Past production constrains current energy demands: persistent scaling in global energy consumption and implications for climate change mitigation

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Abstract

Climate change has become intertwined with the global economy. Here, we describe the importance of inertia to continued growth in energy consumption. Drawing from thermodynamic arguments, and using 38 years of available statistics between 1980 to 2017, we find a persistent time-independent scaling between the historical time integral W of world inflation-adjusted economic production Y, or $W(t) = \int_0^t Y(t')\,dt'$, and current rates of world primary energy consumption $\mathcal{E}$, such that $\lambda = \mathcal{E}/W = 5.9 \pm 0.1$ Gigawatts per trillion 2010 US dollars. This empirical result implies that population expansion is a symptom rather than a cause of the current exponential rise in $\mathcal{E}$ and carbon dioxide emissions C, and that it is past innovation of economic production efficiency $Y/\mathcal{E}$ that has been the primary driver of growth, at predicted rates that agree well with data. Options for stabilizing C are then limited to rapid decarbonization of $\mathcal{E}$ through sustained implementation of over one Gigawatt of renewable or nuclear power capacity per day. Alternatively, assuming continued reliance on fossil fuels, civilization could shift to a steady-state economy that devotes economic production exclusively to maintenance rather than expansion. If this were instituted immediately, continual energy consumption would still be required, so atmospheric carbon dioxide concentrations would not balance natural sinks until concentrations exceeded 500 ppmv, and double pre-industrial levels if the steady-state was attained by 2030.

1 Thermodynamic overview of civilization growth

Like other biological systems [1], the human economy interacts with its surroundings through flows of energy and matter [2,3]. Collectively, we mine primary energy resources, and use the energy to power civilization and convert raw materials into the material make-up of civilization or Earth's "technosphere" [4]. The circulations of our lives include the back-and-forth material exchange of people, goods, and information along transportation and communication networks, and our cardiovascular, pulmonary and nervous systems [5–8]. These require a power source, and any impulse of energy that passes through these networks is ultimately dissipated through frictional losses as waste heat. Concurrently, our infrastructure, bodies, and even our memories undergoes decay. So, without a continual drawdown of primary energy and raw material resources to continually rebuild civilization, it would inevitably fall apart.

While the entirety of humanity seems infinitely complex in its range of activities, what all social phenomena have in common is their need for energy. The sum total
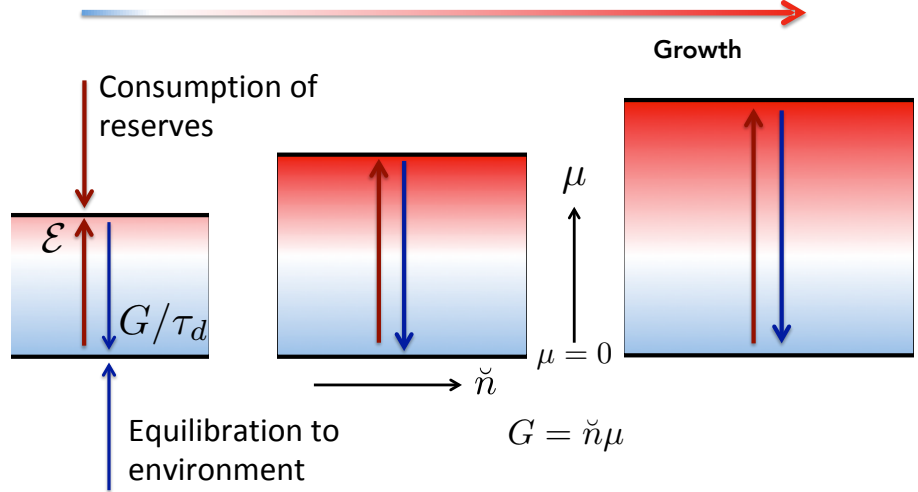
Fig 1. Illustration of the short-term thermodynamic balance between primary energy consumption $\mathcal{E}$ and dissipation at rate G/τ_d (Eq. 2) in a civilization experiencing long-term, proportionate material $\breve{n}$ and specific potential μ growth (Eq. 14).

of their power consumption is constrained by the world total, or the rate of primary energy consumption $\mathcal{E}$. By consuming primary energy, for example through combustion, civilization is stretched to a potential G, higher than an equilibrium with the environment characterized by $G = 0$ (Fig. 1).

This potential energy G is converted into civilization circulations by doing reversible work $\mathcal{W}^{rev}$ to move civilization elements along pathways that align with the potential gradient ∇G [9] and, through frictional losses, work is converted to waste heat that is radiated to space. If the average velocity of the circulations is $\vec{v}$, and no primary energy is consumed, then the rate at which potential energy is dissipated is

$$\mathcal{W}^{rev} = -\frac{dG}{dt} = \vec{v} \cdot \nabla G = \frac{G}{\tau_d} \tag{1}$$

where the dissipation time is $\tau_d = 1/(\vec{v} \cdot \nabla \ln G)$.

The reason civilization does not relax to an environmental equilibrium is that sustained primary energy consumption continually elevates G. For example, oil is extracted from a well for consumption, and there is back-pressure in the reserves to replenish supplies. Available statistics [10] suggest that civilization has been consuming energy nearly as fast as it has been produced. Since 1980, the mean ratio of global production to consumption has been 0.998 with an annual standard deviation smaller than 1%. So, while G may fluctuate because consumption and dissipation are out of phase over rapid timescales shorter than τ_d, averaged over timescales a bit longer, say one week, these variations are no longer apparent. Then, the quasi-equilibrium condition for civilization sustenance is:

$$\left(\frac{dG}{dt}\right)^{sust} = \mathcal{E}^{sust} - \frac{G}{\tau_d} \simeq 0 \tag{2}$$

implying that $\mathcal{E}^{sust} = G/\tau_d$ is the required energy consumption to sustain civilization circulations, or in terms of the thermodynamic First Law, to maintain a short-term balance between external heating and doing reversible work $\mathcal{W}^{rev}$.

Civilization has a wide range of activities, each with their own energetic demands and timescales that could be characterized by a power spectrum of consumption $\mathcal{E}_\tau^{sust}$

satisfying

$$\mathcal{E}^{sust} = \int_0^\infty \mathcal{E}_\tau^{sust} \tau d\tau \tag{3}$$

in which case the characteristic timescale for civilization is $\tau_d = \int_0^\infty \mathcal{E}_\tau^{sust} \tau d\tau / \mathcal{E}^{sust}$. Given that approximately 40% of our time is spent alternating between our top two preferred locations, (e.g. home and work) [11], τ_d might be taken to be about 1 day. Currently, the world primary energy consumption rate is roughly 20 TW, so it follows from the quasi-equilibrium relationship Eq. (2) that

$$G = \mathcal{E}^{sust} \tau_d \simeq 20 \times 10^{12} \,\mathrm{J\,s^{-1}} \times 86400 \,\mathrm{s} = 1.7 \times 10^{18} \,\mathrm{J}, \tag{4}$$

equivalent to the energy contained in 270 million barrels of oil.

