

The use of information theory in nonequilibrium statistical mechanics was introduced by Jaynes in the late 1950's. Despite some successes for near-equilibrium systems, this "entropy maximization" approach does not enjoy the same level of acceptance as its equilibrium counterpart—Gibbs's formalism. As the author of this paper points out, a reason for this state of affairs is the "paucity of published results . . . particularly with regard to new testable predictions of behaviour far from equilibrium". He then proceeds towards remedying the situation by deriving a general form of entropy maximization over paths in phase space (rather than just states) and applying it to three specific problems of nonequilibrium theory: maximal entropy production (MaxEP), the fluctuation theorem (FT) and self-organized criticality (SOC).

The implementation of the Jaynes formalism, that is, maximizing the path information entropy  $S_I = -\sum_{\Gamma} p_{\Gamma} \ln p_{\Gamma}$  with respect to  $p_{\Gamma}$  subject to constraints, is done in two steps. In the first, one maximizes  $S_I$  subject to fixed initial internal energy and mass density configurations, including the constraints of energy and mass conservations. The resulting quantity is denoted by  $S_{I,\max}(\lambda)$  and is a functional of the Lagrange multipliers  $\lambda(\mathbf{x})$ . The associated distribution is denoted by  $p_{\Gamma}(\lambda)$ . In the second step,  $S_{I,\max}(\lambda)$  is maximized with respect to  $\lambda(\mathbf{x})$  subject to the external constraints.

As part of the work for the first step, the author is led to introduce a time averaged entropy production rate, as well as nonequilibrium definitions of temperature and chemical potential. Aware of the existing criticism to such definitions for systems which are not in equilibrium, he remarks that they appear as Lagrange multipliers conjugate to the internal energy and the mass density, and are therefore independent of any (local) equilibrium hypothesis.

The last section of the paper contains the announced derivation of the FT, MaxEP and SOC as

corollaries of the general Jaynes formalism just obtained. The FT is explained by a time-reversal argument in step 1, whereas the MaxEP and SOC are obtained somewhat more qualitatively in a "mean field approximation". For example, the emphasis is in showing that the Jaynes formalism can in principle be used to calculate the precise behaviour of systems constrained by a slow driving flux.

The discussion is peppered with several examples from real physical systems, such as climate and atmospherical problems. It ends with the remark that the generality of the maximum entropy principle (being based on information theory rather than on the details of the underlying physics of the problems) makes it applicable to more general dynamical systems, such as in economics and biological populations.

Reviewed by M. R. Grasselli

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