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Household debt: The missing link between inequality and secular stagnation☆

Gaël Giraud[a,1], Matheus Grasselli[b,2,*]

[a] Centre d'Economie de la Sorbonne, University Paris-1 Panthéon-Sorbonne, AFD, France

[b] Department of Mathematics and Statistics, McMaster University, 1280 Main Street West, Hamilton ON, L8S 4K1, Canada

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ABSTRACT

How do inequality and growth evolve in the long run and why? We address this question by analyzing the interplay between household debt, growth and inequality within a monetary, stock-flow consistent framework. We first consider a Goodwin–Keen model where household consumption, rather than investment by firms, is the key behavioural driver for the dynamics of the economy. Whenever consumption exceeds current income, households can borrow from the banking sector. The resulting three dimensional dynamical system for wage share, employment rate, and household debt exhibits the characteristic asymptotic equilibria of the original Keen model, namely the analogue of Solow's balanced-growth path, where all state variables converge to an interior point, in addition to deflationary equilibria with explosive debt and collapsing employment. We then extend this set-up by separating the household sector into workers and investors, obtaining a four-dimensional system with analogous types of asymptotic behaviour. Our main result is that long-run increasing inequality between these two classes of households occurs if and only if the system approaches one of the equilibria with unbounded debt ratios. More specifically, we find that one essential channel of increased inequality is the wealth transfer from workers to investors due to interest paid on debt from the former to the latter. Finally, when properly rewritten, the celebrated inequality $r > g$ turns out to be a necessary condition for the asymptotic stability of long-run debt-deflation. Our findings shed new light on the relationships between fairness and efficiency, and have implications for public economic policy.

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* Corresponding author.
E-mail addresses: giraudg@afd.fr (G. Giraud), grasselli@math.mcmaster.ca (M. Grasselli).

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1. Introduction

This article addresses what is arguably one of the most basic questions in economics: how do income inequality and growth evolve in the long run and what are the determinants of this co-evolution?

Until recently, this question had hardly been properly addressed because of the alleged sharp distinction between efficiency and fairness. Most of the literature used to argue for letting market prices take care of efficiency, while addressing fairness with a separate policy of transfers of wealth through taxes and public expenditures. Provided growth and inequality are indeed unrelated, this seems to follow the Tinbergen rule on the equality between the number of policy goals and instruments (see Tinbergen, 1967). Three related developments, however, challenge this view. First, as of July 2018, liabilities of the U.S. household sector amounted to $15,489 billions according to Federal Reserve data[3]. This is higher than the peak of $14,449 billions reached during the third quarter of 2008, which coincided with the onset of the global financial crisis induced, at least in part, by subprime mortgage loans mostly contracted by poor North-American households. Second, this unprecedented increase in household debt coincides with high levels of income and wealth inequality, as it is now amply documented in the empirical literature that we review below. Third, the recovery from the Great Recession has been markedly slow when compared to the aftermath of previous financial crises (see Reinhart and Rogoff, 2009). These observations raise two questions which force us to go beyond the conventional separation between efficiency and fairness. First, to which extent are these recurring accumulations of household debt due to the increase of income inequality? Second, will they impede growth in the future? This paper provides an analytical framework where these two questions can be addressed by shedding light on the conditions under which income inequalities can become harmful to growth.

We begin by reviewing the literature on the links between the three interconnected strands mentioned above, namely inequality, household debt, and slow growth. In his influential book (Piketty, 2014), Piketty uses an extensive dataset to document the marked rise in income and wealth inequality observed in most of the developed world since the 1980s, as measured by the increase in the share of income and wealth in the top quantiles of their corresponding distributions. Although the patterns of capital accumulation and accompanying inequality discussed in the book are unequivocally present in the data, the theoretical underpinnings used by Piketty to extrapolate these data regularities into the distant future have been fiercely contested (for example, see Acemoglu and Robinson, 2015 and Stiglitz, 2015). Relatedly, Stiglitz (2012) and Rajan (2010) suggested that growing income and wealth inequalities may be one of the leading factors that fuelled household debt in the last four decades. Other possible explanatory factors include the rise of top incomes in the financial industry (see Boustanifar et al., 2017), increased flexibility in the labour market (see Checchi and García-Peñalosa, 2008), and financial deregulation (see Bazillier et al., 2017). The latter most probably also had an impact on inequality: Financialization may reduce poverty by providing credit to vulnerable people who previously had no access to financial markets, but can also increase inequality if the distribution of credit is twisted in favour of a minority of privileged borrowers (see Claessens and Perotti, 2007). This potential endogeneity issue makes identifying such a relationship a tricky task and might explain why the empirical literature on this issue remained ambiguous until recently: Bordo and Meissner (2012) do not find any role played by inequality in the rise of indebtedness whereas Perugini et al. (2016) witness a causal link from inequality to increasing debt. Despite this difficulty, Bazillier et al. (2017) provides a careful analysis of the link between inequality and credit over 44 countries, between 1970 and 2012, relying on data from the World Bank and the World Income Inequality Database (WIID). It essentially confirms that the rise of income inequality is responsible for the increase of household debt in a number of countries.

In a parallel development, Summers (see for example Summers, 2016) rekindled the interest of the economic profession in the theory of secular stagnation first proposed in Hansen (1939) (see also Gordon, 2014). Although Summers (2016) mentions inequality as one of many factors contributing to secular stagnation, the explicit mechanism connecting them is not discussed. The missing link can be found, however, in the work of Steindl and his followers. As early as in Steindl (1952), it was posited that monopoly power allows firms to increase their markup, leading to an increase of the share of profits, and therefore savings, while at the same time having less pressure to invest as they enjoy captive markets and reduced competition. In later work, Steindl takes into account the distribution of income between workers and capitalists, rather then just wages and profits (see Dutt, 2006 and references therein). The model in Dutt (2006) builds on this Steindlian analysis by explicitly considering the effects of growing levels of consumer debt observed since the 1950s, a phenomenon conspicuously unexplored by both Piketty and Summers. The conclusion in Dutt (2006) is that consumer debt is expansionary at first, as borrowing provides an additional source of income for workers, but might lead to stagnation in the long run, as interest payments are a transfer of income from workers, who have higher propensity to spend, to capitalists, who tend to spend less —similarly to a result also obtained in Palley (1994). That household debt might be the missing channel that links income inequality to secular stagnation is confirmed, for example, in Cynamon and Fazzari (2016).

We address the links between household debt, inequality and long term economic growth using (Akerlof and Stiglitz, 1969; Goodwin, 1967; Keen, 1995) as starting points, instead of the Steindlian framework used in Dutt (2006). We do so because these models put the dichotomy between labor and capital at the centre of the analysis, with the wage share and employment rates as key variables and a labour market dynamics interacting directly with the other financial variables in

[3] Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; Credit Market Instruments; Liability, Level [CMDEBT], retrieved in March 2019 from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CMDEBT.

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the model, such as the debt of firms. This seems like a more natural setting to study inequality than a model where the profit share is assumed to be constant, as is the case in Dutt (2006). We extend the Lotka–Volterra dynamics between employment and wages introduced in the papers just cited in three ways.

First, we consider in Section 2 a stock-flow consistent, monetary, version of the standard Keen model, where households may borrow money to finance their expenditures, while firms adjust their production so as to satisfy aggregate demand. We call this the *dual* Keen model, because in the original model of Keen (1995) it is household consumption that adjusts to the investment decisions of firms. We model the behaviour of households by assuming that the consumption to output ratio is given by an increasing function of their disposable income as a proportion of nominal GDP. This is general enough to include, among others, the types of consumption motives analyzed in Ryoo and Kim (2014) and Kapeller and Schütz (2014a). We recover results analogous to those obtained in the original Keen model, namely that there exist essentially two long-run steady states: an interior one, corresponding to a finite debt-to-output ratio and nonzero wage share and employment rate, and an explosive one, characterized by an infinite debt-to-income ratio and collapsing wages and employment. This should be compared with the full characterization of static equilibria obtained in Giraud and Pottier (2016). There, within an Arrow–Debreu framework with endogenously incomplete markets, it is demonstrated that three, and only three, types of equilibria can occur: an inflationary one, a deflationary one, and a last one leading to a crash. Here, courtesy of our dynamic framework, we can describe the path followed by the state of the economy along the transition leading to a long-term equilibrium, and analyze the stability of the latter. As it happens, within the present setting, the last two types of equilibria of Giraud and Pottier (2016) coincide: debt-deflation always leads to a final collapse.

As a second contribution, we extend this dual model in Section 3 by dividing the household sector into workers, whose income arises solely from wages, and investors, whose income stems solely from capital and the profits of the banking sector. Our main result is that, in this setting, income inequality accelerates in the neighbourhood of the explosive equilibrium, while it stabilizes around the interior one. As in Dutt (2006) and Palley (1994), this allows us to analyze the impact of income inequality beyond the conventional trade-off between efficiency and equity, according to which efficiency is primarily concerned with the issue of increasing the size of the economic pie, while fairness deals with its distribution. In case of conflict, which should have priority? A familiar argument in favour of efficiency goes as follows: emphasizing the growth of the pie may lead those with lower income to a better state than by focusing on fairness in dividing it, as their absolute income might be larger in the first case than in the second. We challenge this traditional wisdom by showing that the trade-off between efficiency and fairness does not necessarily hold: in our set-up, increasing inequality is an unequivocal characterization of paths leading to an economic collapse – a radical shrink of the pie. In other words, putting an upper-bound on inequality improves long-run efficiency.