Obviously, such high potential is a far cry from where we started in the Stone Age. How did we become so strong? There has been no external hand to turn up the civilization flame. Purely mathematically, reaching our current state meant accumulating successive increments in potential G over civilization's history:

$$G(t) = \int_0^t \frac{dG(t')}{dt'} dt' \tag{5}$$

Eq. 2 implies that the primary energy supply $\mathcal{E}$ must have been greater than humanity's collective metabolic needs $\mathcal{E}^{sust}$ so that

$$\frac{dG}{dt} = \frac{G}{\tau_{long}} = \mathcal{E} - \mathcal{E}^{sust} > 0 \tag{6}$$

An important point here is that civilization growth timescales τ_{long} are decades to centuries, that is, much longer than τ_d. For example, recent growth rates of global primary energy consumption are approximately 2% per year (or equivalent to $\tau_{long} \simeq 50$ years), so the implied difference between $\mathcal{E}$ and $\mathcal{E}^{sust}$ is only about 0.01%. Such growth might be imperceptible in our daily lives, but it does slowly accumulate. We can therefore write

$$\mathcal{E}^{sust} = (1 - \epsilon)\mathcal{E}, \tag{7}$$

where $\epsilon = \tau_d/(\tau_d + \tau_{long}) \ll 1$, so that the approximation $\mathcal{E} \simeq \mathcal{E}^{sust}$ can be made that at any given time. For the remainder of $\mathcal{E}$, we have

$$\mathcal{W}^{irr} = \frac{dG}{dt} = \epsilon \mathcal{E}, \tag{8}$$

so that ϵ can be seen to be the efficiency of converting primary energy consumption $\mathcal{E}$ to the irreversible work $\mathcal{W}^{irr}$ that enables growth. Note how the sign on $\mathcal{W}^{irr}$ is opposite to that in Eq. 1, where potential energy is dissipated to do the reversible work $\mathcal{W}^{rev}$ that maintains civilization circulations. Instead, irreversible work is done to grow the civilization potential G. Moreover, it follows that the growth rate of the potential is given by

$$\eta_G := \frac{d\ln G}{dt} = \frac{1}{G}\frac{dG}{dt} = \frac{1}{\tau_{long}} = \frac{\epsilon}{(1-\epsilon)\tau_d} \simeq \frac{\epsilon}{\tau_d} \tag{9}$$

Similarly, taking the derivative of

$$\mathcal{E} = \frac{\mathcal{E}^{sust}}{1 - \epsilon} = \frac{G}{(1-\epsilon)\tau_d} \tag{10}$$

we find that the growth rate of primary energy consumption is

$$\eta_\mathcal{E} := \frac{d\ln \mathcal{E}}{dt} = \frac{1}{\mathcal{E}}\frac{d\mathcal{E}}{dt} = \frac{\epsilon}{(1-\epsilon)\tau_d}\left(1 + \tau_d \eta_\epsilon\right) \simeq \frac{\epsilon}{\tau_d} \tag{11}$$

where

$$\eta_\epsilon := \frac{d\ln\epsilon}{dt} \tag{12}$$

is the growth rate of ϵ and $\tau_d\eta_\epsilon \ll 1$.

The mechanism through which an imbalance leading to $\mathcal{W}^{irr} > 0$ should emerge is not discussed in detail here. It follows from discovering accessible energy resources faster than they are consumed [12, 13]. The gradient ∇G becomes steeper nearer the energy source than to the dissipative sink, so that there is a net convergence of energy in civilization. Very generally, this process is analogous to the heat equation $(dG/dt)_{long} = \nabla \cdot (\mathcal{D}\nabla G) = \mathcal{D}\nabla^2 G$, where $\mathcal{D}$ is a constant diffusivity.

For more intuitive insight, consider a child as a more familiar complex system. From Eq. 2, a child with potential G has quasi-equilibrium metabolic needs of $\mathcal{E}^{sust} = G/\tau_d$ – say about 50 Watts, or about 500 kJ per day per kg. Any food energy the child consumes is used to do reversible work $\mathcal{W}^{rev}$ to maintain rapid internal neurological, respiratory, cardiovascular, and nervous circulations and to reconstitute food nutrients into its material makeup. If the energy in the proteins, carbohydrates and fats of accessible food is only just sufficient to offset decay and keep the child alive, then the child is at steady-state and G and $\mathcal{E}^{sust}$ do not change. A healthy child, however, does net irreversible work at rate $\mathcal{W}^{irr} = \epsilon\mathcal{E}$ to convert some small portion of the energy in food into accumulation of body matter through a conversion factor of about 30 MJ kg^{-1}. The child eventually becomes a robust adult with higher daily energy demands and the growth rate (hopefully) stabilizes. But even then, a typical rate of weight gain translates to a value of W^{irr}/W^{rev} of about 0.2%, while seemingly tiny, results in a 10 kg gain in mass, or 300 MJ of energy, over the 50 years span of a typical adult life [14].

This treatment suggests how to consider the coupling of energy and matter in an open system such as civilization. G is a total potential, so it can be decomposed into any arbitrary number of sub-components with $G = \sum \breve{n}_i\mu_i$, depending on how closely civilization is resolved. Here we take the simplest possible approach which is to suppose, as illustrated in Fig. 1, that the accessibility of energy by civilization can be defined by an interface with resources composed of $\breve{n}$ material elements each with average potential μ, so that

$$G(t) = \breve{n}(t)\mu(t). \tag{13}$$

Thus, civilization elements are not a purely additive summation of civilization "things" n. Rather they represent a number of network nodes, defined in terms of people, firms, or nations that collectively do work to dissipate energy at rate $\mathcal{E}^{sust}$.

It therefore follows from 8 and 13 that any long-term net convergence $\epsilon\mathcal{E}$ is a surplus that can be partitioned between manufacturing more civilization nodes or increasing their average potential μ:

$$\epsilon\mathcal{E} = \mathcal{W}^{irr} = \frac{dG}{dt} = \mu\frac{d\breve{n}}{dt} + \breve{n}\frac{d\mu}{dt} \tag{14}$$

By increasing the average potential at rate $d\mu/dt$, existing civilization elements $\breve{n}$ can go farther and faster. On the other hand, production of new civilization elements $d\breve{n}/dt$ with average potential μ implies a phase change. Just as an excess 30 MJ of energy is required for the chemical transformation of food into a kilogram of flesh, stationary raw materials such as forests, fish stocks, and iron ore are rearranged into familiar forms such as cars, roads, and communications systems.

