 is the total variance—the sum of "fundamental" dividend variance, noise trader–generated price variance, and any covariance terms—associated with holding the risky asset u for one period. Alternatively, let noise traders set the quantity of the risky asset that they buy—whatever its price—as $\lambda_t^n = 1 + [\rho_t/(2\gamma)\sigma^2]$ or let the noise traders misperceive the variance of returns on the risky asset, taking as the variance

$$\sigma^{2*} = \sigma^2\left(\frac{\gamma\sigma^2 - \rho_t}{\gamma\sigma^2 + \rho_t}\right).$$

[4] An appendix of our working paper (De Long et al. 1987) presents an example in which asset prices and consumption are always positive.

turns, the demands for the risky asset are proportional to its perceived excess return and inversely proportional to its perceived variance. The additional term in the demand function of noise traders comes from their misperception of the expected return. When noise traders overestimate expected returns, they demand more of the risky asset than sophisticated investors do; when they underestimate the expected return, they demand less. Sophisticated investors exert a stabilizing influence in this model since they offset the volatile positions of the noise traders.

The variance of prices appearing in the denominators of the demand functions is derived solely from noise trader risk. Both noise traders and sophisticated investors limit their demand for asset u because the price at which they can sell it when old depends on the uncertain beliefs of next period's young noise traders. This uncertainty about the price for which asset u can be sold afflicts all investors, no matter what their beliefs about expected returns, and so limits the extent to which they are willing to bet against each other. If the price next period were certain, then noise traders and sophisticated investors would hold with certainty different beliefs about expected returns; they would therefore try to take infinite bets against each other. An equilibrium would not exist. Noise trader risk limits all investors' positions and in particular keeps arbitrageurs from driving prices all the way to fundamental values.

B. The Pricing Function

To calculate equilibrium prices, observe that the old sell their holdings, and so the demands of the young must sum to one in equilibrium. Equations (7) and (8) imply that

$$p_t = \frac{1}{1+r}\left[r + {}_tp_{t+1} - 2\gamma({}_t\sigma^2_{p_{t+1}}) + \mu\rho_t\right]. \tag{9}$$

Equation (9) expresses the risky asset's price in period t as a function of period t's misperception by noise traders (ρ_t), of the technological (r) and behavioral (γ) parameters of the model, and of the moments of the one-period-ahead distribution of p_{t+1}. We consider only steady-state equilibria by imposing the requirement that the unconditional distribution of p_{t+1} be identical to the distribution of p_t. The endogenous one-period-ahead distribution of the price of asset u can then be eliminated from (9) by solving recursively:[5]

[5] The model cannot have stationary bubble equilibria, for the safe asset is formally equivalent to a storage technology that pays a rate of return r greater than the growth rate of the economy. The number of stationary equilibria in the model does, however,

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1 + r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}({}_t\sigma^2_{p_{t+1}}). \tag{10}$$

Inspection of (10) reveals that only the second term is variable, for γ, ρ^*, and r are all constants, and the one-step-ahead variance of p_t is a simple unchanging function of the constant variance of a generation of noise traders' misperception ρ_t:

$${}_t\sigma^2_{p_{t+1}} = \sigma^2_{p_{t+1}} = \frac{\mu^2\sigma^2_\rho}{(1 + r)^2}. \tag{11}$$

The final form of the pricing rule for u, in which the price depends only on exogenous parameters of the model and on public information about present and future misperception by noise traders, is

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1 + r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\mu^2\sigma^2_\rho}{r(1 + r)^2}. \tag{12}$$

C. *Interpretation*

The last three terms that appear in (12) and (10) show the impact of noise traders on the price of asset u. As the distribution of ρ_t converges to a point mass at zero, the equilibrium pricing function (12) converges to its fundamental value of one.

The second term in (12) captures the fluctuations in the price of the risky asset u due to the variation of noise traders' misperceptions. Even though asset u is not subject to any fundamental uncertainty and is so known by a large class of investors, its price varies substantially as noise traders' opinions shift. When a generation of noise traders is more bullish than the average generation, they bid up the price of u. When they are more bearish than average, they bid down the price. When they hold their average misperception—when $\rho_t = \rho^*$—the term is zero. As one would expect, the more numerous noise traders are relative to sophisticated investors, the more volatile asset prices are.

The third term in (12) captures the deviations of p_t from its fundamental value due to the fact that the average misperception by noise traders is not zero. If noise traders are bullish on average, this "price pressure" effect makes the price of the risky asset higher than it

depend on the primitive specification of noise traders' behavior. For example, if noise traders randomly pick each period the price p_t at which they will buy and sell unlimited quantities of the risky asset, then (trivially) there is only one equilibrium. If the noise traders randomly pick the quantity λ^i_t that they purchase, then the fundamental solution in which p_t is always equal to one is an equilibrium in addition to the equilibrium in which noise traders matter.

would otherwise be. Optimistic noise traders bear a greater than average share of price risk. Since sophisticated investors bear a smaller share of price risk the higher ρ^* is, they require a lower expected excess return and so are willing to pay a higher price for asset u.

The final term in (12) is the heart of the model. Sophisticated investors would not hold the risky asset unless compensated for bearing the risk that noise traders will become bearish and the price of the risky asset will fall. Both noise traders and sophisticated investors present in period t believe that asset u is mispriced, but because p_{t+1} is uncertain, neither group is willing to bet too much on this mispricing. At the margin, the return from enlarging one's position in an asset that everyone agrees is mispriced (but different types think is mispriced in different directions) is offset by the additional price risk that must be run. Noise traders thus "create their own space": the uncertainty over what next period's noise traders will believe makes the otherwise riskless asset u risky and drives its price down and its return up. This is so despite the fact that both sophisticated investors and noise traders always hold portfolios that possess the same amount of fundamental risk: zero. Any intuition to the effect that investors in the risky asset "ought" to receive higher expected returns because they perform the valuable social function of risk bearing neglects to consider that noise traders' speculation is the only source of risk. For the economy as a whole, there is no risk to be borne.

The reader might suspect that our results are critically dependent on the overlapping generations structure of the model, but this is not quite accurate. Equilibrium exists as long as the returns to holding the risky asset are always uncertain. In the overlapping generations structure, this is assured by the absence of a last period. For if there is a last period in which the risky asset pays a nonstochastic dividend and is liquidated, then both noise traders and sophisticated investors will seek to exploit what they see as riskless arbitrage. If, say, the liquidation value of the risky asset is $1 + r$, previous-period sophisticated investors will try to trade arbitrarily large amounts of asset u at any price other than one, and noise traders will try to trade arbitrarily large amounts at any price other than

$$p_t = 1 + \frac{\rho_t}{1 + r}. \tag{13}$$

The excess demand function for the risky asset will be undefined and the model will have no equilibrium. But in a model with fundamental dividend risk the assumption that there is no last period and, hence, the overlapping generations structure are not necessary. With fundamental dividend risk, no agent is ever subjectively certain what the return on the risky asset will be, and so the qualitative properties of equilibrium in our model are preserved even with a known terminal date. The overlapping generations structure is therefore not needed when fundamental dividend risk is present.