In a wage-led economy, where a decrease in wage share leads to a deceleration of growth, the observation that more income inequality impairs growth or macroeconomic stability does not come as a surprise, as shown by a fair body of literature (see Lavoie and Stockhammer, 2013 for a survey or, more recently, Ryoo, 2014). This is why we focus, instead, on a demand-driven but profit-led type of economy, where a reduction of the wage share should have a boosting effect.[4] We argue that, in such a configuration, the dynamics of debt is key to understand why growing income inequality may not only slow down GDP growth but even lead to a deflationary collapse. To the best of our knowledge, this is the first paper to analyze the pivotal role of household debt in linking income and wealth inequality with macro-instability and debt-deflationary de-growth in a stock-flow consistent out-of-equilibrium, profit-led dynamics.

Closely related to this second contribution in our paper are the results of Kapeller and Schütz (2014b), which extends a stock-flow-consistent post Keynesian model with conspicuous consumption and where an increase in income inequality also leads to a corresponding increase in debt-financed consumption. In their model, as the solvency of households decreases and interest rates move up, banks reduce lending, triggering household bankruptcies and, ultimately, a recession. What follows is a period of consolidation, where past debts are repaid and financial stability is regained. In our model, by contrast, a Minskyan dynamics also comes into play even without credit restriction on the banking side. Moreover, the recession may not be followed by a consolidation phase, as endogenous forces are not sufficient for any recovery to take place. Rather, the economy may end up attracted towards a liquidity trap from which it has no way to escape, absent any public intervention.

As mentioned above, Cynamon and Fazzari (2016) also explore the effect of debt-financed consumption on growth. According to their findings, had the spending rate of the bottom 95% remained stable (or even risen like the top 5%), the demand drop that caused the recession in the aftermath of the GFC would have been much less severe. Here, we show that, in the medium run, debt-financed consumption may help overcome the recessive impact of not-smoothing consumption, which confirms the standpoint defended by Cynamon and Fazzari (2016), as well as the evidence put forward by Belabed et al. (2017). However, we also show that this may occur at the cost of incurring the risk that workers enter into a debt spiral which leads to a collapse in the long-run. This is in line with the empirical finding of Belabed et al. (2017): In the early 1980s, the bottom 95% responded to slower income growth and higher real interest rates by taking on more debt rather than by reducing consumption enough to keep its debt-income ratio stable. This outcome, in a sense, temporarily rescued the U.S. economy from the demand drag that many theories predict as a result of rising inequality. But the deteriorating balance sheets of the bottom 95 % would eventually set the stage for the Great Recession.

[4] As an example of the empirical relevance of profit-led economies, Onaran and Galanis (2013) found that Canada, Australia, Argentina, Mexico, China, India, and South Africa were profit-led on the 1970-2007 period —a sample of the G20 which represented 20% of the world's GDP in 2007 and 36% of the population on Earth at the time.

The third contribution of our paper consists in a reinterpretation of the relationship $r > g$ that has been emphasized by Piketty (2014) and the target of considerable criticism. On the one hand, as shown in Acemoglu and Robinson (2015), and contrary to Piketty's claim, this inequality does not necessarily imply a rise in inequality. On the other hand, Piketty conflates two concepts in the variable r: the interest rate on debt and the return on capital. The recent empirical evolution of these two variables forces us to disentangle these two interpretations of r. Taking this distinction into account, we show that, in our set-up, $r > g + i$ (where r is the nominal short-run interest rate and i is inflation, as we deal with a monetary setting) is a necessary condition for the local stability of the explosive equilibrium. Therefore, an economy may converge towards the interior equilibrium, hence stabilizing inequality, even though $r > g + i$ is satisfied (in accordance with the result in Acemoglu and Robinson, 2015). But, if $r \leq g + i$, the bad equilibrium is no longer locally stable, which considerably reduces its chance to be reached.

To wrap up, there is ample empirical evidence of the link between the rise of income inequality and that of household debt, as well as their potentially destabilizing effect. In the remainder of this paper, we analyze this channel within a stock-flow consistent dynamical framework where the impact on growth can be made explicit. As already alluded to, our main finding is that, even though we confine ourselves to a profit-led setting, too much inequality may lead to an overhang of private debt and subsequent deflationary trend and, ultimately, a deep depression. It is worth noticing that, as acknowledged by Bazillier et al. (2017), variations in inequality have no observable effect on the private debt of firms. Therefore, for the sake of simplicity, we focus primarily on household debt in the sequel.

Before leaving this introduction, we want to make two remarks on methodology. First, although the qualitative outputs of our dynamics look more familiar to the Cambridge tradition, our contention is that our simple model can be viewed as a starting point for both a neo-classical development *à la* Solow–Swan or a more Kaleckian/post-Keynesian oriented set-up (see Remark 2.2 below). This might be viewed as a preliminary step towards a theoretical synthesis of both strands of the literature which, so far, have remained widely separated.

Second, our full-fledged dynamic approach should be contrasted with alternative ways of modelling time within an Arrow–Debreu setting. For instance, in Giraud and Tsomocos (2010) (see also the references therein), an out-of equilibrium path is viewed as a succession (in continuous time) of infinitesimal short-run equilibria in the spirit of Smale (1976b) and Smale (1976a). Here, we go one step further in departing from the equilibrium constraint: at each point of time, households and firms behave in a myopic and possibly irrational way. Under circumstances that we carefully elucidate, the state of the economy will converge towards a long-run equilibrium equivalent to the celebrated steady-state of the Solow model. But if these conditions are not met, the economy may cycle or converge to another type of steady-state. Actually, we care only about the aggregate behavior of economic agents. Due to the celebrated Sonnenschein–Mantel–Debreu theorem (see, e.g., Debreu, 1974; Sonnenschein, 1972 and Kirman, 1989), indeed, even if economic agents were fully rational, the aggregation of their behavior might lead to emerging phenomena that cannot be deduced from the microeconomic scale. One way to deal with this form of complexity consists in simulating agent-based models (see, for example, Caiani et al., 2016). Here, for simplicity, and as a first attempt, we abstract from any microfoundation and deal only with the macroeconomic level.

2. The dual Keen model

We first set the scene by considering an economy with a single type of households, who can borrow money from the banking sector to fulfil their consumption plans, and firms that adjust their investment so that total output matches aggregate demand.

2.1. Preliminaries

On the production side, we adopt the same Leontieff function with total capital utilization adopted in the Goodwin (1967) and Keen (1995) models, namely

$$Y = \frac{K}{\nu} = a\ell, \tag{1}$$

where Y and K denote, respectively, output and capital in real terms, $\nu > 0$ is a constant capital-output ratio, $a \geq 0$ is productivity per worker and ℓ is the number of employed workers. Capital accumulates according to

$$\dot{K} = I - \delta K, \tag{2}$$

where I is real gross investment and $\delta \geq 0$ is a constant depreciation rate. It follows that the growth rates of the economy and of capital are the same and given by

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{I}{\nu Y} - \delta. \tag{3}$$

We denote the wage rate by $\mathrm{w} = W/\ell$, where W is the total nominal wage bill. Consequently, the wage share of nominal output is given

$$\omega := \frac{W}{pY} = \frac{\mathrm{w}}{pa}.$$

Table 1
Balance sheet, transactions and flow of funds for a three-sector economy.

	Households	Firms		Banks	Row sum
Balance sheet					
Capital stock		$+pK$			pK
Deposits	$+M_h$	$+M_f$		$-M$	0
Loans	$-L_h$	$-L_f$		L	0
Equity	E	$-E_f$		$-E_b$	0
Column sum (Net worth)	X_h	0		0	pK
Transactions		Current	Capital		
Consumption	$-pC_h$	$+pC$		$-pC_b$	0
Investment		$+pI$	$-pI$		0
Accounting memo [GDP]		$[pY]$			
Depreciation		$-p\delta K$	$+p\delta K$		0
Wages	$+\mathrm{w}\ell$	$-\mathrm{w}\ell$			0
Interest on loans	$-rL_h$	$-rL_f$		$+rL$	0
Interest on deposits	$+rM_h$	$+rM_f$		$-rM$	0
Bank dividends	$+\Delta_b$			$-\Delta_b$	0
Column sum (balances)	S_h	S_f	$-pI+p\delta K$	S_b	0
Flows of funds					
Change in capital stock		$+p\dot{K}$			$p\dot{K}$
Change in deposits	$+\dot{M}_h$	$+\dot{M}_f$		$-\dot{M}$	0
Change in loans	$-\dot{L}_h$	$-\dot{L}_f$		$+\dot{L}$	0
Column sum (savings)	S_h	S_f		S_b	$p\dot{K}$
Change in firm equity	$+\dot{E}_f$	$-(S_f+\dot{p}K)$			0
Change in bank equity	$+\dot{E}_b$			$-S_b$	0
Change in net worth	$\dot{E}+S_h$	0		0	$\dot{p}K+p\dot{K}$

If N denotes total workforce, which we assume to be exponentially growing at rate $\beta > 0$, then $\lambda = \ell/N$ is the employment rate and evolves according to:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} - \frac{\dot{N}}{N} = \frac{I}{\nu Y} - \delta - \alpha - \beta. \tag{4}$$