We assume the following thermodynamic proportionality:

$$\frac{d\mu}{d\breve{n}} = (k-1)\frac{\mu}{\breve{n}} \tag{15}$$

for a constant k. Substituting this into Eq. 14 leads to

$$\mathcal{W}^{irr} = k\mu\frac{d\breve{n}}{dt} \tag{16}$$

or rearranging, net production of network nodes is related to current energy consumption through

$$\frac{d\breve{n}}{dt} = \frac{\epsilon}{k\mu}\mathcal{E} \tag{17}$$

where $\epsilon/(k\mu)$ is the efficiency of converting primary energy consumption to network growth.

To summarize, we consume energy at rate $\mathcal{E}$. Most of it, namely $\mathcal{E}^{sust} = (1-\epsilon)\mathcal{E}$, is used to do reversible work at rate $\mathcal{W}^{rev}$ to sustain circulations along civilization networks. The surplus $\epsilon\mathcal{E}$ is used to do irreversible work at rate $\mathcal{W}^{irr}$ to convert raw materials into the stuff of humanity and extend existing network nodes at rate $d\breve{n}/dt$. These form the fabric of society, including roads, telecommunications networks and even neural pathways encapsulating memories within our brains. We grow fastest if the efficiency ϵ is high. And through expansion of the physical interface at rate $d\breve{n}/dt$ with reserves of energy and matter, civilization grows, leading recursively to further expansion and higher consumption.

2 Thermodynamics of global economic value

Can these strictly thermodynamic concepts be linked to the economy expressible in financial terms? Suppose for the moment that they can, and that there exists a hypothetical global quantity expressible in units of real currency (or "widgets") W that expresses the size of civilization and is presumed to be proportional to energy consumption and the civilization potential through a constant scaling factor λ:

$$\frac{G}{\tau_d} \simeq \mathcal{E} = \lambda W \tag{18}$$

In this expression, civilization elements of whatever kind contribute to W only insofar as they contribute to the overall network capacity to dissipate primary potential energy at rate $\mathcal{E}$, requiring a thermodynamic potential G.

From a purely dimensional perspective, the simplest possible economically quantifiable definition of W is that it is an integration over time of a global quantity with units of widgets per time. The approach previously taken in [15] was to suppose that the most obvious candidate is the world economic production Y (or gross domestic product GDP) calculated at market exchange rates (MER) and adjusted for inflation, in which case W is the world cumulative production:

$$W = \int_0^t Y(t')\,dt' \tag{19}$$

From Eqs. 18 and 19, the testable hypothesis is that economic production aggregated for all nations and integrated over all of history is tied through a constant λ to energy consumption through:

$$\mathcal{E}(t) = \lambda \int_0^t Y(t')\,dt' \tag{20}$$

Note the similarity here with Eq. 5. If the expression in Eq. 20 can be shown to empirically justified, it would imply that there is a constant relationship between

inflation-adjusted economic production Y and a <u>rise</u> in global energy consumption demands $\mathcal{E}$:

$$Y = \frac{dW}{dt} = \frac{1}{\lambda}\frac{d\mathcal{E}}{dt} \qquad (21)$$

From Eqs. 11 and 17, we can see that economic production is tied to the use of primary energy to do irreversible work to convert raw materials into civilization matter. Namely,

$$Y = \frac{\mathcal{E}\eta_{\mathcal{E}}}{\lambda} \simeq \frac{\epsilon\mathcal{E}}{\lambda\tau_d} = \frac{k\mu}{\lambda\tau_d}\frac{d\check{n}}{dt} = \frac{\mathcal{W}^{irr}}{\lambda\tau_d} \qquad (22)$$

In other words, with a surplus of energy $\epsilon\mathcal{E}$, the extraction of raw materials can be tied to the production of economically useful material goods (namely "widgets") as has been noted previously in [16]. These add value by speeding up human activity and increasing civilization size. Economic production provides the recipe for growth by expanding our collective interface with energy and material reserves, leading to positive increments in our capacity to consume.
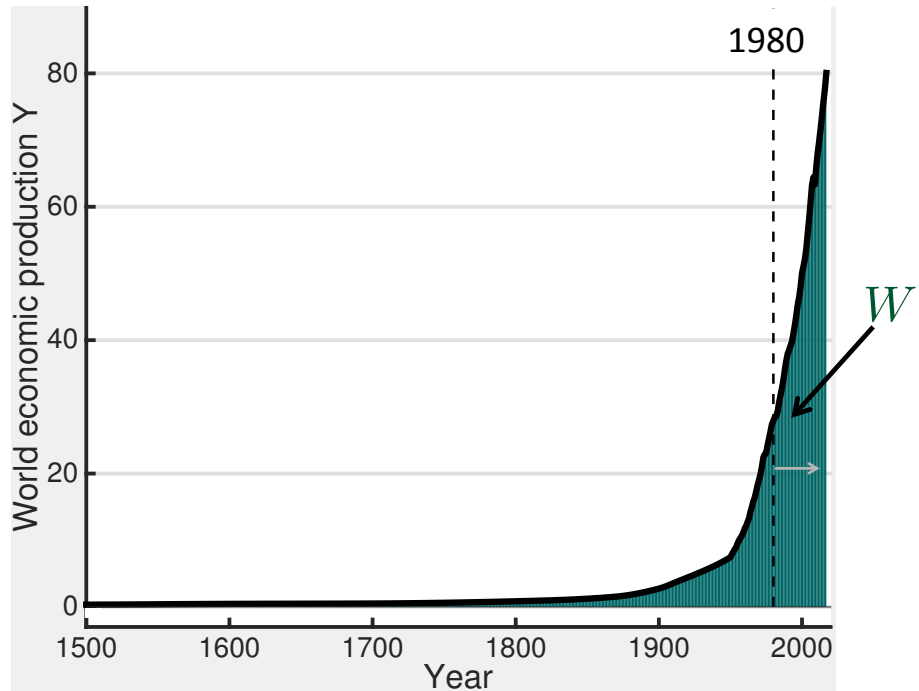

Fig 2. Evolution of world economic production Y in trillion 2010 USD per year (solid line) and the integrated contribution to the world cumulative production W in trillion 2010 USD proportional to the shaded area under the curve. The period between 1980 to 2017 that is used for comparison with world primary energy consumption $\mathcal{E}$ as described in the text is delineated by the dashed line and shown by the gray arrow.