The infinitely extended overlapping generations structure of the basic model does play another function. It assures that each agent's horizon is short. No agent has any opportunity to wait until the price of the risky asset recovers before selling. Such an overlapping generations structure may be a fruitful way of modeling the effects on prices of a number of institutional features, such as frequent evaluations of money managers' performance, that may lead rational, long-lived market participants to care about short-term rather than long-term performance. In our model, the horizon of the typical investor is important. If sophisticated investors' horizons are long relative to the duration of noise traders' optimism or pessimism toward risky assets, then they can buy low, confident that they will be able to sell high when prices revert to the mean. As we show below, as the horizon of agents becomes longer, arbitrage becomes less risky and prices approach fundamental values. Noise trader risk is an important deterrent to arbitrage only when the duration of noise traders' misperceptions is of the same order of magnitude as or longer than the horizon of sophisticated investors.

II. Relative Returns of Noise Traders and Sophisticated Investors

We have demonstrated that noise traders can affect prices even though there is no uncertainty about fundamentals. Friedman (1953) argues that noise traders who affect prices earn lower returns than the sophisticated investors they trade with, and so economic selection works to weed them out. In our model, it need not be the case that noise traders earn lower returns. Noise traders' collective shifts of opinion increase the riskiness of returns to assets. If noise traders' portfolios are concentrated in assets subject to noise trader risk, noise traders can earn a higher average rate of return on their portfolios than sophisticated investors.

A. *Relative Expected Returns*

The conditions under which noise traders earn higher expected returns than sophisticated investors are easily laid out. All agents earn a certain net return of r on their investments in asset s. The difference between noise traders' and sophisticated investors' total returns given equal initial wealth is the product of the difference in their holdings of the risky asset u and of the excess return paid by a unit of the risky asset u. Call this difference in returns to the two types of agents ΔR_{n-i}:

$$\Delta R_{n-i} = (\lambda_t^n - \lambda_t^i)[r + p_{t+1} - p_t(1 + r)]. \tag{14}$$

The difference between noise traders' and sophisticated investors' demands for asset u is

$$\lambda_t^n - \lambda_t^i = \frac{\rho_t}{(2\gamma)_t\sigma_{p_{t+1}}^2} = \frac{(1 + r)^2\rho_t}{(2\gamma)\mu^2\sigma_\rho^2}. \tag{15}$$

Note that as μ becomes small, (15) becomes large: noise traders and sophisticated investors take enormous positions of opposite signs because the small amount of noise trader risk makes each group think that it has an almost riskless arbitrage opportunity. In the limit in which $\mu = 0$, equilibrium no longer exists (in the absence of fundamental risk) because the two groups try to place infinite bets against each other.

The expected value of the excess return on the risky asset u as of time t is

$$_t[r + p_{t+1} - p_t(1 + r)] = (2\gamma)_t\sigma_{p_{t+1}}^2 - \mu\rho_t = \frac{(2\gamma)\mu^2\sigma_\rho^2}{(1 + r)^2} - \mu\rho_t. \tag{16}$$

And so

$$_t(\Delta R_{n-i}) = \rho_t - \frac{(1 + r)^2(\rho_t)^2}{(2\gamma)\mu\sigma_\rho^2}. \tag{17}$$

The expected excess total return of noise traders is positive only if both noise traders are optimistic (ρ_t is positive, which makes [15] positive) and the risky asset is priced below its fundamental value (which makes [16] positive).

Taking the global unconditional expectation of (17) yields

$$E(\Delta R_{n-i}) = \rho^* - \frac{(1 + r)^2(\rho^*)^2 + (1 + r)^2\sigma_\rho^2}{(2\gamma)\mu\sigma_\rho^2}. \tag{18}$$

Equation (18) makes obvious the requirement that for noise traders to earn higher expected returns, the mean misperception ρ^* of returns on the risky asset must be positive. The first ρ^* on the right-hand side of (18) increases noise traders' expected returns through what might be called the "hold more" effect. Noise traders' expected returns relative to those of sophisticated investors are increased when noise traders on average hold more of the risky asset and earn a larger share of the rewards to risk bearing. When ρ^* is negative, noise traders' changing misperceptions still make the fundamentally riskless asset u risky and still push up the expected return on asset u, but the rewards to risk bearing accrue disproportionately to sophisticated investors, who on average hold more of the risky asset than the noise traders do.

The first term in the numerator in (18) incorporates the "price pressure" effect. As noise traders become more bullish, they demand more of the risky asset on average and drive up its price. They thus reduce the return to risk bearing and, hence, the differential between their returns and those of sophisticated investors.

The second term in the numerator incorporates the buy high–sell low or "Friedman" effect. Because noise traders' misperceptions are stochastic, they have the worst possible market timing. They buy the most of the risky asset u just when other noise traders are buying it, which is when they are most likely to suffer a capital loss. The more variable noise traders' beliefs are, the more damage their poor market timing does to their returns.

The denominator incorporates the "create space" effect central to this model. As the variability of noise traders' beliefs increases, the price risk increases. To take advantage of noise traders' misperceptions, sophisticated investors must bear this greater risk. Since sophisticated investors are risk averse, they reduce the extent to which they bet against noise traders in response to this increased risk. If the create space effect is large, then the price pressure and buy high–sell low effects inflict less damage on noise traders' average returns relative to sophisticated investors' returns.

Two effects—hold more and create space—tend to raise noise traders' relative expected returns. Two effects—the Friedman and price pressure effects—tend to lower noise traders' relative expected returns. Neither pair clearly dominates. Noise traders cannot earn higher average returns if they are on average bearish, for if ρ^* does not exceed zero, there is no hold more effect and (18) must be negative. Nor can noise traders earn higher average returns if they are too bullish, for as ρ^* gets large the price pressure effect, which increases with $(\rho^*)^2$, dominates. For intermediate degrees of average bullishness, noise traders earn higher expected returns. And it is clear from (18) that the larger γ is, that is, the more risk averse agents are, the larger is the range of ρ^* over which noise traders earn higher average returns.

B. *Relative Utility Levels*

The higher expected returns of the noise traders come at the cost of holding portfolios with sufficiently higher variance to give noise traders lower expected utility (computed using the true distribution of wealth when old). Since sophisticated investors maximize true expected utility, any trading strategy alternative to theirs that earns a higher mean return must have a variance sufficiently higher to make it unattractive. The average amount of asset s that must be given to old noise traders to give them the ex ante expected utility of sophisticated investors can be shown to be

$$\frac{(1 + r)^2}{(4\gamma)\mu^2}\left(1 + \frac{\rho^{*2}}{\sigma_\rho^2}\right). \tag{19}$$

This amount is decreasing in the variance and increasing in the square of the mean of noise traders' misperceptions. The size of their mistakes grows with ρ^*, but the risk penalty for attempting to exploit noise traders' mistakes grows with σ_ρ^2. Noise traders receive the same average realized utility when $\rho^* = x$ as when $\rho^* = -x$, but when $\rho^* > 0$, they may receive higher average returns. When $\rho^* < 0$, noise traders receive both lower realized utility levels and lower average returns.