The evolution of the wage rate w is provided by a bargaining equation of the form

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \gamma i(\omega), \tag{5}$$

where $\Phi : [0, 1) \to \mathbb{R}$ is a short-run Phillips curve (see, e.g., Gordon, 2011; Gordon, 2013; Gregory, 2001, and Gregory, 2014) and $\gamma \in [0, 1]$ is a parameter measuring the extent to which inflation $i(\omega)$ is incorporated in the bargaining for nominal wages, with $\gamma = 1$ corresponding to no money illusion, in which case (5) is equivalent to the bargaining in terms of real wages assumed in Goodwin (1967) and Akerlof and Stiglitz (1969). Here inflation is given by

$$i(\omega) := \frac{\dot{p}}{p} = \eta_p(m\omega - 1), \tag{6}$$

where $m \geq 1$ is target markup towards which the price level p adjusts in the (imperfectly competitive) goods market with a relaxation time $1/\eta_p$. Labour productivity $a = Y/\ell$ is assumed to grow exponentially at an exogenous rate $\alpha \geq 0$, so that the dynamics of the wage share is given by

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{a}}{a} - \frac{\dot{p}}{p} = \Phi(\lambda) - \alpha - (1-\gamma)i(\omega). \tag{7}$$

Remark 2.1. As seen above, our modelling approach utilizes a short-run Phillips curve, which some consider to be an outdated economic concept, especially in view of its failure to capture the strong positive relation between inflation and unemployment observed in the stagflation episode in the 1970s. However, as argued in Gordon (2013), rumours of the death of the Philips curve have been greatly exaggerated. In any event, the Phillips curve adopted in (5) postulates a relationship between employment and changes in wage rate, instead of inflation. This is much more in line with the original proposal by Phillips (1958) and has been shown in Grasselli and Maheshwari (2018) to hold up to data reasonably well.

2.2. Accounting structure

The structure of the three-sector economy considered in this section is the same as in the Keen (1995) model and is described in Table 1.

We assume that firms and banks are privately owned by a subset of the households, so that the assets of the aggregate household sector consist of firm equity E_f, bank equity E_b, and deposits M_h, whereas their liabilities consist of loans L_h, resulting in net worth (i.e., wealth) equal to $X_h = E + M_h - L_h$, where $E := E_f + E_b$. The assets of the aggregate firm sector are the capital stock pK and deposits M_f, whereas their liabilities are the loans L_f and equity $E_f = pK + M_f - L_f$, here treated as a balancing variable so that the net worth of firms is $X_f = 0$. Similarly, the loans are the assets of the banking sector, and its liabilities are the deposits, whereas bank equity, also treated as a balancing variable, is $E_b = (L_h + L_f) - (M_h + M_f)$. Hence, the net worth of banks is $X_b = 0$. We therefore obtain that

$$\begin{aligned} X = X_h &= E_f + E_b + M_h - L_h \\ &= pK + M_f - L_f + (L_h + L_f) - (M_h + M_f) + M_h - L_h \\ &= pK, \end{aligned}$$

that is to say, the total wealth in the economy equals the wealth of the households, which in turn corresponds to the non financial assets of the firm sector. Observe that we do not consider the case where firms and banks are owned by shareholders through publicly traded stocks. In particular, firms and banks in this model do not issue or buy back any shares.

The budget constraint for the household sector implies that whenever nominal household consumption exceeds household disposable income, the difference needs to be financed by an increased in household net debt $D_h = L_h - M_h$. Conversely, if household disposable income exceeds nominal household consumption, the difference, which consists of household savings S_h, is used to decrease household net debt. In other words, we have that

$$\dot{D}_h = \dot{L}_h - \dot{M}_h = -S_h = pC_h - (\mathrm{w}\ell - rD_h + \Delta_b). \tag{8}$$

Here, C_h denotes real household consumption, whereas household disposable income consists of $(\mathrm{w}\ell - rD_h + \Delta_b)$, where $r > 0$ is a constant nominal short-run interest rate paid to banks on net debt D_h and Δ_b denotes dividends received from banks, which are assumed to be privately owned by a subset of households.

As in the original Keen (1995) model, we assume in this section that firms retain all their profits in order to finance investment. That is why Table 1 does not have a row representing dividends paid by firms. If the amount to be invested exceeds profits, then firms finance the difference by increasing their loans from the banking sector. Conversely, when profits exceed investment, the difference is used to either repay existing loans or accumulate deposits. Denoting the net debt of firms by $D_f := L_f - M_f$, we see from Table 1 that net profits for firms, after paying wages, interest on net debt, and accounting for depreciation (i.e., consumption of fixed capital) are given by

$$\Pi = pY - \mathrm{w}\ell - rD_f - p\delta K, \tag{9}$$

where we assumed, for simplicity, that firms pay the same constant interest rate r on net debt D_f as households. In the absence of distributed profits paid to shareholders in the form of dividends, the savings of the firm sector are given by $S_f = \Pi$. It therefore follows from the budget constraint of the firm sector that

$$\dot{D}_f = p(I - \delta K) - S_f = pI - (pY - \mathrm{w}\ell - rD_f). \tag{10}$$

Finally, savings of the banking sector are given by

$$S_b = r(L_h + L_f) - r(M_h + M_f) - \Delta_b - pC_b = r(D_f + D_h) - \Delta_b - pC_b, \tag{11}$$

where C_b denotes real consumption by banks. Using the fact that

$$Y = C + I = (C_h + C_b) + I, \tag{12}$$

we can then verify that

$$S_h + S_f + S_b = (\mathrm{w}\ell - rD_h + \Delta_b - pC_h) + (pY - \mathrm{w}\ell - rD_f - p\delta K) \tag{13}$$

$$+ r(D_f + D_h) - \Delta_b - pC_b = p(I - \delta K), \tag{14}$$

so that savings always equal net investment in the economy.

2.3. Aggregate behavioural rules

We briefly recall that the original Keen (1995) model is based on the assumption that investment is given by

$$I = \kappa(\pi)Y, \tag{15}$$

where $\kappa : \mathbb{R} \to \mathbb{R}$ is an increasing function of the pre-depreciation profit share

$$\pi = \frac{pY - \mathrm{w}\ell - rD_f}{pY} = 1 - \omega - rd_f, \qquad d_f = \frac{D_f}{pY}. \tag{16}$$

Using (3) we find that

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1-\omega-rd_f)}{\nu} - \delta, \tag{17}$$

from which we obtain that

$$\begin{aligned}\frac{\dot{d}_f}{d_f} &= \frac{\dot{D}_f}{D_f} - \frac{\dot{Y}}{Y} - \frac{\dot{p}}{p}\\ &= \frac{\kappa(1-\omega-rd_f) - (1-\omega-rd_f)}{d_f} - \frac{\kappa(1-\omega-rd_f)}{\nu} + \delta - i(\omega)\\ &= \left[r - \frac{\kappa(1-\omega-rd_f)}{\nu} + \delta - i(\omega)\right] + \frac{\kappa(1-\omega-rd_f) - (1-\omega)}{d_f}\end{aligned} \tag{18}$$

The dynamical system consisting of (7), (4) and (18) then corresponds to what we call the primal Keen model, which is driven by investment decisions and has the debt ratio of firms as its key state variable, in addition to the wage share and employment. The primal model implicitly assumes that total consumption adjusts to investment according to

$$C = C_h + C_b = Y - I = (1 - \kappa(1-\omega-rd_f))Y. \tag{19}$$

We can then see that several alternative specifications of bank and household behaviour are compatible with the general structure of the model. For example, one can assume that

$$\Delta_b = r(D_h + D_f), \quad C_b = 0, \quad C_h = C, \tag{20}$$

that is, all bank profits are paid to households in the form of dividends and there is no consumption by banks, in which case $S_b = 0$ and the equity of banks remain constant. Moreover, it follows from (8) and (10) that

$$\begin{aligned}\dot{D}_h &= pC_h - \mathrm{w}\ell + rD_h - r(D_h + D_f)\\ &= pY - pI - \mathrm{w}\ell - rD_f = -\dot{D}_f.\end{aligned} \tag{21}$$

Another alternative with $S_b = 0$ consists of setting

$$\Delta_b = 0, \quad C_b = r(D_h + D_f), \quad C_h = C - r(D_h + D_f), \tag{22}$$

that is, banks retain and consume all profits, in which case it is easy to see that (21) still holds. For either (20) or (22), or any other specification of bank and household behaviour leading to $S_b = 0$, we see that the equity E_b of the banking sector remains constant. It should be clear that more general specifications of Δ_b, C_b and C_h would also be compatible with the primal Keen model, provided that total consumption satisfies (19).

We are now ready to complete the specification of the dual Keen model. It consists of assuming that (20) holds and that, in addition, total consumption is given by

$$C = C_h := c(\omega - rd_h)Y, \tag{23}$$

where $c : \mathbb{R} \to \mathbb{R}_+$ is an increasing function determining the household consumption to output ratio and $d_h = D_h/(pY)$ is the ratio of net debt of households to output. Observe that the disposable income of the aggregate household sector consists of

$$\mathrm{w}\ell - rD_h + \Delta_b = \mathrm{w}\ell + rD_f = \mathrm{w}\ell - rD_h + rE_b, \tag{24}$$

where we have used that $E_b = D_h + D_f$. Consequently, the variable $(\omega - rd_h)$ measures the share of disposable income of households, apart from the constant rE_b.