Available data for testing Eq. 20 include annual market exchange rate estimates of GDP, inflation-adjusted to "real" units (namely constant 2010 US dollars) from the World Bank [17] and the United Nations [18] for the years 1970 to 2017, and reconstructions from the Maddison database of the real GDP adjusted for purchasing power parity (PPP) 1990 USD for each year between 1950 and 1992, with more sparse estimates extending back to 1 CE [19] (see details in Appendix A). Annual rates of global primary energy consumption $\mathcal{E}$ are available from British Petroleum and the U.S. Department of Energy (DOE) Energy Information Administration (EIA) for the time period 1980 to 2017 [10, 20]. For initialization of the integration in Eq. 19, it is estimated that

world cumulative production in 1 C.E. was 250 trillion 2010 USD, a number that is $_{159}$
obtained iteratively so that there is consistency for that period between growth of W $_{160}$
and population growth rates of about 6% per century [21]. This reconstruction is about $_{161}$
7.3% of the value obtained for 2017, suggesting the ancient world had already evolved $_{162}$
non-negligible wealth in its Western, Middle Eastern, and Eastern empires [22]. The $_{163}$
Maddison database is sparse, and presumably increasingly uncertain the farther one $_{164}$
goes back in time. However, the value of W accumulated over the period 1 CE to 1000 $_{165}$
CE is just 4.6% of the value in the year 2017. It is not until the last century that $W(t)$ $_{166}$
grows appreciably (Figure 2). The world cumulative production between 1980 and 2017 $_{167}$
comprises a remarkable 60% of the historically accumulated total.
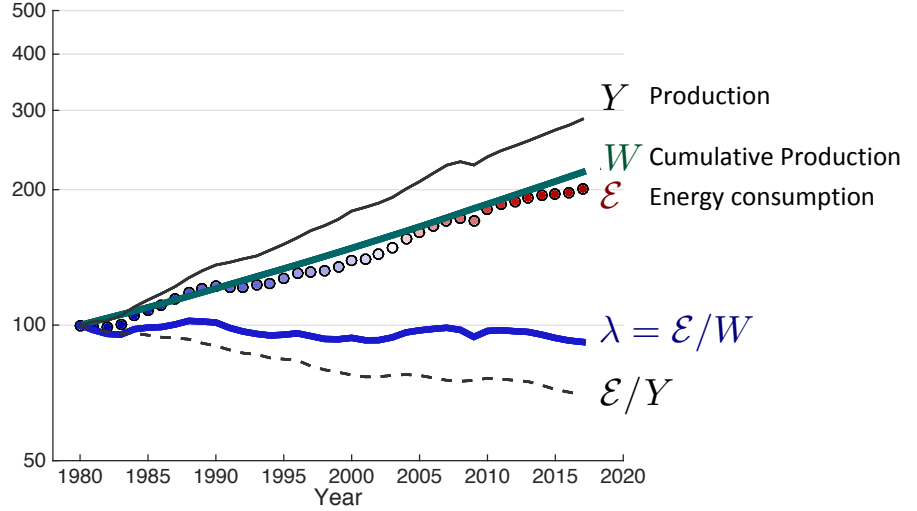

Fig 3. Relative evolution since 1980 of the world real GDP Y, economic potential
$W = \int_0^t Y(t') dt'$ (Eq. 19), primary energy consumption $\mathcal{E}$, $\lambda = \mathcal{E}/W$ (Eq. 18) and the
energy intensity of production $\mathcal{E}/Y$.

Table 1. The global value of λ ((gigawatts per trillion 2010 US dollar with standard
deviation) defined by Eq. 18 for various time periods.

Period	1980-1990	1990-2000	2000-2010	2010-2017	1980-2010	1980-2017
$\lambda = \mathcal{E}/W$	6.04±0.14	5.83±0.15	5.83±0.14	5.79±0.14	5.90±0.16	5.88±0.17

$_{168}$

The relative evolution since 1980 of $\mathcal{E}$, Y, W, $\lambda = \mathcal{E}/W$ and $\mathcal{E}/Y$ is shown in Figure $_{169}$
3 and summarized in Table 1. Expressing Eq. 20 as a summation of yearly data, $_{170}$
$\lambda_j = \mathcal{E}_j / \sum_0^j Y_i$, the mean value[1] of λ is 5.9 with a standard deviation of 0.1 (2%) and an $_{171}$
uncertainty in the mean at the 95% confidence level of 0.05 (0.8%). As an indicator of $_{172}$
the sensitivity to uncertainty in the data, assuming for the initializing of the integration $_{173}$
a value of $W(1)$ double the previously derived value of 250 trillion 2010 USD, then $_{174}$
$\lambda = 5.2 \pm 0.2$. If half as much, then $\lambda = 6.2 \pm 0.2$. While there is arguably a small $_{175}$
secular downward trend of 0.1% yr^{-1}, the temporal variation in λ is sufficiently small $_{176}$
and insensitive to assumptions that it appears useful as a scaling factor relating an $_{177}$
economic quantity W to a physical measure $\mathcal{E}$. If $\tau_d \simeq 1$ day, then the implication is $_{178}$
that $G/W = \lambda \tau_d \simeq 510 \pm 9$ Joules per 2010 US dollar. $_{179}$
The relationship in Eq. 20 appears to hold empirically. But it might seem surprising $_{180}$
given that it lacks any explicit representation of the spontaneous appreciation or of con- $_{181}$

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[1]All values for λ in this paragraph have units of gigawatts per trillion 2010 USD

sumption, decay, and depreciation. Obviously, trade agreements and resource discovery add value without being tabulated in production. And not all of what has been produced remains as old technologies turn obsolete and networks fray due to physical destruction by storms, rust, and forgetting. A cook may produce food in a restaurant, but other than temporary body sustenance and lingering memories, the meal is gone. The total "wealth" of civilization cannot be simply a summation of production. Equation 20 is easier to justify, however, if there is exists a steady fraction of production that contributes to civilization expansion through irreversible work, as in Eq. 22.

In traditional macroeconomic growth models, there is also a quantity with units of widgets, termed real capital K that relates to real economic output Y according to some production function $Y = f(K, L)$ where L represent labor. A common example is the Cobb-Douglas production function $Y = AK^\alpha L^{1-\alpha}$ where α is determined from a fit to data and A is termed a "total factor productivity" that can itself evolve with time according to, for example, investments in research and development [23] (see also [24] for a different interpretation of A in the context of a Cobb-Douglas production function taking into account energy inputs). One could argue that the appropriate theoretical relationship to draw is not between $\mathcal{E}$ and W as defined in Eq. 19, but between $\mathcal{E}$ and capital K.