Sophisticated investors are necessarily better off when noise traders are present in this model. In the absence of noise traders, sophisticated investors' opportunities are limited to investing at the riskless rate r. The presence of noise traders gives sophisticated investors a larger opportunity set, in that they can still invest all they want at the riskless rate r, but they can also trade in the unsafe asset. Access to a larger opportunity set clearly raises sophisticated investors' expected utility.[6]

Noise traders receive higher average consumption than sophisticated investors, and sophisticated investors receive higher average consumption than in fundamental equilibrium. Yet the productive resources available to society—its labor income per period, its ability to create the productive asset s, and the unit amount of asset u yielding its dividend r per period—are unchanged by the presence of noise trading. The source of extra returns is made clear by the following thought experiment. Imagine that before some date τ there are no noise traders. Up until time τ, both assets sell at a price of one. At τ it is unexpectedly announced that in the next generation noise traders will appear. The price p_τ of the asset u drops; those who hold asset u in period τ suffer a capital loss. This capital loss is the source of the excess returns and of the higher consumption in the equilibrium with noise. The period τ young have more to invest in s because they pay less to the old for the stock of asset u. If at time ω it became known

[6] If the stock of the risky asset is endogenous—if there is a nontrivial capital supply decision—sophisticated investors can be worse off with noise traders present. If noise traders make capital riskier and reduce the price of risk, they reduce the opportunity set of sophisticated investors and their welfare (De Long et al. 1989).

that noise traders had permanently withdrawn, then those who held u at time ω would capture the present value of what would otherwise have been future excess returns as p_ω jumped to one. The same supernormal return would also be received by a generation that suddenly acquired the opportunity to "bust up" the risky asset by turning it into an equivalent quantity of the safe asset. The fact that the generation that suffers from the arrival of noise traders is pushed off to negative infinity in the model creates the appearance of a free lunch.[7]

C. *A Comparison with Other Work*

The fact that bullish noise traders can earn higher returns in the market than sophisticated traders implies that Friedman's simple "market selection" argument is incomplete.[8] Since noise traders' wealth can increase faster than sophisticated investors', it is not possible to make any blanket statement that noise traders lose money and eventually become unimportant. One should not overinterpret our result. The greater variance of noise traders' returns might give them in the long run a high probability of having low wealth and a low probability of having very high wealth. Market selection might work against such traders even if their expected value of wealth is high since they would be poor virtually for certain. A more appropriate selection criterion would take this into account, but we have not found a tractable way to implement such a selection criterion in a model in which noise traders affect prices.[9]

At this point we can compare our results with recent discussions of Friedman's argument that destabilizing speculation is unprofitable, and so profitable speculation must be stabilizing. Hart and Kreps (1986) point out that an injection of rational investors able to perform profitable intertemporal trade could destabilize prices. In our model, rational speculation is always stabilizing, but average returns earned by rational speculators need not be as high as those earned by noise traders. In Stein (1987), speculators' access to private information allows for profitable destabilizing speculation. In our model, arbitrageurs know exactly the way in which noise traders are confused today, and noise traders have no private information. The uncer-

[7] In practice, the cost of future noise trader risk in a security will be paid for by whoever sells it to the public. In the case of a stock, the cost will be paid by the entrepreneur.

[8] The key difference from Friedman's (1953) model is that here the demand curve of sophisticated investors shifts in response to the addition of noise traders and the resulting increase in risk. Because of this shift, sophisticated investors' expected returns may fall even though their expected utility rises.

[9] De Long et al. (1988) consider the evolution of the wealth distribution in a model in which noise traders have no effect on prices.

tainty that affects noise traders and sophisticated investors equally concerns the behavior of noise traders tomorrow. Haltiwanger and Waldman (1985) and Russell and Thaler (1985) study the effects of irrational behavior on prices in the presence of externalities and of restrictions on trade, respectively. Our model is related to Russell and Thaler's, in that the short horizon of arbitrageurs can be interpreted as a form of restriction on trade.

III. Imitation of Beliefs

We have already observed that noise traders earn higher expected returns than sophisticated investors. This at least raises the possibility that their importance does not diminish over time. Our two-period model does not permit us to examine the accumulation of wealth by noise traders. As an alternative approach, we consider two rules describing the emulative behavior of new generations of traders. While it is possible to think of the succession of generations of investors in our model as families, a more relevant image of a new investor entering the market is that of a pension fund searching for a new money manager. Our new investors collect information about the performance of the past generation and decide which strategy to follow. The first approach is to postulate that new investors respond only to recent returns achieved by different investment strategies and are not able to accurately assess the ex ante risks undertaken. For this case, we show that noise traders' effects on prices do not inevitably diminish over time. In our second approach, new investors select their investment strategies on the basis of recent utility levels realized by these strategies. For this case, we show that noise traders' influence necessarily diminishes over time. We stress, however, that even readers preferring the second imitation rule should consider the *empirical* implications of our model. Under the P. T. Barnum rule that a noise trader is born every minute, a steady supply of new noise traders enters the market every period (as in our basic model) even if their strategies are not imitated.

A. *A Model of Imitation Based on Realized Returns without Fundamental Risk*

Each generation of investors earns exogenous labor income when young and consumes all its wealth when old. Each generation has the same number of investors following noise trader and sophisticated investor strategies as the previous one, except a few investors in each generation change type on the basis of the past relative performance of the two strategies. If noise traders earn a higher return in any period, a fraction of the young who would otherwise have been sophisticated investors become noise traders, and vice versa if noise traders earn a lower return. Moreover, the higher the difference in realized returns in any period, the more people switch. Letting μ_t be the share of the population that are noise traders and R_t^n and R_t^s be the realized returns of noise traders and sophisticated investors, we assume that

$$\mu_{t+1} = \max\{0, \min[1, \mu_t + \zeta(R_n - R_s)]\}, \tag{20}$$

where ζ is the rate at which additional new investors become noise traders per unit difference in realized returns.[10]

Equation (20) says that success breeds imitation: investment strategies that made their followers richer win converts. Underlying this imitation rule is the idea that new money entering the market is not completely sure which investment strategy to pursue. If sophisticated investors have earned a high return recently, new investors try to allocate their wealth mimicking sophisticated investors, or perhaps even entrusting their wealth to sophisticated money managers. If noise trader strategies have earned a higher return recently, new investors imitate those strategies to a greater extent. One way to interpret this imitation rule is that *some* new investors use what Black (1986) calls pseudosignals, such as the past return, to decide which strategy to follow.

This model can be easily solved only if ζ is very close to zero. If ζ is significantly different from zero at the scale of any one generation, then those investing in period t have to calculate the effect of the realization of returns on the division of those young in period $t + 1$ between noise traders and sophisticated investors. If ζ is sufficiently small, then returns can be calculated under the approximation that the noise trader share will be unchanged.