Notice that we do not strive for microfoundations of the aggregate consumption function, $C(\cdot)$. This is consistent with the celebrated Sonnenschein-Mantel-Debreu theorem already mentioned in Section 1. Thus we content ourselves with taking $C(\cdot)$ as given. For practical applications, $C(\cdot)$ should be empirically estimated as, e.g., in Bastidas et al. (2017b) and Bastidas et al. (2017a).

Differently from the primal Keen model, we now assume that investment is the adjusting variable instead, so that the capital accumulation dynamic now reads

$$\dot{K} = Y - C - \delta K = (1 - c(\omega - rd_h))Y - \delta K. \tag{25}$$

Combining (25) and (1), we obtain that the growth rate of real output is

$$g(\omega, d) := \frac{\dot{Y}}{Y} = \frac{1 - c(\omega - rd_h)}{\nu} - \delta. \tag{26}$$

Notice that Eq. (26) confirms that the dynamics under scrutiny is profit-led: g is a decreasing function of ω.

We then have the following dynamics for the household debt ratio $d_h = D_h/pY$:

$$\frac{\dot{d}_h}{d_h} = \frac{\dot{D}_h}{D_h} - \frac{\dot{Y}}{Y} - \frac{\dot{p}}{p} = \frac{\dot{D}_h}{D_h} - g(\omega, d) - i(\omega)$$

$$= \left[r - \frac{1 - c(\omega - rd_h)}{\nu} + \delta - i(\omega)\right] + \frac{c(\omega - rd_h) - \omega - re_b}{d_h}, \tag{27}$$

where $e_b = E_b/(pY)$. To summarize our set-up so far, we end up with the following 3-dimensional non-linear system:

$$\begin{aligned} \dot{\omega} &= \omega[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega)] \\ \dot{\lambda} &= \lambda\left[\frac{1 - c(\omega - rd_h)}{\nu} - (\alpha + \beta + \delta)\right] \\ \dot{d_h} &= d_h\left[r - \frac{1 - c(\omega - rd_h)}{\nu} + \delta - i(\omega)\right] + c\big(\omega - rd_h\big) - \omega - re_b. \end{aligned} \tag{28}$$

Remark 2.2. The link between our model and the neo-classical tradition that emerged in the wake of Solow (1956) is easy to understand. It suffices to replace the Leontieff production function in (1) by a CES one, in the same way as van der Ploeg (1985) and Bastidas et al. (2018). The main consequence of this change is that the capital-output ratio, ν, now becomes endogenous. Nevertheless, it can be shown for the primal model that allowing for capital-labor substitution does not qualitatively change the phase space: it basically broadens the basin of attraction of the balanced-growth path, and induces trajectories that dampen and converge more rapidly to the Solovian steady state than in the non-substitutable case. There is no reason to believe that a similar change in our dual model would lead to significantly different consequences.

The link between our model and the Kaleckian tradition is somewhat less straightforward.[5] Start with the markup rule driving inflation (6): instead of treating the relaxation parameter η_p as an exogenous parameter, turn it into an endogenous variable which depends upon the profit share:

$$1 + \eta_p = \frac{1}{1 - \pi}.$$

Next, drop Say's law in the same way as it is done by Grasselli and Nguyen-Huu (2018) for the primal model, by adding a dynamics for inventories and an endogenous capacity utilisation rate $u = \nu Y/K$. Since supply does no more automatically adjust to demand, one now needs to specify an investment function as in (15). Putting together these extensions, one would recover a rendition of Kaleckian macroeconomics in an economy closed for foreign trade where saving rates for workers and capitalists, are endogenously determined by the gap between income and consumption (resp. profit and investment). In order to include Kaldor's and Steindl's further extension (Kaldor, 1940, Steindl, 1952) of Kalecki's seminal contribution, all one needs to do is to extend the argument of the investment function $\kappa(\cdot)$ appearing in (15) so as to include not only the profit share, π, but also the utilization rate u. In other words, our current model can be viewed as a particular case of a Kalecki/Steindl setting in the post-Keynesian guise. At variance with the modifications needed to recover the neo-classical setting, however, such extensions would profoundly modify the resulting dynamical system. Since our minimal model turns out to be already rich and subtle, we leave these developments for further research.

Of course, one could as well add Kalecki's further idea, namely that there is a lag between investment and capital change in the accumulation equation (25). Similarly, the Phillips curve might exhibit delays in order to better account for empirical data. For sufficiently large delays, this would induce Hopf bifurcations that would potentially deeply perturb the underlying dynamics, whereas for small enough lags, the perturbation would keep the phase space qualitatively intact as in Yu and Peng (2016).

2.4. Long-run equilibria

Being the dual to Keen's seminal model, our model exhibits a similar pattern of dynamics. Let us make the following set of assumptions similar to Grasselli and Nguyen-Huu (2015).

Assumption 2.1.

(a) The consumption function $c : \mathbb{R} \to \mathbb{R}_+$ is $\mathcal{C}^1$, strictly increasing over $\mathbb{R}_+$, and verifies
  - (i) $y \le 0 \Rightarrow c(y) = c_- > 0$
  - (ii) $\lim_{z\to+\infty} c(y) := c_+ \le 1$.
  - (iii) $c_- < 1 - \nu(\alpha + \beta + \delta) < c_+$

(b) The Phillips curve $\Phi : [0, 1) \to \mathbb{R}$ is $\mathcal{C}^1$, strictly increasing and admits a vertical asymptote at $\lambda = 1$. Moreover, $\Phi'(0) = 0$ and

$$\Phi(0) < \min\{\alpha, \gamma(\alpha + \beta) - \beta\} \tag{29}$$

The first condition on the consumption function expresses the fact that households need a minimum level of subsistence consumption even at negative income, whereas the second condition corresponds to the fact that total consumption cannot exceed total output. The third condition, on the other hand, is related to the existence of an interior equilibrium.

[5] See Taylor (2004) pp. 124 sqq for an in-depth account.

We first observe that the dynamical system (28) admits a class of trivial equilibria of the form $(\overline{\omega}_0, \overline{\lambda}_0, \overline{d}_0) = (0, 0, \overline{d}_0)$ for arbitrary $\overline{d}_0 \geq 0$, provided that the identity

$$r + \delta - \frac{1 - c_-}{\nu} + \eta_p = 0$$

is satisfied (recall that $c(-r\overline{d}_0) = c_-$). Since this is structurally unstable, we can disregard this family of equilibria.

2.4.1. The balanced-growth path

We now consider the existence of an interior equilibrium for (28). Begin by defining

$$\eta_1 := c^{-1}\big(1 - \nu(\alpha + \beta + \delta)\big),$$

which exists because of condition (a)–(iii) in Assumption 2.1, and take $\overline{\omega}_1 - r\overline{d}_1 = \eta_1$ so that $\dot{\lambda} = 0$ in the second equation of (28). It follows that the growth rate of real output (26) at this equilibrium is

$$g(\overline{\omega}_1, \overline{d}_1) = \frac{1 - c(\overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha + \beta, \tag{30}$$

so that $e_b = E_b/(pY) \to 0$, since E_b is constant[6]. Since $g(\overline{\omega}_1, \overline{d}_1)$ also equals the growth rate of capital (recall (3)), it follows that an equilibrium with this growth rate describes the analog of the balanced-growth path in Solow (1956), albeit with different stability properties. Using (30), we can now verify that

$$\overline{\omega}_1 = \eta_1 + r\left[\frac{1 - \nu(\alpha + \beta + \delta) - \eta_1}{\alpha + \beta + i(\overline{\omega}_1)}\right] \tag{31}$$

$$\overline{\lambda}_1 = \Phi^{-1}\big(\alpha + (1 - \gamma)i(\overline{\omega}_1)\big) \tag{32}$$

$$\overline{d}_1 = \frac{1 - \nu(\alpha + \beta + \delta) - \eta_1}{\alpha + \beta + i(\overline{\omega}_1)} \tag{33}$$

is an interior equilibrium for (28).

Lemma 2.1. *A sufficient condition for the existence of $\overline{\lambda}_1 > 0$ in (32) is a strictly positive nominal growth rate, that is, $\alpha + \beta + i(\overline{\omega}_1) > 0$. Moreover, in this case, we have that $\overline{d}_1 \geq 0$ if, and only if, $c(\overline{\omega}_1 - r\overline{d}_1) \geq \overline{\omega}_1 - r\overline{d}_1$.*

Proof. See Appendix A. □

Observe that $i(\overline{\omega}_1) = \eta_p(m\overline{\omega}_1 - 1)$, so (31) is a quadratic equation. In what follows, we assume that this equation has at least one solution with $\overline{\omega}_1 > 0$. As in Grasselli and Nguyen-Huu (2015), whenever the markup m is relatively high, namely $m \geq 1/\overline{\omega}_1$, the economy is asymptotically inflationary and we observe a trade-off between long-run inflation and employment according to (32). Notice that this holds even when $r = 0$. Indeed, according to (31), the latter simply means that a change in inflation does not affect the wage share $\overline{\omega}_1$.