To explore this possibility further, observe that capital K evolves as

$$\frac{dK}{dt} = I - \delta K = Y - \mathcal{C} - \delta K \tag{23}$$

where $I = Y - \mathcal{C}$ is gross investment, δ is a depreciation rate, and $\mathcal{C}$ refers to consumption of goods that are assumed to depreciate much faster than capital itself. If we define γ as the fraction of production used for consumption plus depreciation, that is

$$\gamma Y = \mathcal{C} + \delta K, \tag{24}$$

then Eq. 23 becomes $dK/dt = (1 - \gamma)Y$, or equivalently,

$$K(t) = \int_0^t (1 - \gamma(t')) Y(t') \, dt'. \tag{25}$$

Provided γ does not vary greatly in time, we can approximate the factor $(1 - \gamma)$ by its time average $\langle 1 - \gamma \rangle$ so that

$$K \simeq \langle 1 - \gamma \rangle \int_0^t Y(t') \, dt'. \tag{26}$$

In other words, if there were a fundamental scaling between $\mathcal{E}$ and K, this would also imply the scaling expressed in Eq. 20.

Then, if society experienced a sudden collapse in its growth toward a steady-state economy with constant $\mathcal{E}$, economic production Y could remain positive, even adjusting for inflation, but the implication would be that all production would be used simply for consumption and to sustain capital K at its current state, that is ot say, $\gamma(t) = 1$, and both K and $\mathcal{E}$ are stationary.

Unfortunately long-term global time series to test whether K scales with $\mathcal{E}$ are scant, in part because there is disagreement among economists about how to appropriately aggregate the value of items as different as houses and tractors [25], and also because of difficulties with estimating a starting point for the corresponding time series [26] similarly to the difficulty we mentioned above related to $W(1)$. Also, it is unclear what to include because economic capital does not normally consider for example people or their culture, even though these are core elements of the human dissipative engine. In other words, whereas capital K and the global quantity W in Eq. 18 may be numerically

related, they are not conceptually analogous. The value expressed by W lies not so much in an aggregation of inert "things", but in their summed capacity to form interconnected networks through a thermodynamic potential $G = \check{n}\mu$ that sustains the dissipation of potential energy at rate $\mathcal{E}$, whether it is from international trade, housework, or the firing of neuronal networks in human brains.

In the meantime, even if W calculated from Eq. 19 yields an overestimate of the true aggregated wealth of civilization, λ as calculated in Eq. 20 has been nearly a constant for sufficiently long that it seems reasonable to assume that this regularity will carry into the future.

3   Implications for economic growth

Adopting a nearly fixed relationship between W and $\mathcal{E}$ offers a testable basis for exploring thermodynamic constraints on economic growth. From Eqs. 19 and 20, both the world cumulative production and primary energy consumption grow at the exponential rate:

$$\eta_{\mathcal{E}} = \eta_W := \frac{1}{W}\frac{dW}{dt} = \frac{Y}{W} = \lambda\frac{Y}{\mathcal{E}} = \lambda\varepsilon \tag{27}$$

where $\varepsilon := Y/\mathcal{E}$ is the energy efficiency of economic production or energy productivity. That is to say, higher energy productivity is related with increased energy demands.

This result may seem counter-intuitive, but it rests only on the empirical result that λ is a constant. It should not be confused with the concept of backfire or Jevons' Paradox [27–29], which also argues that efficiency gains lead to consumption growth, but is stated within the context of traditional economic models for specific economic sectors or nations and is often refuted. Here, Eq. 27 applies strictly within a global context so that complications due to trade do not play a role. Interpreted physically, Eq. 27 can be compared with Eq. 22, suggesting that $\varepsilon \simeq \epsilon/\lambda\tau_d$, or that energy productivity is related to the efficiency of doing irreversible work $\mathcal{W}^{irr} = \epsilon\mathcal{E}$ that further expands the capacity to consume. The implication is that civilization is an emergent phenomenon that grew spontaneously to its currently high state of production through an energy surplus.

The decades following 1950 – known as the "great acceleration" – stand out in particular, when a relative ease of access to oil led to rapid innovations [13, 30]. To characterize acceleration in growth, η_W can be considered to have its own exponential growth rate

$$\eta_I := \frac{d\ln\eta_W}{dt} \tag{28}$$

We term this an "innovation rate" because From Eq. 27, if λ is a constant, η_I can also be expressed as

$$\eta_I = \eta_\varepsilon := \frac{d\ln\varepsilon}{dt} \tag{29}$$

Whenever conditions support an increase in energy productivity ε (or thermodynamic efficiency ϵ), for example due to energy reserve discoveries [13], then $\eta_I > 0$ and civilization growth is superexponential.

The world GDP growth rate $\eta_Y := d\ln Y/dt$ follows a slightly different pathway. From $Y = \varepsilon\mathcal{E}$ and Eq. 27, it follows that:

$$\eta_Y = \eta_{\mathcal{E}} + \eta_\varepsilon = \lambda\varepsilon + \eta_\varepsilon \tag{30}$$

At global levels, by reducing the energy required for manufacture of valuable goods, civilization bootstraps through innovation to higher GDP growth and accelerating rates of energy consumption.

Table 2. Measured average growth rates (%/yr) compared with rates derived assuming λ is a constant in bold. Pertinent equations are in parentheses.

	Wealth	Energy		Innovation		GDP	
	η_W	$\eta_{\mathcal{E}}$	$\lambda\varepsilon$ (27)	η_I (28)	η_ε (29)	η_Y	$\lambda\varepsilon + \eta_\varepsilon$ (30)
1980-2010	2.06	1.98	**2.09**	0.82	**0.91**	2.88	**3.0**
2010-2017	2.33	1.60	**2.40**	0.45	**1.18**	2.78	**3.58**
1980-2017	2.14	1.84	**2.15**	0.73	**1.0**	2.84	**3.15**

Table 2 shows a check of the applicability of the derived growth equations 27 to 30 for the period between 1980 and 2010 and a shorter more recent period from 2010 to 2017. Overall, there is close agreement between observations and calculated rates of change based on the constancy of λ suggesting the relationship offers a useful tool for making simplified predictions. However, there are also some discrepancies after 2010. For example, as stated in Eq. 27, a consequence of a constant value for λ is that the primary energy consumption growth rate $\eta_{\mathcal{E}}$ should be equivalent to both the growth rate of the world cumulative production η_W and the product $\lambda\varepsilon$. For the period 1980 to 2010 over which energy consumption increased by 80%, the three calculations differ by at most 6%.