Equation (12), the pricing rule with a constant number of noise traders, with μ changed to μ_t, gives the limit as ζ converges to zero of the pricing rule for the model with imitation:

$$p_t = 1 + \frac{\mu_t(\rho_t - \rho^*)}{1 + r} + \frac{\mu_t\rho^*}{r} - \frac{(2\gamma)\mu_t^2\sigma_\rho^2}{r(1 + r)^2}. \tag{21}$$

The expected return gap between noise traders and sophisticated investors is equation (17) when the proportion of noise traders is fixed

[10] An alternative learning rule, studied by Bray (1982), would make the conversion parameter ζ a function of time: $\zeta_t = \zeta_0/t$. Under this alternative conversion rule, the noise trader share would converge to an element of the set $\{0, 1\}$ in the model without fundamental risk and to an element of the set $\{\mu_L, 1\}$ in the model with fundamental risk studied in the following subsection.

at μ. With the proportion μ_t variable, the limit of the expected return gap as ζ converges to zero is given by

$$E_t(\Delta R_{n-i}) = \rho_t - \frac{(1 + r)^2(\rho_t)^2}{(2\gamma)\mu_t\sigma_\rho^2}. \tag{22}$$

Over time, μ_t tends to grow or shrink as (22) is greater or less than zero. It is then clear that although there is a value for μ_t at which $E_t(\mu_{t+1}) = \mu_t$, this value is unstable. As the share of noise traders declines, sophisticated investors' willingness to bet against them rises. Sophisticated investors then earn more money from their exploitation of noise traders' misperceptions, and the gap between the expected returns earned by noise traders and those earned by sophisticated investors becomes negative. If the noise trader share μ_t is below

$$\mu^* = \frac{(\rho^{*2} + \sigma_\rho^2)(1 + r)^2}{2\rho^*(\gamma\sigma_\rho^2)}, \tag{23}$$

then μ_t tends to shrink. If μ_t is greater than μ^*, noise traders create so much price risk as to make sophisticated investors very reluctant to speculate against them. Noise traders then earn higher average returns than sophisticated investors and grow in number. In the long run, noise traders dominate the market or effectively disappear, as shown in figure 1.

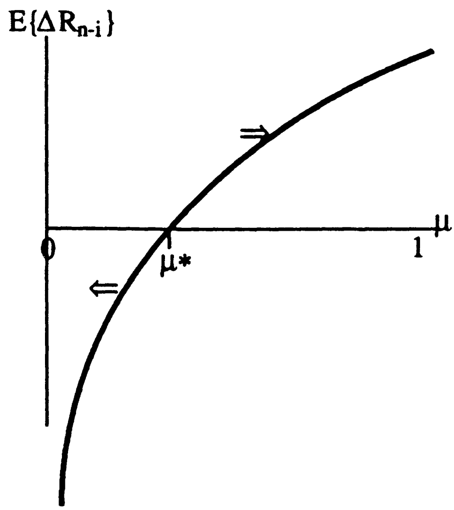

FIG. 1.—Dynamics of the noise trader share with no fundamental risk

B. An Extension with Fundamental Risk

This subsection extends our model of imitation to the case of fundamentally risky returns on the unsafe asset. We show that the long-run distribution of the share of noise traders is very different from the case without fundamental risk. Specifically, for sufficiently small values of ζ, the expected noise trader share for the steady-state distribution of μ_t is always bounded away from zero.

Let asset u pay not a certain dividend r but an uncertain dividend

$$r + \epsilon_t, \tag{24}$$

where ϵ_t is serially independent, normally distributed with zero mean and constant variance, and, for simplicity, uncorrelated with noise traders' opinions ρ_t. Asset demands then become

$$\lambda_t^i = \frac{r + E_t p_{t+1} - (1 + r)p_t}{2\gamma(\sigma^2_{p_{t+1}} + \sigma^2_\epsilon)} \tag{25}$$

and

$$\lambda_t^n = \frac{r + E_t p_{t+1} - (1 + r)p_t}{2\gamma(\sigma^2_{p_{t+1}} + \sigma^2_\epsilon)} + \frac{\rho_t}{2\gamma(\sigma^2_{p_{t+1}} + \sigma^2_\epsilon)} \tag{26}$$

instead of (7) and (8). The only change is the appearance in the denominators of the asset demand functions of the total risk involved from asset u—the sum of noise trader price risk and fundamental dividend risk—instead of simply noise trader–generated price risk.

The pricing function if there is fundamental risk is transformed from (21) into

$$p_t = 1 + \frac{\mu_t\rho^*}{r} - \frac{2\gamma}{r}\left[\sigma^2_\epsilon + \frac{\mu^2\sigma^2_\rho}{(1 + r)^2}\right] + \frac{\mu_t(\rho_t - \rho^*)}{1 + r} \tag{27}$$

in the limit as ζ converges to zero. The noise trader risk term is replaced by the total risk associated with holding u. The difference between expected returns of noise traders and those of sophisticated investors becomes

$$E[\Delta R_{n-i}(\mu)] = \rho^* - \frac{\rho^{*2} + \sigma^2_\rho}{2\gamma\left[\dfrac{\sigma^2_\rho\mu}{(1 + r)^2} + \dfrac{\sigma^2_\epsilon}{\mu}\right]} \tag{28}$$

if μ is greater than zero and

$$E[\Delta R_{n-i}(0)] = \rho^*. \tag{29}$$

While the hold more, average price pressure, and Friedman effects are not changed by the addition of fundamental risk, the create space effect—the denominator of the second term on the right-hand side of (28)—is increased. Since holding asset u is now more risky, sophisticated investors are less willing to trade in order to exploit noise traders' mistakes. We continue to assume that $\zeta = 0$ in the calculation of prices, so that (27) is the pricing rule for this model and (28) is the difference in expected returns.

Equation (12) shows that, in the absence of fundamental risk, a sequence of economies in which μ approaches zero also has $E(\Delta R_{n-i})$ approach negative infinity. By contrast, equation (28) shows that, with fundamental risk present, $E(\Delta R_{n-i})$ approaches ρ^* as μ approaches zero. There is an intuitive explanation for the substantially different dynamics for $\sigma_\epsilon^2 = 0$ and $\sigma_\epsilon^2 > 0$. If $\sigma_\epsilon^2 > 0$, then noise traders' and sophisticated investors' demands remain bounded as μ approaches zero. For a sufficiently small noise trader share, therefore, sophisticated investors must have positive holdings of the risky asset—the very small number of noise traders cannot hold it all—and so the risky asset must offer an expected return higher than the safe rate in equilibrium. If $\sigma_\epsilon^2 = 0$, then noise traders' and sophisticated investors' demands become unbounded as μ approaches zero and the unsafe asset loses its risk. Noise traders' positions then lose them arbitrarily large amounts each period.

For parameter values that satisfy both $\rho^* > 0$ and

$$\sigma_\epsilon^2 > \frac{(1 + r)^2(\rho^* + \sigma_\rho^2)^2}{16\gamma^2\rho^{*2}\sigma_\rho^2}, \tag{30}$$

equation (28) has no real roots and noise traders always earn higher expected returns. In this case, for sufficiently small values of ζ the expected long-run noise trader share is close to one.

For parameter values such that (30) fails, (28) has two positive real roots. If the lower root $\mu_L < 1$, noise traders do not always earn higher expected returns and the expected long-run noise trader share is not in general close to one. For this case, we have proved the following proposition.

PROPOSITION. Let the pricing rule be given by (27) and the imitation rule by (20). Suppose that the equation $E[\Delta R(\mu)] = 0$ has at least one real root for $\mu \in [0, 1]$. Consider a sequence of economies indexed by n, differing only in their values of the imitation parameter ζ_n, such that $\zeta_n \rightarrow 0$ as $n \rightarrow \infty$. Then there is a $\delta > 0$ such that $E(\mu_t) \rightarrow \mu \geq \delta$ as $n \rightarrow \infty$, where the expectation is taken over the steady-state distribution of μ_t.

An appendix containing a proof is available from the authors on request.