Regarding the speed of convergence towards the interior equilibrium, the eigenvalues of the Jacobian matrix of system (28) allow us to provide a "quick and dirty" estimation. In the vicinity of the long-run steady state, a Taylor expansion yields

$$\lambda(t) = \overline{\lambda}_1 + e^{-\varepsilon_\lambda t}(\lambda(0) - \overline{\lambda}_1) \tag{34}$$

Since $\varepsilon_\lambda = \frac{1-c}{\nu} - (\alpha + \beta + \delta)$ (see Appendix A1), we can calibrate (34) to check how quickly actual economies are likely to approach this balanced-growth path, provided they are already in its neighbourhood. Typically, ε_λ is about 3% per annum (this would arise with 1 to 2% population growth, 1 to 2% growth in productivity, and 3 to 4% depreciation, while $\nu \simeq 3$ and $c \simeq 0.7$). Therefore, λ moves 3% of the remaining distance towards $\overline{\lambda}_1$ each year, and takes approximately 23 years to get halfway of its steady-state level.[7] What is more important, an increase of ν or c actually slows down the speed at which the economy converges towards its structural (un)employment rate.

We end this subsection with two remarks comparing properties of the interior equilibrium with well-known concepts in macroeconomics.

Remark 2.3. By normalizing capital through the effective labor force, $k := K/aN$, one can ask whether the balanced-growth path in our model does satisfy Solow's celebrated golden rule. Namely, we ask whether the equilibrium normalized capital

[6] Observe that, had we assumed that banks pay proportional dividends $\Delta_b = \delta_b(D_f + D_h)$, in which case the equity E_b grows at constant rate $(r - \delta_b)$, then $e_b = E_b/(pY) \to 0$ would still hold provided $r - \delta_b < \alpha + \beta + i(\overline{\omega}_1)$.

[7] The half-life t^* is the solution of $e^{-\varepsilon t^*} = 1/2$, which yields: $t^* = -\ln(1/2)/\varepsilon_\lambda \simeq 23$.

$\overline{k}_1$ maximizes welfare, defined here as the flow of consumption per unit of effective labor force, $\mathcal{C} := C/aN$. It is easy to see that the answer is positive, since the dynamics of normalized capital is

$$\dot{k} = \frac{\kappa(1-\omega-rd_f)}{\nu}k - (\alpha+\beta+\delta)k,$$

while, at the interior equilibrium, $\dot{k} = \dot{\mathcal{C}} = 0$. The latter equality follows in a straightforward way from the fact that $C = Y - I$ and $\dot{\kappa}(1-\omega-rd_f) = 0$ at the steady state.

Remark 2.4. Because of the non-trivial trade-off between long-run inflation and employment embodied in (32), the analog of a "long-run Phillips curve" cannot be vertical in our setting. Hence, the structural long-run unemployment rate $1-\overline{\lambda}_1$ is not to be confused with the "natural rate of unemployment" introduced in Friedman (1968) and Phelps (1968). Moreover, as will be clear from our stability analysis of the balanced-growth path, monetary policy (captured here through the setting of r, the short-run nominal interest rate) does play a role, even in the long run, since it influences the asymptotic local stability of the interior steady state. Similarly, if the markup m is linked to the monopoly power of firms (as suggested, among others, by Kalecki, 1971), the institutional rules governing the consumption good market will have an influence on employment (via the impact of m on $\overline{\lambda}_1$). This contrasts with the very concept of "natural unemployment", which embodies the idea that monetary policy is ineffective in reducing underemployment. Actually, our structural unemployment rate, $1-\overline{\lambda}_1$, is closer to the NAIRU ("Non Accelerating Inflation Rate of Unemployment") introduced in Tobin (1980), since, at the balanced-growth path, inflation, $i(\overline{\omega}_1)$, remains constant. As for the NIRU ("Non Inflationary Rate of Unemployment") of Modigliani and Papademos (1975), it is given by $1-\lambda_1^*$, where $\lambda_1^* := \Phi^{-1}(\alpha)$, and corresponds to the special case $\overline{\omega}_1 = 1/m$. Consequently, our concept of "structural unemployment" departs from the hybrid New Keynesian Phillips curve (NKPC) as developed in Galí and Gertler (1999), where the inflation rate is related to lagged inflation, inflation expectations and a measure of excess demand, which results in a short run trade-off between inflation and unemployment but a unique long-run equilibrium, with steady inflation.

On the other hand, reducing m by improving "perfect competition" on the consumption market, or expanding the profit share π (and consequently reducing $\overline{\omega}_1$), are alternate ways to reduce the gap $m\overline{\omega}_1 - 1$, so that the long-run Solovian steady-state will be less inflationary. As will be explained in Remark 2.6, flattening $\Phi(\cdot)$ should also decrease the long-run "natural" unemployment rate. Thus, most of the conventional policy tools on the supply-side that are usually advocated in neo-classical analysis can be dealt with in our set-up. Finally, dropping Say's law along the lines of Remark 2.2. would obviously bring as well all the policy tools on the demand side favoured by post-Keynesian economists.

2.4.2. Equilibria with collapsing employment

As in Grasselli and Costa Lima (2012), it is straightforward to verify that an explosive equilibrium of the form $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, \pm\infty)$ arises from the change of variable $u := 1/d_h$ in (28). In addition, as in Grasselli and Nguyen Huu (2015), when $\gamma < 1$ the introduction of inflation gives rise to another class of economically undesirable equilibria of the form $(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_3) = (\overline{\omega}_3, 0, \overline{d}_3)$ where

$$\overline{\omega}_3 = \frac{\Phi(0)-\alpha}{m(1-\gamma)\eta_p} + \frac{1}{m}, \tag{35}$$

and $\overline{d}_3$ is either a finite solution of

$$d\left[r+\delta - \frac{1-c(\overline{\omega}^3 - rd)}{\nu} - i(\overline{\omega}^3)\right] = \overline{\omega}^3 - c(\overline{\omega}^3 - rd) \tag{36}$$

or else $\overline{d}_3 = \pm\infty$. As observed in Grasselli and Nguyen Huu (2015), it follows from the second equation in (28) that

$$i(\overline{\omega}_1) = \frac{\Phi(\overline{\lambda}_1)-\alpha}{1-\gamma} > \frac{\Phi(0)-\alpha}{1-\gamma} = i(\overline{\omega}_3), \tag{37}$$

so that inflation at an equilibrium of this form is necessarily lower than inflation at the interior equilibrium. Moreover, in view of (29), we have the $i(\overline{\omega}_3) < 0$, so that any equilibrium of the form $(\overline{\omega}_3, 0, \overline{d}_3)$ is, in fact, strictly deflationary.

Similarly to what we have done for the interior equilibrium, we can estimate the speed at which the economy is likely to reach an explosive equilibrium. Consider a collapse where the ratio of the household debt to output increases to infinity (i.e., $d_h \to +\infty$). Then, $\varepsilon_\lambda = \frac{1-c_-}{\nu} - (\alpha+\beta+\delta)$. If, say, $c_- \simeq 0.03$, then λ moves each year approximately 25% of the remaining distance towards 0, and it takes roughly 6 years for the zero employment rate to be reached, provided $\lambda(0)$ is already in the neighborhood of the explosive steady state, where the Taylor-series approximation is reliable. Of course, meanwhile, a number of political complications are likely to occur, such as social protests, political turmoil, etc. But at least, this provides an intuition of the forces that accelerate or decelerate the process towards a collapse. For instance, an increase in ν or c_- would slow down the fall of the economy.

The local stability properties of these different classes of equilibria are analyzed in Appendix B. Let us notice here that, according to (77) and (78), a necessary condition for the stability of any equilibrium with $\lambda = 0$ and $d_h \to +\infty$ is

$$1 - \nu(\alpha+\beta+\delta) < c_-. \tag{38}$$

But this is not compatible with Assumption 2.1 (a)-(iii), which in turn is a necessary condition in order to establish the existence of an interior equilibrium. Thus we can only have one of the two following situations: either there exists an interior equilibrium and the equilibria with $\lambda = 0$ and $d_h \to +\infty$ are locally unstable, or these undesirable equilibria are stable and there exists no interior equilibrium. Notice that, for typical parameter values, namely a capital-output ratio ν close to 3, population plus productivity growth $\alpha + \beta$ close to 3% and depreciation close to 4% one obtains $1 - \nu(\alpha + \beta + \delta) \approx 0.79$, which is comfortably above c_-, which in most countries should be of the order of a few percentage points of the output.[8] In other words, for parameter values within the range currently observed in most advanced economies, it is much more likely for the interior equilibrium to exist than for the explosive equilibria with $\lambda = 0$ and $d_h \to +\infty$ to be locally stable. This contrasts with the primal Keen model, where both types of equilibria are stable for typical parameters. On the other hand, notice that the condition

$$1 - \nu(\alpha + \beta + \delta) < c_+, \tag{39}$$

which is necessary for the stability of an explosive equilibrium with $\lambda = 0$ and $d_h \to -\infty$, is always compatible with assumption (a)-(iii). Observe further that if the capital-to-output ratio were to grow significantly, then both (38) and (39) would be satisfied, whereas assumption (a)-(iii) would be violated. In other words, the interior equilibrium would cease to exist and the explosive equilibria with $\lambda = 0$ and $d_h \to \pm\infty$ would likely become stable.