Similarly the GDP growth rate calculated from the expression $\lambda\varepsilon + \eta_\varepsilon$ (rightmost in Eq. 30) agrees to within 4% with the directly calculated value η_Y. However, for the shorter period between 2010 and 2017, the agreement is less precise, and also for the time period 1980 to 2017. The reason is unknown, although note from Table 2 and the mathematical equivalency $\eta_Y = \eta_{\mathcal{E}} + \eta_\varepsilon$ that any discrepancy between η_Y and $\lambda\varepsilon + \eta_\varepsilon$ is due entirely to discrepancies between $\eta_{\mathcal{E}}$ and $\lambda\varepsilon$. Energy consumption increased by just 11% between 2010 and 2017 so there is greater susceptibility to quantification errors. Also, nominal GDP is what is measured, and any calculation of real Y requires an accurate assessment of the GDP deflator that attempts to account for inflation, and its true magnitude may have been underestimated. Indeed, $\lambda = \mathcal{E}/W$ was 2% lower in the latter period (Table 1), and any departure from constancy affects calculation of higher order derivatives such as the growth rates $\eta_{\mathcal{E}}$ and η_Y.

4 Implications for carbon dioxide emissions and concentrations

4.1 Simplifications to socio-economic drivers

The anthropogenic contribution to atmospheric carbon dioxide emissions can be conveniently decomposed into the product of population P, affluence expressible as the gross domestic product (GDP) per person $g := Y/P$, the energy intensity of economic production $i =: \mathcal{E}/Y = 1\varepsilon$, and the amount of carbon dioxide emitted by the choice of energy source $c := C/\mathcal{E}$, leading to the Kaya identity $C = P \times g \times i \times c$ [31]. In terms of growth rates, we have

$$\eta_C = \eta_P + \eta_g - \eta_\varepsilon + \eta_c \tag{31}$$

The most recent IPPC report [32] lists rapid increases in population and standard of living as primary drivers of past emissions growth but focuses on innovations that improve production efficiency and reductions in the carbon intensity of fuels as targets for future reductions.

Eq. 31 helps frame issues surrounding climate change mitigation. But it is only a mathematical identity and, as such, it does not directly allow for dynamic interactions between terms. What the link expressed in Eq. 20 between current consumption and the

economy provides is an added strong constraint on interrelationships between population, standard of living, and production efficiency. For example, inserting Eq. 20 in $C = c\mathcal{E}$ yields that current carbon dioxide emissions can be related to <u>past</u> accumulated economic production through

$$C(t) = \lambda c \int_0^t Y(t')\,dt' \tag{32}$$

A revised expression then follows for Eq. 31, namely

$$\eta_C = \eta_c + \eta_{\mathcal{E}} = \eta_c + \lambda\varepsilon \tag{33}$$

where we used Eq. 27.

A test of Eq. 33 is shown in Table 3, based on the values for $\lambda\varepsilon$ shown in Table 2 and data from the Global Carbon Atlas [33]. Between 1980 and 2010, the observed average annual rate of emissions growth η_C was 1.77 % yr^{-1}, within 6% of the calculate value of 1.88 % yr^{-1} for $\eta_c + \lambda\varepsilon$. For more recent years, the discrepancy is 20%.

Table 3. Average growth rates in carbonization and CO_2 emissions (%/yr). Rates derived assuming λ is a constant are shown in bold, and pertinent equations in parentheses. The units of C/W are Gt C yr^{-1} per quadrillion 2010 USD.

	Scaling (Eq. 32)	Carbonization $c = C/\mathcal{E}$	CO_2 emissions C	
	$C/W = \lambda c$ ($\pm$std. dev.)	η_c	η_C	$\eta_c + \lambda\varepsilon$ (33)
1980-2010	1.50$\pm$0.06	-0.21	1.77	**1.88**
2010-2017	1.45$\pm$0.05	-0.36	1.25	**2.04**
1980-2017	1.49$\pm$0.06	-0.25	1.59	**1.90**

Similarly, the revised identity for CO_2 emissions given by Eq. 33 implies that

$$\eta_P + \eta_g = \lambda\varepsilon + \eta_\varepsilon \tag{34}$$

A test of this relation is shown in Table 4. For the 1980 to 2017 time period, the difference between both sides of the equality in Eq. 34 is about 10%. Also, note that over the longer term η_p and η_g are remarkably similar. If $\breve{n}$ can be related to population and μ to standard of living, this empirical result is consistent with the thermodynamic relationship given by Eq. 15 with $k = 2$.

Table 4. Average growth rates of population and standard of living (%/yr). Summed rate derived assuming λ is a constant from efficiency estimates are shown in bold.

	Population	Standard of living	Summation	
	η_P	η_g	$\eta_p + \eta_g$	$\lambda\varepsilon + \eta_\varepsilon$ (34)
1980-2010	1.45	1.43	2.88	**3.0**
2010-2017	1.10	1.68	2.78	**3.58**
1980-2017	1.38	1.46	2.84	**3.15**

As discussed, the production efficiency can be related to the energy efficiency through $\varepsilon \simeq \epsilon/(\lambda\tau_d)$. This suggests that, whether it is CO_2 emissions, population, or standard of living, it is the fractional imbalance between energy consumption and dissipation that drives growth. A surplus enables irreversible work to be done to make more of everything, including people, and speeding it all up. Moreover, current efficiency levels arose from an accumulation of prior innovations:

$$\varepsilon = \int_0^t \frac{d\varepsilon(t')}{dt'}\,dt' \tag{35}$$

so a conclusion might be reached that it is current and past improvements in production efficiency that have driven current growth in emissions, population, and standard of

living. Population and emissions growth rates have inertia because the world has memory of its past innovations.

4.2   Emissions stabilization and climate change mitigation

So what can be done to reverse the course of growing CO_2 emissions? Eq. 33 suggests that stabilizing carbon dioxide emissions will require the economy to decarbonize at a rate $\eta_c = -\lambda\varepsilon$ that is as fast as the rate of energy consumption growth $\eta_{\mathcal{E}} = \lambda\varepsilon$. In the period 2010-2017, we observed $\lambda\varepsilon$ to be about 2.4% per year (Table 2). For a sense of what this implies, consider that 2.4% of the current global rate of energy consumption $\mathcal{E} \approx 20$ TW corresponds to 480 GW. That is to say, for energy consumption to grow at this rate without increasing carbon dioxide emissions would require over 1 GW of new power capacity in nuclear or renewable energy to be added online each day, the approximate size of a large central power plant[2].

Furthermore, stabilized emissions do not lead to stabilized atmospheric CO_2 concentrations, not until there is a balance with natural uptake by the land and oceans. As a crude approximation, land and ocean sinks are linearly proportional to the perturbation $\Delta\left[CO_2\right]$ from pre-industrial concentrations of approximately 275 ppmv [34]. More sophisticated approaches are possible that permit accurate projections over multi-century timescales [35].