When imitation is based on realized returns, for some parameter values the expected noise trader share of the population approaches

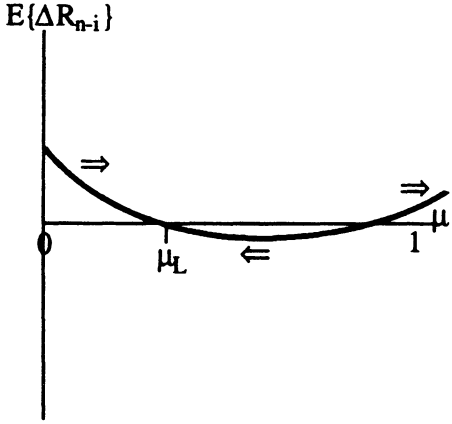

FIG. 2.—Dynamics of the noise trader share with fundamental risk

one as ζ approaches zero. The proposition above shows, and figure 2 illustrates, that if asset u is fundamentally risky, there are no parameter values for which the expected noise trader share of the population approaches zero as ζ becomes small. This result suggests that at least one plausible form of dynamics ensures that noise traders matter and affect prices in the long run.

C. *Imitation Based on Utility*

The imitation rule (20) is based on the assumption that the rate of conversion depends on the difference in realized *returns* and not on the difference in realized *utilities*. It implicitly assumes that converts do not take account of the greater risk that noise traders bear to earn higher returns. This form of imitation requires investors to use past investors' realized returns as a proxy for success even though their own objective is to maximize not wealth but utility.

An alternative imitation rule is to make the number of new noise traders depend on the difference in *utilities* realized last period from sophisticated investor and noise trader strategies. This rule is different from (20): with concave utility, there is more switching away from a strategy in response to past low returns than switching toward a strategy in response to past high returns. Under this imitation rule, the share of noise traders in the economy in fact converges to zero as ζ approaches zero, in contrast to our result under (20). Since sophisticated investors maximize true expected utility, on average the realized utility of a sophisticated investor is higher than the realized utility of a noise trader. That is, under this imitation rule the higher variance of noise traders' returns costs them in terms of winning converts because it costs them in terms of average utility. For each initial state of the system, the noise trader share tends to fall under a utility difference–based imitation rule until it reaches the neighborhood of the reflecting barrier at $\mu = 0$. The expected noise trader share for the steady-state distribution of μ is no longer bounded away from zero as ζ approaches zero.

This alternative rule has considerable appeal in that imitation is based on the realization of agents' true objectives. Nonetheless, there are two reasons to prefer the wealth-based imitation rule (20). First, we find it plausible that many investors attribute the higher return of an investment strategy to the market timing skills of its practitioners and not to its greater risk. This consideration may be particularly important when we ask whether individuals change their own investment strategies that have just earned them a high return. When people imitate investment strategies, they appear to focus on standard metrics such as returns relative to market averages and do not correct for ex ante risk. As long as enough investors use the pseudosignal of realized returns to choose their own investment strategy, noise traders will persist. The second reason to focus on returns-based imitation is that Friedman (1953) argued that noise traders must earn lower average *returns* and so become unimportant. He did not argue that money-making noise traders would fail to attract imitators because potential imitators would attribute their success to luck rather than to skill. Our focus on an imitation rule in which higher wealth wins converts is closer to Friedman's argument.

IV. Noise Trading and Asset Market Behavior

This section describes some implications of our model for financial markets (see also Black 1986). We show that in the presence of noise trader risk, asset returns exhibit the mean reversion documented by a great deal of empirical work, asset prices diverge on average from fundamental values as suggested by Mehra and Prescott (1985) and by the comparison of the portfolio and market values of closed-end mutual funds, and long-term investors stabilize prices. Finally, we discuss the effects of noise trader risk on corporate finance.

A. *Volatility and Mean Reversion in Asset Prices*

In our model with noise traders absent—with both ρ^* and σ_ρ^2 set equal to zero—the price of u is always equal to its fundamental value of one. When noise traders are present, the price of u—identical to s in all fundamental respects—is excessively volatile in the sense that it moves more than can be explained on the basis of changes in fundamental values. None of the variance in the price of u can be justified by changes in fundamentals: there are no changes in expected future dividends in our model or in any fundamental determinant of required returns.

Accumulating evidence suggests that it is difficult to account for all the volatility of asset prices in terms of news. Although Shiller's (1981) claim that the stock market wildly violated variance bounds imposed by the requirement that prices be discounted present values relied on controversial statistical procedures (Kleidon 1986), other evidence that asset price movements do not all reflect changes in fundamental values is more clear-cut. Roll (1984) considers the orange juice futures market, where the principal source of relevant news is weather. He demonstrates that a substantial share of the movement in prices cannot be attributed to news about the weather that bears on fundamental values. Campbell and Kyle (1987) conclude that a large fraction of market movements cannot be attributed to news about future dividends and discount rates.

Such excess volatility becomes even easier to explain if we relax our assumption that all market participants are either noise traders or sophisticated investors who bet against them. A more reasonable assumption is that many traders pursue passive strategies, neither responding to noise nor betting against noise traders. If a large fraction of investors allocate a constant share of their wealth to stocks, then even a small measure of noise traders can have a large impact on prices. When noise traders try to sell, only a few sophisticated investors are willing to hold extra stock, and consequently prices must fall considerably for them to do so. The fewer sophisticated investors there are relative to the noise traders, the larger is the impact of noise.[11]

If asset prices respond to noise and if the errors of noise traders are temporary, then asset prices revert to the mean. For example, if noise traders' misperceptions follow an AR(1) process, then the serial correlation in returns decays geometrically as in the "fads" example of Summers (1986), who stresses that even with long time series it is difficult to detect slowly decaying transitory components in asset prices. Since the same problems of identification that plague econometricians affect speculators, actual market forces are likely to be less effective in limiting the effects of noise trading than in our model, where rational investors fully understand the process governing the behavior of noise traders.

[11] A simple example may help to make our point. Suppose that all investors are convinced that the market is efficient. They will hold the market portfolio. Now suppose that one investor decides to commit his wealth disproportionately to a single security. Its price will be driven to infinity.

Moreover, even if sophisticated investors accurately diagnose the process describing the behavior of noise traders, if misperceptions are serially correlated, they will not be willing to bet nearly as heavily against noise traders: the risk of a capital loss remains and is balanced by a smaller expected return since the next-period price is not expected to move all the way back to its fundamental value. A high unconditional variance of prices can coexist with only a small opportunity to exploit noise traders.

For an example of how rapidly unconditional price variance grows as misperceptions become persistent, assume that misperceptions follow an AR(1) process with innovation η_t and autoregressive parameter ϕ. In this case the unconditional variance of the price of u is[12]

$$\sigma_p^2 = \frac{\mu^2\sigma_\rho^2}{[r + (1 - \phi)]^2} = \frac{\mu^2\sigma_\eta^2}{[r + (1 - \phi)]^2(1 - \phi^2)}. \tag{31}$$

Noise traders who earn higher expected returns than sophisticated investors can thus cause larger deviations of prices from fundamental values if misperceptions are serially correlated. The difference in expected returns is given by

$$E(\Delta R_{n-i}) = \rho^* - \frac{[r + (1 - \phi)]^2(\rho^*)^2}{(2\gamma)\mu\sigma_\eta^2} - \frac{[r + (1 - \phi)]^2}{(2\gamma)\mu(1 - \phi^2)}. \tag{32}$$

Highly persistent transitory components in asset prices can be very large and still consistent with noise traders' earning higher returns than sophisticated investors.