These different scenarios are illustrated in Fig. 1, where we observe first convergence to the interior equilibrium in the top panel, the emergence of business cycles with slowly increasing oscillations in the middle panel, and convergence to the explosive equilibrium in the bottom panel, with the parameter values and specific functional forms for the Phillips curve and consumption as described in Appendix D. The mechanism behind the cycles observed in this Figure correspond to a straightforward modification of Goodwin growth cycles. For example, at low wage share and relatively strong employment (top-left quadrant in the $\omega \times \lambda$ phase diagram), firms experience an investment-driven boom (as consumption is low and the factor $1 - c(\omega - rd_h)$ is high), leading to even higher employment (according to the second equation in (28)) and higher wages (according to the first equation in (28)). As wages increase, so does consumption, leading to a gradual decrease in investment (recall that investment is an adjusting variable in our model). Eventually the wage share rises to a point where $1 - c(\omega - rd_h)$ is lower than $\nu(\alpha + \beta + \gamma)$ and employment starts to decrease, as a result of contracting investment. Because employment is still high, however, wages continue to increase (top-right quadrant in the $\omega \times \lambda$ phase diagram), until employment because sufficiently low that workers begin to accept wage cuts, while employment continues to decrease (bottom-right quadrant). When wage come sufficiently down, consumption is also low and consequently investment is high again, making employment start to increase (bottom-left quadrant), although wages continue to go down. At sufficiently high employment the economy is back at the top-left quadrant and wages begin to increase again. Differently from the Goodwin model, however, the wage share and employment rate do not cycle back to exactly the same value, because the dynamics of private debt affects the factor $1 - c(\omega - rd_h)$. A convergent or explosive private debt ratio d_h determines whether the economy ultimately converges to the interior equilibrium as in the top row of Figure 1 or an equilibrium with collapsing wages and employment as in the bottom row.

Finally, observe that, since $g_\pm := (1 - c_\pm)/\nu - \delta$ is the growth rate for any equilibrium with $d_h \to \pm\infty$, we also see from (77) and (78) that another necessary condition for the stability of any equilibrium with $\lambda = 0$ and $d_h \to \pm\infty$ is

$$r > g_\pm + i(\overline{\omega}), \tag{40}$$

where either $i(\overline{\omega}) = i(\overline{\omega}_2) = -\eta_p$ or $i(\overline{\omega}) = i(\overline{\omega}_3)$. In either case, (40) is reminiscent of the controversial condition $r > g$ emphasized in Piketty (2014). Some differences, however, are worth mentioning:

(1) Here, we deal with a monetary non-linear dynamical system *à la* Keen with endogenous saving rate and private debt, while Piketty (2014) deals with a non-monetary Solow model with exogenous saving rate and no debt, where in fact the condition $r > g$ does *not* imply a divergence between the incomes from work and capital, as observed in Acemoglu and Robinson (2015).
(2) In Piketty (2014), r denotes the average return of capital, which includes the interest paid on government bonds, but also the return on many other types of financial assets. Here, r denotes the average rate of interest paid on private debt in the form of bank loans.
(3) Piketty (2014) argues that money should be neutral in the long run, even though no convincing argument is provided that would sustain this statement. Here, by contrast, money is neither neutral in the short-run, nor in the long-run. Indeed, as already emphasized, the nominal rate, r, and inflation, i, deeply shape the configuration space of system (28). Hence, following the present analysis, the entire dynamical landscape of an actual economy is likely to be affected by monetary policy.

Nevertheless, the connection between (40) and the stability of explosive equilibria prompts the question as to whether these type of equilibria would induce some divergence among income and wealth for different groups of households, a question to which we shall turn after a few remarks.

[8] In France, for example, the cost of spending the minimal wage to the entire population would amount to $c_- \simeq 0.03$.

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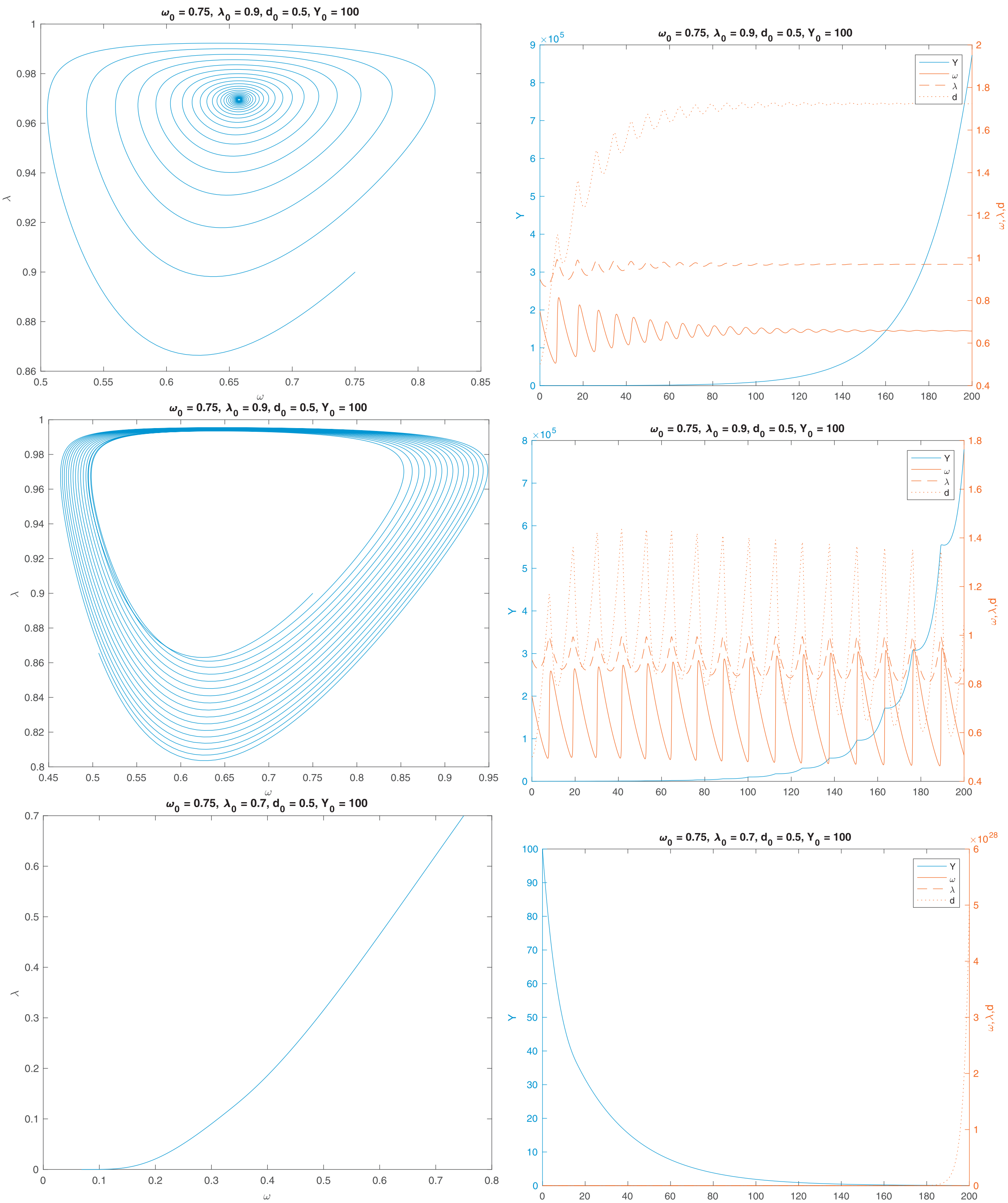

Fig. 1. Simulations of the dual Keen model with parameter values as in Table 3. Top row shows convergence to an interior equilibrium using all baseline parameters. Middle row shows business cycles with slowly increased oscillations when $\eta_p = 0.45$ and $\gamma = 0.96$. Bottom row shows convergence to an explosive equilibrium when $\nu = 15$.

Remark 2.5. The current debate on secular stagnation leads one to ask what would happen in our model when productivity weakens, that is to say, when the parameter α decreases. First, a smaller α makes it easier for the condition in (38) to be violated and consequently easier for the condition (a)–(iii) in Assumption 2.1 to be satisfied. Therefore, according to Lemma 2.1 and the discussion in Appendix B.2, low productivity makes it easier for the interior equilibrium to exist and harder for the explosive equilibria to be locally stable.[9] Second, it would increase the speed at which a steady state is reached (see (34)). Next, it would lead to a slower long-run real growth pace $g(\bar{\omega}_1, \bar{d}_1) = \alpha + \beta$ for the interior equilibrium, together with a higher level of steady state private debt $\bar{d}_1$, higher unemployment, and a larger wage share $\overline{\omega}_1$ as follows from (33), (32), and (31).