Nonetheless, a simple model readily lends insight to near term evolution while remaining consistent with multiple decades of observations. Data show that the linear sink rate σ to the land and oceans with respect to the decadally averaged perturbation $\Delta\left[CO_2\right]$ was 2.3 ± 0.5 % yr^{-1} in the 1980s, 2.4 ± 0.4 % yr^{-1} in the 1990s and 2.2 ± 0.4 % yr^{-1} in the 2000s [36]. There is no inter-decadal trend and the uncertainty is greater than the variability. Thus, an average value of 2.3 ± 0.4% yr^{-1} is assumed here.

The effect of carbon emissions on atmospheric CO_2 concentrations can be obtained by normalizing by the atmospheric mass. Every gigaton of emitted carbon corresponds to 0.47 parts per million by volume (ppmv) of increased CO_2 concentration [37]. The approximate balance equation is then

$$\frac{d\Delta\left[CO_2\right]}{dt} = \kappa C - \sigma\Delta\left[CO_2\right]. \tag{36}$$

Substituting Eq. 32, carbon dioxide emissions can then be related to past economic production and the carbonization of the energy source c through the integro-differential equation

$$\frac{d\Delta\left[CO_2\right]}{dt} = \kappa\lambda c \int_0^t Y\left(t'\right)dt' - \sigma\Delta\left[CO_2\right] \tag{37}$$

Taking κ and λ as constants then stabilization or reduction of concentrations at any given perturbation value $\Delta\left[CO_2\right]$ requires the following limits on the carbonization of emissions

$$c \leq \frac{\sigma}{\kappa\lambda}\frac{\Delta\left[CO_2\right]}{\int_0^t Y\left(t'\right)dt'} \simeq 0.26\frac{\Delta\left[CO_2\right]}{W} \tag{38}$$

where c has units of Gt C EJ^{-1}, W has units of trillion 2010 USD and the numerical coefficient $\sigma/\left(\kappa\lambda\right) \simeq 0.26$ has units of $(\text{Gt C}) \times (\text{trillion 2010 USD}) \times (\text{ppmv}^{-1}) \times (\text{EJ}^{-1})$.

Recent values for c are close to 0.017 Gt C EJ^{-1}, and despite a recent surge in renewables, c is not rapidly declining [38]. As shown in Table 3, the annual decarbonization rate η_c in recent years is just 0.36% yr^{-1} and the correspondence between CO_2 emissions C and cumulative global production W expressed as $C/W = \lambda c$ has effectively

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[2]The corresponding figure using $\eta_{\mathcal{E}} = 1.6$% directly (Table 2), instead of the value $\lambda\varepsilon = 2.4$%, is 320 GW, or just under 1 GW of new power capacity in renewable energy per day.

been unchanged over the past four decades. Based on these observations, to develop an intuition for constraints on the future, we take the baseline assumption that λc will remain fixed. In this case, Eq. 37 implies a straightforward proportionality relating the world cumulative production W and the equilibrium value for the concentration perturbation $\Delta [CO_2]_{eq}$. Setting $d\Delta [CO_2] /dt = 0$ it follows that:

$$W = \frac{\sigma}{\kappa \lambda c} \Delta [CO_2]_{eq} \simeq 15.4 \Delta [CO_2]_{eq} \tag{39}$$

Here, the numerical coefficient $\sigma / (\kappa c \lambda) \simeq 15.4$ is obtained from the years 1980 to 2010 and has units of trillion 2010 USD ppmv^{-1}. Values of this coefficient for different periods are shown in Table 5.

Table 5. Average values of the scaling $W/\Delta [CO_2]_{eq} = \sigma / (\kappa c \lambda)$ defined by Eq. 39 for various time periods, in trillion 2010 USD ppmv^{-1}

Period	1980-1990	1990-2000	2000-2010	1980-2010
$\sigma / (\kappa c \lambda)$	15.0	16.3	14.9	15.4

Comparing Eq. 39 with Eq. 18 we obtain

$$\Delta [CO_2]_{eq} \simeq \frac{\kappa c}{\sigma \tau_d} G$$

Effectively, at equilibrium with land and ocean sinks, civilization's combustion garbage heap in the form of an atmospheric CO_2 perturbation is linearly proportional to how far thermodynamically it has stretched itself away from the environmental base state of $G = 0$. Both rise as our collective historical achievement.

Figure 4 shows the relationship between the world cumulative production W and CO_2 concentrations, both for the past 2000 years and for the future assuming no future decarbonization and that energy consumption persists at its current growth trend of 2.4% yr^{-1}, a conservative estimate perhaps given that it implies an equivalent rate of world GDP growth (Eq. 30). That is to say, the open circles for each future date correspond to the non-equilibrium values for CO_2 concentrations derived from Eq. 37 assuming that W grows at the constant rate $\eta_W = \lambda \varepsilon$. In addition, for each value of W the set of solutions is provided for the CO_2 concentration at which emissions stabilize with land and ocean sinks by applying Eq. 39, namely the equilibrium concentration to which civilization is committed even for the mathematically extreme case that W were to remain constant (in other words, not only zero GDP growth, but effectively zero inflation-adjusted production).

The equilibrium CO_2 concentration is approached asymptotically with timescale $\tau_{CO_2} = 1/\sigma$, so that the difference is halved in about 30 years. For example, Figure 4 shows that stabilizing concentrations at a nominal value of 350 ppm would require that the current world cumulative production shrink by two thirds to a value not seen since 1960.

It is probably safe to assume that civilization will not willingly engage in such drastic pruning. Looking to the future, Figure 4 shows that without rapid decarbonization, we have already committed ourselves to CO_2 concentrations above 500 ppmv, well in excess of the 450 ppmv threshold that has been deemed "dangerous" [42]. At current growth rates, the commitment is to a doubling of pre-industrial levels by 2030, and to levels close to 650 ppmv by 2040.