There is significant evidence that stock prices indeed exhibit mean-reverting behavior. Fama and French (1988*b*) and Poterba and Summers (1988) demonstrate that long-horizon stock returns exhibit negative serial correlation. The fact that prices revert to the mean also implies that measures of scale have predictive power for asset returns: when prices are above p^*—that is, are high relative to their historical average multiple of dividends—prices are likely to fall in our model. In fact, Campbell and Shiller (1987), Fama and French (1988*a*), and other studies find that dividend/price and earnings/price ratios appear to contain substantial power for detecting transitory components in stock prices.

Many studies including Mankiw and Summers (1984) and Mankiw

[12] Demand for assets depends not on the unconditional price variances but on the conditional one-step-ahead price risk. The variance of the price of u about its one-step-ahead anticipated value is

$${}_t\sigma_{p_{t+1}}^2 = \frac{\mu^2\sigma_\eta^2}{[r + (1 - \phi)]^2}$$

in the case of serially correlated misperceptions.

(1986) note that anomalies exactly paralleling the dividend/price ratio anomaly are present in the bond market. Long rates have predictive power for future short rates, but it is nonetheless the case that when long rates exceed short rates, they tend to fall and not to rise as predicted by the expectations hypothesis. While convincing stories about changing risk factors are yet to be provided, this behavior is exactly what one would expect if noise trading distorted long bond yields. Specifically, if we think of the short-term bond as asset s and the long-term bond as asset u, then the price of u exhibits the mean-reverting behavior observed in the data on long-term bonds.

In a world with mean-reverting noise traders' misperceptions, the optimal investment strategy is very different from the buy and hold strategy of the standard investment model. The optimal strategy for sophisticated investors is a *market timing* strategy that calls for increased exposure to stocks after they have fallen and decreased exposure to stocks after they have risen in price. The strategy of betting against noise traders is a contrarian investment strategy: it requires investment in the market at times when noise traders are bearish, in anticipation that their sentiment will recover. The fundamentalist investment strategies of Graham and Dodd (1934) seem to be based on largely the same idea, although they are typically described in terms of individual stocks. The evidence on mean reversion in stock returns suggests that, over the long run, such contrarian strategies pay off.

As our model shows, successful pursuit of such contrarian investment strategies can require a long time horizon, and such strategies are by no means safe because of the noise trader risk that must be run (see the quotation from Keynes at the beginning of this article). In fact, our model shows precisely why apparent anomalies such as the high dollar of the mid 1980s and the extraordinary price/earnings ratios on Japanese stocks in 1987–89 can persist for so long even when many investors recognize these anomalies. Betting against such perceived mispricing requires bearing a lot of risk. Even if the price is too high now, it can always go higher in the short run, leading to the demise of an arbitrageur with limited resources or a short time horizon.

Contrarian investment strategies work because arbitrageurs can take advantage of mean reversion in noise traders' beliefs. An alternative rational investment strategy would be to gather information about future noise trader demand shifts and to trade in anticipation of such shifts. Such information can come from examining trading volume, price patterns, buy/sell ratios, and other "chartist" indicators. Trading based on forecasting the behavior of others is not modeled here, but we consider it elsewhere (De Long et al. 1990). With short horizons, it may well be more attractive for smart money to pursue these anticipatory strategies than to wait for the reversion of noise traders' beliefs to their mean (Shleifer and Vishny 1990). In this case, we anticipate that many sophisticated investors will try to "guess better than the crowd how the crowd will behave" rather than pursue contrarian long-term arbitrage.

B. *Asset Prices and Fundamental Values: Closed-End Mutual Funds*

The efficient markets hypothesis states that assets ought to sell for their fundamental values. In most cases, fundamental value is difficult to measure, and so this prediction cannot be directly tested. But the fundamental value of a closed-end fund is easily assessed: the fund pays dividends equal to the sum of the dividends paid by the stocks in its portfolio and so should sell for the market price of its portfolio. Yet closed-end funds sell and have sold at large and substantially fluctuating discounts (Malkiel 1977; Herzfeld 1980), which have been relatively small during the bull markets of the late 1960s and the 1980s and large during the bear markets of the 1970s.

Available explanations of discounts on closed-end funds are not completely satisfactory. Two of the most prominent explanations rely on the agency costs of fund management and on the miscalculation of net asset value because of a failure to deduct the fund's capital gains tax liability. The agency theory for discounts, however, cannot explain how closed-end funds are ever rationally formed since the original investors throw away the present value of future agency costs without earning higher returns. The agency explanation is also inconsistent with the evidence that funds with higher transaction costs and stock turnover do not sell at higher discounts (Malkiel 1977) and with the correlated variability of discounts across funds (Herzfeld 1980). With respect to tax-based theories, Brauer (1984) and Brickley and Schallheim (1985) find that prices of closed-end funds rise on the announcement of open-ending or of liquidation. This result is difficult to interpret if the closed-end fund's discount reflects its unrealized capital gain tax liability since, if anything, discounts should *widen* when the fund is open-ended and tax payments can no longer be deferred. Nor can the capital gains story explain how funds trade at a premium when they get started.

The concept of noise trader risk can explain both the persistent and variable discounts on closed-end funds and the creation of such funds.[13] Think of the safe asset s in our model as the stocks in the closed-end fund and of the unsafe asset u as the fund itself. As in our

[13] It does not explain why such funds are not broken up immediately once a discount appears.

model, the two securities are perfect substitutes as far as dividends are concerned and so should sell at the same price in equilibrium without noise traders. Note that it does not matter for our purposes if there is noise trading in the stocks themselves and therefore a mispricing of s as well. All we need is additional noise trader misperception of returns on the closed-end fund u that is separate from their misperception of returns on the underlying stocks. Finally, we need to assume that noise traders' misperceptions of returns on closed-end funds are correlated with other (possibly irrational) sources of systematic risk since idiosyncratic noise trader risk is not priced in our model.

Under these assumptions, the results from our basic model can be directly applied to closed-end funds. Noise traders' misperceptions about the returns on the funds become a source of risk for any short-horizon investor trying to arbitrage the difference between the fund and its underlying assets. Thus when an investor buys the fund u and sells short the underlying stocks s, he bears the risk that at the time he wants to liquidate his position the discount will be wider. Just as in our model, noise traders can become more bearish on the fund in the future than they are today, and so an arbitrageur will suffer a loss. Such risk of changes in noise traders' opinions of closed-end funds leads to the market's discounting of their price on average relative to the net asset value even if noise traders themselves are neither bullish nor bearish on average, that is, $\rho^* = 0$. The discount arises solely because holding the fund entails additional noise trader risk: we do not assume that noise traders are on average bearish about closed-end funds.[14]

This theory of discounts on closed-end funds makes several accurate predictions. First, it explains how the funds can get started even when on average they will be underpriced. Closed-end funds get started when noise traders are unusually optimistic about the returns