If the demographic trend also decreases to zero, as is expected by some UN demographic scenarios (according to which world population would plateau around 11 billion people toward the end of this century) or as it is already the case in most countries of the developed world, then the interior equilibrium would exhibit stagnation, that is, $g(\bar{\omega}_1, \bar{d}_1) = \alpha + \beta \approx 0$. This, however, provides an interesting case where credit creation does not create a “growth imperative” even though the short-run interest rate r may be strictly positive at the long-run (stagnating) steady state.[10]

Remark 2.6. The last decades in the U.S. and the U.K. have witnessed a sharp fall in the bargaining power of workers together with a slowdown of labor productivity. Because the bargaining power of workers is represented by the Phillips curve $\Phi(\cdot)$ in our model, this raises the question of what happens when $\Phi(\cdot)$ becomes flatter, especially in the regime when productivity is also low (namely small α) and the markup power of firms is high (namely high m), as it seems to be the case in the present. In this case, condition (b) in our structural Assumption 2.1 implies that the Phillips curve would reach the value $\alpha + (1-\gamma)i(\bar{\omega}_1)$ only at very high levels of employment, as it would otherwise remain flat for lower levels of employment, but nevertheless needs to increase rapidly as $\lambda \to 1$ in order to approach a vertical asymptote at that limit. It then becomes apparent that the sole impact of this extreme situation is to increase the long-run employment level $\overline{\lambda}_1$ according to (31) – which is consistent with the low unemployment rates recently observed both in the U.S. and the U.K. (provided these economies are indeed on the way to the interior long-run steady state).

Remark 2.7. It is apparent from the discussion above that deflation is most readily associated with the equilibria exhibiting collapsing employment, namely $i(\bar{\omega}_2) = -\eta_p < 0$ and $i(\bar{\omega}_3) < 0$. While it is possible to observe $i(\bar{\omega}_1) < 0$, it is nevertheless true that if $\alpha + \beta + i(\bar{\omega}_1) < 0$, then we cannot guarantee that $\bar{\lambda}_1 > 0$ exists. In other words, the combination of deflation, low growth, and low productivity make it exceedingly unlikely for the economy to converge to the balanced-growth path.

Remark 2.8. Within a hybrid New Keynesian–Phillips Curve (NKPC) setting, feedback effects between labour productivity and unemployment as in Phelps' structural slumps (Phelps, 1994) and, accordingly, hysteresis of unemployment (e.g., Stiglitz, 1997) may cause the New Keynesian NAIRU to shift. More radically, the empirical evidence supporting the existence of the NAIRU in today's economies is, at best, weak (e.g., Stanley, 2004 or Storm and Naastepad, 2012). If there is no such thing as a long-run horizon independent of short-run fluctuations and demand, the system then becomes path-dependent, and hysteresis again enters the picture (see, e.g., Cross, 1993 and Cross et al., 2000). In our model, the dynamics is driven by an autonomous dynamical system (28). Hence, there is no path-dependence in the mathematical sense of the word.[11] However, the very fact that there are multiple equilibria — among which, the interior equilibrium is the unique one potentially exhibiting a NAIRU, see Remark 2.7 above— means that initial conditions do play a role in determining whether the economy will ultimately converge towards anything akin to a NAIRU or not. In other words, the past matters. Similarly, policy interventions discussed in Remarks 2.4 and 2.6 above obviously alter the long-run level of $\bar{\lambda}_1$ through (32), so that a “time-varying” NAIRU (as described e.g., in Gordon, 1997) can be perfectly accommodated for in our set-up.

3. A model with two classes of households

3.1. Workers and investors

We now consider a household sector divided into two classes: workers and investors. Workers are employed by the firm sector and hold deposits M_w and loans L_w in the banking sector. Their income therefore consists solely of wages and the difference between the interest received on deposits and paid on loans. Investors, on the other hand, have private ownership of both the firm and baking sector. Therefore, in addition to deposits M_i and loans L_i, their balance sheet consist of firm equity, E_f, and bank equity, E_b. Accordingly, the income of investors consists of dividends earned from their ownership of firms and banks, in addition to the interest rate differential between deposits and loans. The accounting structure of the economy with two classes of households is given by Table 2. We therefore see that the net worth (i.e., wealth) of workers is $X_w = -D_w$ and that of investors is $X_i = E_f + E_b - D_i$, where, as before, $E_f = pK + M_f - L_f = pK - D_f$ is the equity of firms and $E_b = (L_f + L_w + L_i) - (M_f + M_w + M_i)$ is the equity of banks. Once more we obtain

$$X = X_i + X_w = E_f + E_b - D_i - D_w$$

[9] In both cases, one would also need $\Phi(0) < 0$, according to (29), which is perfectly reasonable.
[10] See Jackson and Victor (2015) for an analogous point.
[11] Although introducing delays as suggested in Remark 2.2 would lead to a path-dependent dynamics.

Table 2
Balance sheet, transactions and flow of funds for a three-sector economy with two types of households.

	Workers	Investors	Firms		Banks	Sum
Balance sheet						
Capital stock			$+pK$			pK
Deposits	$+M_w$	$+M_i$	$+M_f$		$-M$	0
Loans	$-L_w$	$-L_i$	$-L_f$		$+L$	0
Equity		E	$-E_f$		$-E_b$	0
Column sum (Net worth)	X_w	X_i	0		0	pK
Transactions			Current	Capital		
Consumption	$-pC_w$	$-pC_i$	$+pC$		$-pC_b$	0
Investment			$+pI$	$-pI$		0
Accounting memo [GDP]			[pY]			
Wages	$+\mathrm{w}\ell$		$-\mathrm{w}\ell$			0
Depreciation			$-p\delta K$	$+p\delta K$		0
Interest on loans	$-rL_w$	$-rL_i$	$-rL_f$		$+rL$	0
Interest on deposits	$+rM_w$	$+rM_i$	$+rM_f$		$-rM$	0
Dividends		$+r_k pK + \Delta_b$	$-r_k pK$		$-\Delta_b$	0
Column sum (balances)	S_w	S_i	S_f	$-pI + p\delta K$	S_b	0
Flows of funds						
Change in capital stock			$+p\dot{K}$			$p\dot{K}$
Change in deposits	$+\dot{M}_w$	$+\dot{M}_i$	$+\dot{M}_f$		$-\dot{M}$	0
Change in loans	$-\dot{L}_w$	$-\dot{L}_i$	$-\dot{L}_i$		$+L$	0
Column sum (savings)	S_w	S_i	S_f		S_b	$p\dot{K}$
Change in firm equity		$+\dot{E}_f$	$-(S_f + \dot{p}K)$			0
Change in bank equity		$+\dot{E}_b$			$-S_b$	0
Change in net worth	S_w	$\dot{E} + S_i$	0		0	$\dot{p}K + p\dot{K}$

$$= pK - D_f + (D_f + D_i + D_w) - D_i - D_w$$
$$= pK,$$

but now observe that

$$X_i = pK - D_f + (D_f + D_i + D_w) - D_i = pK + D_w.$$

In other words, whereas the total wealth of the aggregate household sector (and therefore of the entire economy) still consists of the non-financial assets of the firm sector, the wealth of the investor class consists of these non-financial assets *plus* the net debt of workers.

The budget constraint for workers implies that their net debt evolves as

$$\dot{D}_w = -S_w = pC_w - \mathrm{w}\ell + rD_w, \tag{41}$$

where C_w is the consumption of workers. On the other hand, the net debt of investors D_i evolves as

$$\dot{D}_i = -S_i = pC_i - r_k pK + rD_i - \Delta_b, \tag{42}$$

where C_i is the consumption of investors, $r_k pK$ corresponds to dividends paid by the firms with a rate of return on capital r_k (in general distinct from the loan interest rate r) and Δ_b are the dividends paid by the banking sector. Let us assume, as in Section 2, that the net equity E_b of the banking sector remains constant because its consumption is zero and all of its profits are distributed to investors, that is $C_b = 0$ and $\Delta_b = r(D_f + D_w + D_i)$. As we have seen in the previous section, the constant E_b plays no role in the dynamics of the system (or its equilibria), and from now on we assume that $E_b = 0$ without loss of generality, so that $D_w + D_f = -D_i$. It then follows that (42) reduces to

$$\dot{D}_i = pC_i - r_k pK + rD_i. \tag{43}$$

Extending the notation of the previous section, we assume that consumption of workers and investors is determined by functions $c_w(\cdot)$ and $c_i(\cdot)$ as follows

$$C_w := c_w(\omega - rd_w)Y \tag{44}$$

$$C_i := c_i(r_k \nu - rd_i)Y, \tag{45}$$

where $y_\omega = \omega - rd_w$ and $y_i = r_k \nu - rd_i$ are the shares of nominal output corresponding to the disposable income of workers and investors. Here $d_w = D_w/(pY)$, $d_i = D_i/(pY)$, and we used the fact that $r_k pK = r_k \nu pY$. Even though the aggregate consumption of both classes depend upon their private debt servicing, they are independent of their balance sheet. As already alluded to in Section 1, a rationale for this behaviour may be provided by a Veblen-type of conspicuous consumption nurtured by the desire to "keep with the Joneses". In addition, we assume that investors have a lower consumption propensity with respect to income, that is to say, $c'(y_w) > c'(y_i)$.