5 Conclusions

This article identifies a persistent relationship between global energy consumption and cumulative economic production. It implies that a surprisingly simple description of the
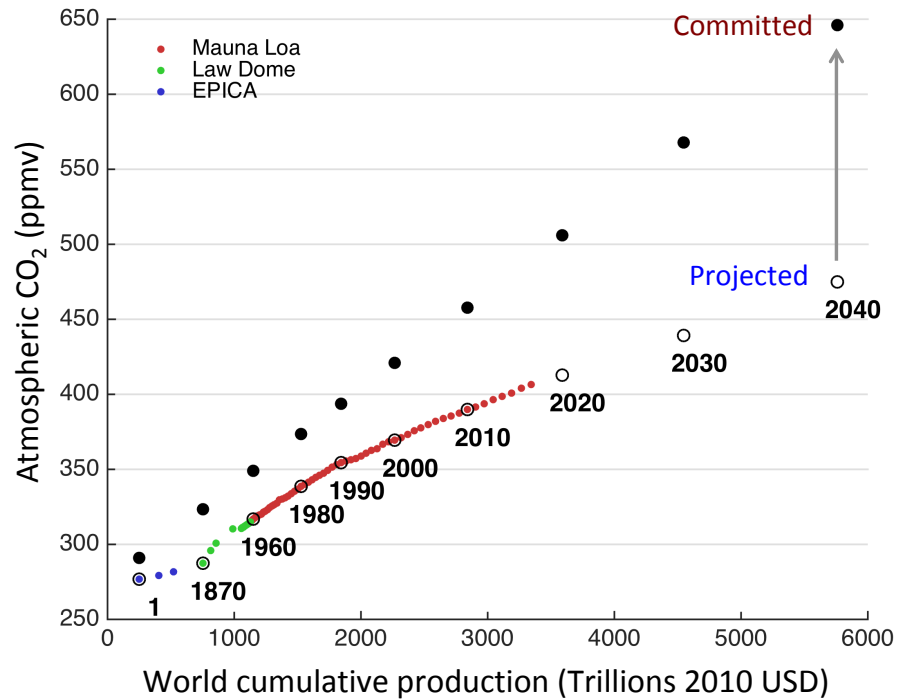
Fig 4. Historical reconstructions of world cumulative production W and atmospheric CO_2 concentrations with projections assuming $\eta_c = 0$ and $\eta_\mathcal{E} = \eta_\mathcal{E}(2017) = 2.4\%\mathrm{yr}^{-1}$ (circles) and corresponding stabilization concentrations from Eq. 39. The halving time between predicted and committed is about 30 years. Concentration data includes flask samples from Mauna Loa [39] and Antarctic ice core data [40, 41].

human system is sufficient to explain past global trends and make robust projections of the aggregated world economy and its waste products. Humanity grows when more energy is available than required for daily needs. Then work can be done not just for sustenance but for expansion. Because current sustenance demands emerge from past growth, inertia plays a much more important role in determining future societal and climate trajectories than has been generally acknowledged, particularly in the physically unconstrained models that are widely used to link the economy to climate [43,44]. We have accumulated over history a long series of innovations in efficiency that continue to propel us forward. Without forgetting these advances, we will maintain a continued ability to expand our interface with the primary resources we consume.

Eventually, of course, the interwoven networks of civilization will unravel and emissions will decline, whether it is through depletion of resources, environmentally forced decay or – as demonstrated recently – pandemics [45]. But the cuts will have to be deep, continuous, and cumulative to overcome the tremendous accumulated growth we have sustained up to this point.

The formulations presented here are intended to help constrain the problem by reducing the number of available targets that can reasonably be expected to lead to avoidance of extreme climate change. Notably, gains in energy efficiency play a critical role in enabling increases in population and prosperity, and in turn growth of energy demands and carbon dioxide emissions, contrary to what would reasonably be assumed if civilization did not grow [31,46,47]. What seems to be required is a peculiar dance between reducing the production efficiency of civilization while simultaneously innovating

new technologies that move us away from combustion. ⁴²⁴

The relationships identified all stem mathematically from the falsifiable identify $\mathcal{E} = \lambda W$ where $W = \int_0^t Y(t')\,dt'$. While the specific value of λ that was identified is 5.9±0.2 gigawatts per trillion 2010 US dollars, what matters from the standpoint of calculating trends is that the ratio $\mathcal{E}/\int_0^t Y(t')\,dt'$ is a constant to within observational uncertainty. Further theoretical work is required to link the relationship to more traditional macroeconomic modeling frameworks. Continued observations will provide a useful check on its validity. Any evidence of a sustained downward trend in λ help to pinpoint where there is a decoupling of economic production from civilization's metabolic needs.

Acknowledgments

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A Calculation of cumulative production

Market exchange rate estimates of Y_i, inflation-adjusted to "real" constant year 2010 dollars, are available from the World Bank and the United Nations for the years between 1970 and 2017 [17, 18]. Estimates of real GDP adjusted for purchasing power parity (PPP) 1990 USD are available for each year between 1950 and 1992, and in larger intervals extending back to 1 CE [19]. To calculate W these estimates are converted to market exchange rate MER inflation-adjusted 2010 values. For the time period between 1970 and 1992 for which concurrent MER and PPP statistics are available, the mean inflation-adjusted ratio PPP/MER is $\kappa = 1.205$ with no clear trend.

A historical reconstruction of the annual global GDP is obtained by dividing the Maddison PPP values by κ between 1 C.E. and 1970 C.E, applying a cubic spline between sparse data points to obtain annual values, and using World Bank statistics for more recent years [17]. The value of world cumulative production W is then

$$W(t) = W(1) + \sum_{1}^{t} Y(t) \tag{40}$$

where $W(1)$ refers to total accumulated world cumulative production to date in 1 C.E. To obtain a value for $W(1)$, it is assumed that W and world population grew equally fast at that time. Available statistics suggest a population in ca. 1 C.E. [21] that was 170 million and growing by 10 million every hundred years, at a rate of $\eta_{pop} =0.059$ % per year. The estimated value for the real MER GDP in 1 C.E. is 0.147 trillion 2010 USD. Assuming that civilization population and wealth grew at the same rate, i.e., $\eta_{pop} = \eta_W$, then from Eq. 19 it follows that $W(1) = 250$ trillion 2010 USD.

One criticism might be that MER dollars should be adjusted to PPP dollars [48] since market exchange rates fail to account for differences in how people in different countries value equivalent baskets of goods. One rebuttal has been that such equivalents do not exist because different cultures value goods differently and that any discrepancies tend to diminish over time with a half life of three to five years due to the pressures of international and domestic trade [49]. In the case of the work here, there is another counter-argument which is that there is no intent to address short-term inequalities between nations, only the global sum of all of civilization and its evolution over they long-run. Effectively, there is only one "basket of goods", and that is humanity taken as a whole, including all its social and physical networks.

Rates of global primary energy consumption from all sources $\mathcal{E}$ are available from the U.S. Department of Energy (DOE) Energy Information Administration (EIA) for the time period 1980 to 2016 and from British Petroleum between 1965 and 2017 [10, 20]. Rates of global primary energy consumption and production provided by the EIA have a mean ratio of 99.83% so here it is assumed that the two are equivalent.