[14] One can see how the fact that closed-end fund shares are subject not only to fundamental risk (risk affecting the value of the fund's portfolio) but also to noise trader risk (risk that the closed-end fund discount might change) affects investment decisions in the investment advice given by Malkiel (1973, 1975, 1985, 1989). He confidently recommended in 1973 that investors purchase then heavily discounted (20–30 percent) closed-end fund shares: such an investor would do better than by picking stocks or investing in an open-end fund unless "the discount widened in the future." The confidence of Malkiel's recommendation stemmed from his belief that "this . . . risk is minimized . . . [since] discounts [now] . . . are about as large as they have ever been historically" (1975, p. 263). And the obverse is his belief that the holder of a closed-end fund should be prepared to sell if the discount narrowed, not only if the discount disappeared, but also if the discount narrowed. The 4th ed. of *A Random Walk down Wall Street* does not recommend the purchase of closed-end fund shares in spite of the fact that many closed-end funds still sell at discounts. The noise trader risk that discounts may widen again in the future is a disadvantage that apparently weighs heavily against the relatively small advantages given by the small then-current discount. The 5th ed. once again recommends the purchase of closed-end funds now that the discount has widened.

on closed-end funds, that is, when ρ_t *for the funds* is unusually high. In such a case, it would pay entrepreneurs to buy stocks (asset s), repackage them as closed-end funds (asset u), and sell the closed-end funds to optimistic noise traders at a premium. This result has the implication, which has not yet been tested, that new closed-end funds are formed in clusters at the times when other closed-end funds sell at a premium.

The fluctuations in noise trader opinion of the expected return on the funds also explain why the discounts fluctuate, widening at some times and turning into premiums at others (when the funds get started). Fluctuations in discounts are in fact the reason that there is an average discount. No other theory of discounts predicts that closed-end funds sometimes sell at a premium, that changes in discounts are correlated across funds, or that new funds are started when old closed-end funds sell at a premium.

Two key assumptions must be made for this theory of closed-end fund discounts to be coherent. First, noise trader risk on the funds should be systematic and not idiosyncratic. Consistent with this assumption, discounts on different closed-end funds do seem to fluctuate together (Herzfeld 1980). Second, investors in the economy must have horizons that are with some probability shorter than the time to liquidation of the fund. If some investors on the contrary have very long horizons, they can buy the closed-end fund and sell short the underlying securities, wait until the fund is liquidated, and so lock in a capital gain without bearing any risk. Consistent with this observation, discounts become much narrower on the announcement of the open-ending of a closed-end fund (Brauer 1984). The application of our model to closed-end funds illustrates the essential role played by the finite horizon of investors.

C. *Asset Prices and Fundamental Values: The Mehra-Prescott Puzzle*

In our model, if noise traders earn higher expected returns than sophisticated investors, then the average price of u must be below its fundamental value. The expected value of p_t is

$$E(p) = p^* = 1 - \frac{2\gamma\mu\sigma_\rho^2}{r(1 + r)^2} + \frac{\mu\rho^*}{r}. \tag{33}$$

Since noise traders hold more of the risky asset and earn negative capital gains on average, they can earn higher expected returns than sophisticated investors only if the dividend on the unsafe asset amounts to a higher rate of return on average than the same dividend on the safe asset. For this to hold, the unsafe asset must sell at an average price below its fundamental value of one.

The result that noise traders earn a higher expected return whenever the unsafe asset is priced below its fundamental value may shed some light on the well-known Mehra-Prescott puzzle. Mehra and Prescott (1985) show that the realized average return on U.S. equities over the last 60 years has been around 8 percent, and the realized real return on safe bonds only around zero. Such a risk premium seems to be inconsistent with the standard representative consumer model applied to U.S. data unless that consumer has an implausibly large coefficient of risk aversion.

If we interpret asset u in our model as the aggregate stock market and asset s as short-term bonds, our model can shed light on the Mehra-Prescott puzzle. Since noise trader risk drives down the price of u, equities yield a higher return in our model than the riskless asset does. Moreover, this difference in yields obtains despite the fact that aggregate consumption does not vary too much with the expected return on equities. The reason is that the consumption of sophisticated investors satisfies the Euler equation with respect to the true distribution of expected returns exactly, but the consumption of noise traders does not. In fact, the share of wealth invested (and thus not consumed) by noise traders is low when the true expected return is high, and high when the true expected return is low. The presence of noise traders thus makes aggregate consumption less sensitive to the variation of true expected returns than it should be. A large equity premium can thus coexist with a low covariance of returns on equities with aggregate consumption. Although the mechanics of our model are very different from the model in Ingram (1987), this particular implication works similarly to her explanation of the equity premium, which relies on the insensitivity of the consumption of a group of rule-of-thumb agents to expected returns.

It is important to stress that our model sheds light on the Mehra-Prescott puzzle only if equities are underpriced, which is itself a necessary condition for noise traders to earn higher expected returns. In other words, the fact that the Mehra-Prescott equity premium obtains in an economy is evidence for the proposition that the expected returns of noise traders are likely to be higher than those of sophisticated investors. In the context of our model, the existence of an equity premium in the U.S. economy suggests that American noise traders are on average bullish on the assets that they disturb and may earn higher average returns than American arbitrageurs.

D. Long Horizons

Noise trader risk makes coherent a widely held view of the relative social merits of "speculation" and "investment" that has found little academic sympathy. Many participants in financial markets have argued that the presence of traders who are looking for only short-term profits is socially destructive. The standard economist's refutation of this argument relies on recursion: If one seeks to buy a stock now to sell in an hour, one must calculate its price in an hour. But its price in an hour depends on what those who will purchase it think its price will be a further hour down the road. Anyone who buys an asset, no matter how short the holding period, must perform the same present value calculation as someone who intends to hold the asset for 50 years. Since a linked chain of short-term "traders" performs the same assessment of values as a single "investor," the claim that trading is bad and investing good cannot be correct. Prices will be unaffected by the horizon of the agent as long as the rate of discount and willingness to bear risk are unchanged.

In our model this analysis does not apply. The horizon of agents matters. If agents live for more than two periods, the equilibrium is closer to the "fundamental" equilibrium than if agents live for two periods. As an example, consider an infinitesimal measure of infinitely lived but risk-averse sophisticated traders. Suppose that p_t is less than one. An infinitely lived agent can sell short a unit of s and buy a unit of u. He collects a gain of $1 - p_t$, and he has incurred no liability in any state of the world. The dividend on u will always offset the dividend owed on s. The fact that an infinitely lived agent can arbitrage assets s and u without ever facing a settlement date implies that any infinitely lived sophisticated investor could push the price of u to its fundamental value of one.

Although arbitrage is not riskless for long but finite-lived agents, their asset demands are more responsive to price movements than those of two-period-lived agents. There are two reasons for this. First, even if an $n > 2$ period-lived sophisticated investor can liquidate his position in asset u only in the last period of his life, he bears the same amount of resale price risk as his two-period-lived counterpart but gets some insurance from dividends. If, for example, he buys an undervalued asset u, he receives a high dividend yield for several periods before he sells. Because as the horizon expands so does the share of dividends in expected returns, agents with longer horizons buy more at the start. Second, a long-lived sophisticated investor has in fact many periods to liquidate his position. Since he makes money on arbitrage if the price reverts to the mean at any time before his death, having several opportunities to liquidate reduces his risk. For these two reasons, raising sophisticated investors' horizons makes them more aggressive and brings the price of u closer to fundamentals.

The embedding of the financial market in an overlapping generations model in which agents die after two periods is a device to give rational utility maximizers short horizons. This device may adequately model institutional features of asset markets—triennial performance evaluations of pension fund money managers, for example—that may lead even fully rational agents to have short horizons. Realistically, even an agent with a horizon long in terms of time may have a horizon "short" in the context of this model. If dividend risk is great enough and if noise trader misperceptions are persistent, then agents might well find it unattractive to buy stocks and hold them for a long time hoping that the market someday recognizes their value. For in the meantime, during which the assets might have to be sold, market prices may deviate even further from fundamental values. The claim that short horizons are bad for the economy is both coherent and true in our model.

E. *Observations on Corporate Finance*

Throughout this paper, we have focused on the implications of marketwide noise trader risk. The reason is that in our model, just as in a standard asset valuation model, idiosyncratic risk is unpriced. A number of implications of noise trading, however, including those stressed by Black (1986), rely on misperceptions of firm-specific returns. To allow such idiosyncratic misperceptions to matter, the model must include transactions costs that limit the universe of stocks that each sophisticated investor holds (Mayshar 1983). Although such a model is beyond the scope of this paper, we mention a few issues that idiosyncratic risk raises in the context of corporate finance.

In a model with noise traders the Modigliani-Miller theorem does not necessarily apply. To see this, consider the standard homemade leverage proof of the theorem. This proof demonstrates that a rational investor can undo any effects of firm leverage and maintain the same real position regardless of a firm's payout policy. It does not suggest that less than rational traders will do so. Given that noise traders in general affect prices, it follows that unless they happen to trade so as to undo the effects of changes in leverage, the Modigliani-Miller theorem will not hold.

It is plausible to think that noise traders do not get confused about the value of assets that have a certain and immediate liquidation value. Noise traders are more likely to become confused about assets that offer fundamentally risky payouts in the distant future. Assets of long duration that promise fundamentally uncertain as opposed to immediate and certain cash payouts may thus be subject to an especially great amount of noise trader risk. In this case, a firm might choose to pay dividends rather than reinvest even if there are tax costs to dividends. If dividends make equity look more like a safe short-term bond to noise traders, then paying dividends can reduce the total amount of noise trader risk borne by a firm's securities. Paying dividends might raise the value of equity if the reduction in the discount entailed by noise trader risk exceeds additional shareholder tax liability. Moreover, dividends are not equivalent to share repurchases unless *noise traders* perceive the two to be complete substitutes. If investors believe that future stock repurchases are of uncertain value because noise traders disturb the price of equity, then the equity of a firm repurchasing shares can be subject to greater undervaluation than that of a firm paying dividends. A bird in the hand is truly better than one in the bush.

Jensen (1986) summarizes evidence showing that the more constrained the allocation of the firm's cash flows, the higher its valuation by the market. For example, share prices rise when a firm raises dividends, swaps debt for equity, or buys back shares. In contrast, share prices fall when a firm cuts dividends or issues new shares. These results are consistent with our model if making the returns to equity more determinate reduces the noise trader risk that it bears. Increases in dividends that make equity look safer to noise traders may reduce noise trader risk and raise share prices. Swaps of debt for equity have the same effect, as do share buybacks. As long as a change in capital structure convinces noise traders that a firm's total capital is more like asset s and less like asset u than they had previously thought, changes in capital structure raise value.

The discussion above suggests that noise trader risk is a cost that an issuer of a security that will be publicly traded must bear. Both traded equity and traded long-term debt will be underpriced relative to fundamentals if their prices are subject to the whims of noise traders' opinions. Why then are securities traded publicly? Put differently, why don't all firms go private to avoid noise trader risk? Presumably firms have publicly traded securities if the benefits, such as a broader base from which to draw capital, a larger pool to use to diversify systematic risk, and liquidity, exceed the costs of the noise trader–generated undervaluation. Assets for which these benefits of public ownership are the highest relative to the costs of noise trader risk are the assets that will be issued into markets with public trading. While the issuers of these securities will try to minimize the costs of noise trader risk by "packaging" the securities appropriately, they will not be able to eliminate such risk entirely.

V. Conclusion

We have shown that risk created by the unpredictability of unsophisticated investors' opinions significantly reduces the attractiveness of arbitrage. As long as arbitrageurs have short horizons and so must worry about liquidating their investment in a mispriced asset, their aggressiveness will be limited even in the absence of fundamental risk. In this case noise trading can lead to a large divergence between market prices and fundamental values. Moreover, noise traders may be compensated for bearing the risk that they themselves create and so earn higher returns than sophisticated investors even though they distort prices. As we discuss in the paper, this result at the least calls for a closer scrutiny of the standard argument that destabilizing speculation must be unprofitable and so noise traders will not persist in the market.

This paper has also argued that a number of financial market anomalies can be explained by the idea of noise trader risk. These anomalies include the excess volatility of and mean reversion in stock market prices, the failure of the expectations hypothesis of the term structure, the Mehra-Prescott equity premium, the undervaluation of closed-end mutual funds, and several others. The essential assumption we use is that the opinions of noise traders are unpredictable and arbitrage requires bearing the risk that their misperceptions become even more extreme tomorrow than they are today. Since "unpredictability" seems to be a general property of the behavior of irrational investors, we believe that our conclusions are not simply a consequence of a particular parameterization of noise trader actions.

Our model suggests that much of the behavior of professional arbitrageurs can be seen as a response to noise trading rather than as trading on fundamentals. Many professional arbitrageurs spend their resources examining and predicting the pseudosignals noise traders follow in order to bet against them more successfully. These pseudosignals include volume and price patterns, sentiment indices, and the forecasts of Wall Street gurus. Just as it pays entrepreneurs to build casinos to exploit gamblers, it pays rational investors to spend considerable resources to exploit noise traders. In both cases, private returns to the activity probably exceed social returns.

Our focus on irrationality in financial markets departs from that of earlier studies of rational but heterogeneously informed investors (Grossman and Stiglitz 1980; Townsend 1983; Varian 1986; Stein 1987). Many of the results in this paper could perhaps be derived using a fully rational model with differentially informed investors, provided that one gets away from the "no-trade" theorems (Milgrom and Stokey 1982).

Apart from the question of tractability, we have focused on models of irrationality for three reasons. First, in the context of fluctuations in the aggregate market, we find the idea of privately informed investors somewhat implausible. While one can always think of a person's opinion as private information, this seems like playing with words. Speaking of the private information of a market timer like Joe Granville—who himself insists that he has a "system" rather than an informational advantage—makes little sense to us. Second, given the traditional argument that the stock market price aggregates information and opinions, it is important to examine the extent to which there is a tendency of prices to reflect "good" rather than "bad" opinions. Even more than Figlewski's (1979) result that "bad" opinions can influence market prices for a long time, our paper suggests skepticism about the long-run irrelevance of "bad" opinions. Third, our analysis illustrates the point that studying irrational behavior does not always require specifying its content. We have shown that something can be learned about financial markets simply by looking at the effect of *unpredictability* of irrational behavior on the opportunities of rational investors. The idea of noise trader risk is much more general than our particular examples. In future research, it would be valuable to consider asset markets with more primitive descriptions of irrationality. One advantage of such an approach would be to generate more restrictive predictions that are easier to reject.

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