3.2. Return on capital and corporate debt

The production side of the economy remains unchanged, given by (12), (25) and (1), with total consumption being $C = C_i + C_w$, except that firms now pay dividends to shareholders, which we assume to be a constant fraction $\Theta \in [0, 1]$ of profits (9). Accordingly, the rate of return on capital, r_k, can be found endogenously as:

$$\begin{aligned} r_k := r_k(\omega, d_w, d_i) &= \frac{\Theta(pY - \mathrm{w}\ell - rD_f - p\delta K)}{pK} \\ &= \frac{\Theta}{\nu}\left(1 - \omega - rd_f - \delta\nu\right) \\ &= \frac{\Theta}{\nu}(1 - \omega + r(d_w + d_i) - \delta\nu), \end{aligned} \tag{46}$$

where we have used again the fact that $D_f = -(D_i + D_w)$, since we assumed that $E_b = 0$. Savings for the firm sector are therefore given by retained profits, that is,

$$S_f = (1 - \Theta)(pY - \mathrm{w}\ell - rD_f - p\delta K) = pY - \mathrm{w}\ell - rD_f - p\delta K - r_k pK. \tag{47}$$

It then follows that the amount that needs to be raised externally to finance investment, namely the difference $[p(I - \delta K) - S_f]$, is given by

$$p(I - \delta K) - S_f = pI - pY + \mathrm{w}\ell + rD_f + r_k pK.$$

As in the Keen (1995) model, we assume that external financing is obtained solely through loans from the banking sector, rather than, for example, a combination of new loans and share issuance. It then follows from (47) that corporate debt evolves according to:

$$\dot{D}_f = pI - pY + \mathrm{w}\ell + rD_f + r_k pK = -(\dot{D}_w + \dot{D}_i), \tag{48}$$

where we have used (41) and (43).

Remark 3.1. Observe that, whenever workers spend exactly their wages for consumption, that is, $pC_w \equiv w\ell$, then the accounting equations (9) and (12) yield

$$\Pi = pI + pC_i - rD_f - p\delta K. \tag{49}$$

If, moreover, $rD_f + p\delta K = 0$, this is but Kalecki's celebrated equation, of which (49) can be read as an extension. Kalecki's reasoning about the sense in which causality runs between the two sides of the equality still holds in its extended version. Indeed, whereas investors cannot choose the level of profit Π, they do decide their own consumption and therefore influence the level of investment and leverage of the firms they own. In other words, while "workers consume what they earn, investors earn what they consume and invest *less* the debt burden of their firms". Now, in actual economies, workers also borrow money in order to finance consumption. Thus, equation (41) leads us to modify (49) so that

$$\Pi = pI + pC_i + \dot{D}_w + rD_w - rD_f - p\delta K.$$

This sheds some light on the incentives of banks to provide loans to workers (e.g., during the decade prior to the subprime crisis of 2007-2009): investors also earn the debt burden of workers. Notice that a trade off shows up in the choice of the sort-run nominal rate, r. In countries where D_w is large, investors are likely to be in favour of increasing r, while in countries where D_f is large (relative to D_w), they are likely to put pressure on the Central Bank to decrease r.

3.3. The main dynamical system

The evolution of wages is still provided by a short-run Phillips curve (5), so that the dynamic of wage share remains given by (7). The real growth rate of output is now

$$g(\omega, d_w, d_i) = \frac{\dot{Y}}{Y} = \frac{1 - c(\omega, d_w, d_i)}{\nu} - \delta, \tag{50}$$

where

$$c(\omega, d_w, d_i) := c_w(\omega - rd_w) + c_i(r_k\nu - rd_i). \tag{51}$$

The dynamics of employment is given by (4) with $g(\omega, d_w, d_i)$ as in (50). Finally, the net debt of workers and investors change according to (41) and (43) (with $E_b = 0$). We therefore end up with a 4-dimensional dynamical system for the state variables $(\omega, \lambda, d_w, d_i)$:

$$\dot{\omega} = \omega[\Phi(\lambda) - \alpha - (1 - \gamma)i(\omega)]$$

$$\begin{aligned}
\dot{\lambda} &= \lambda\left[\frac{1-c(\omega,d_w,d_i)}{\nu} - (\alpha+\beta+\delta)\right] \\
\dot{d_w} &= d_w\left[r+\delta - \frac{1-c(\omega,d_w,d_i)}{\nu} - i(\omega)\right] + c_w(\omega - rd_w) - \omega \\
\dot{d_i} &= d_i\left[r+\delta - \frac{1-c(\omega,d_w,d_i)}{\nu} - i(\omega)\right] + c_i(r_k\nu - rd_i) - r_k\nu
\end{aligned} \tag{52}$$

where $r_k\nu = \Theta(1-\omega+r(d_w+d_i)-\delta\nu)$ was defined in (46).

In order to study long-run steady states in this economy with two classes of households, we replace the Assumption 2.1 with the following.

Assumption 3.1.

1. The consumption function $c_i, c_w : \mathbb{R}^2 \to \mathbb{R}_+$ are locally Lipschitz, increasing on both arguments, and satisfy the following properties, for all $x \in \mathbb{R}$:
   (i) $y \le 0 \Rightarrow c_i(y) = c_{i^-} > 0$ and $c_w(y) = c_{w^-} > 0$, with $c_{i^-} + c_{w^-} = c_-$.
   (ii) $\lim_{y\to+\infty} c_w(y) = c_{w^+}$ and $\lim_{y\to+\infty} c_i(y) = c_{i^+}$, with $c_{w^+} + c_{i^+} = c_+ \le 1$.
   (iii) $c_- < 1 - \nu(\alpha+\beta+\delta) < c_{w^+}$
2. $\Phi : [0,1) \to \mathbb{R}$ is $\mathcal{C}^1$, strictly increasing and admits a vertical asymptote at $\lambda = 1$. Moreover, $\Phi'(0) = 0$ and
$$\Phi(0) < \min\{\alpha, \gamma(\alpha+\beta) - \beta\} \tag{53}$$
3. $r + \eta_p \ge \alpha + \beta$

3.3.1. The balanced-growth path

As in Section 2.4.1, we start with an equilibrium with nonzero wage share and employment rate. As before, the equation for employment implies that the equilibrium growth rate is $\alpha+\beta$. With this in mind, define

$$\omega_0 = \frac{1}{m}\left[\frac{r-(\alpha+\beta)}{\eta_p} + 1\right] \tag{54}$$

and observe that Assumption 3.1 implies that $\omega_0 \ge 0$. Moreover, we can see that $\alpha+\beta+i(\omega_0) = r$ and $\alpha+\beta+i(\omega) > r$ for all $\omega > \omega_0$. Define next d_{w0} as the solution to

$$c_w(\omega_0 - rd) = \omega_0, \tag{55}$$

which exists provided $c_{w-} < \omega_0 < c_{w+}$. Finally, define d_{i0} as the solution to

$$c_i(\Theta(1-\omega_0+rd_{w0}-\delta\nu) - (1-\Theta)rd) = \Theta(1-\omega_0+rd_{w0}+rd-\delta\nu), \tag{56}$$

which always exists, since the right-hand side is a linear function of d, whose image always contains the interval $[c_{i-}, c_{i+}]$.

Lemma 3.1. *Assume that ω_0, d_{w0} and d_{i0} defined as in (54)–(56) satisfy*

$$c_{w-} < \omega_0 < c_{w+} \tag{57}$$

and

$$(1-\Theta)\omega_0 + \Theta r(d_{w0}+rd_{i0}) < 1-\nu(\alpha+\beta+\delta) - \Theta(1-\delta\nu). \tag{58}$$

Then there exist an equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_{w1}, \overline{d}_{i1})$ for (52) with $0 \le \omega_0 < \overline{\omega}_1 < \infty$ and $\overline{\lambda}_1 > 0$.

Proof. See Appendix A. □

Before investigating how inequality evolves along this growth path, let us turn to undesirable equilibria.

3.3.2. Equilibria with collapsing employment

In order to investigate the properties of the system for large values of net debt for workers and investors, we consider first the changes of variables $u_w = 1/d_w$ and $u_i = 1/d_i$, leading to the modified system

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha - (1-\gamma)i] \\
\dot{\lambda} &= \lambda\left[\frac{1-c}{\nu} - (\alpha+\beta+\delta)\right] \\
\dot{u_w} &= u_w\left[\frac{1-c}{\nu} + i - r - \delta\right] - u_w^2[c_w - \omega] \\
\dot{u_i} &= u_i\left[\frac{1-c}{\nu} + i - r - \delta\right] - u_i^2\left[c_i - \Theta\left(1-\omega+\frac{r}{u_w}+\frac{r}{u_i}-\delta\nu\right)\right].
\end{aligned} \tag{59}$$

The appearance of the term u_i^2/u_w in the differential equation for u_i above makes it unclear whether $u_i = u_w = 0$ corresponds to an equilibrium of the modified system. We then follow Grasselli and Costa Lima (2012) and consider the $u_w = 1/d_w$ together with the ratio $\nu = u_i/u_w = d_w/d_i$. This leads to the further modified system

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha - (1-\gamma)i] \\
\dot{\lambda} &= \lambda\left[\frac{1-c}{\nu} - (\alpha+\beta+\delta)\right] \\
\dot{u}_w &= u_w\left[\frac{1-c}{\nu} + i - r - \delta\right] - u_w^2[c_w - \omega] \\
\dot{\nu} &= \nu(1+\nu)r\Theta + \nu u_w[c_w - \omega] - \nu^2 u_w[c_i - \Theta(1-\omega-\delta\nu)]
\end{aligned} \tag{60}$$

We can then see that $(\omega, \lambda, u_w, \nu) = (0, 0, 0, 0)$ and $(\omega, \lambda, u_w, \nu) = (0, 0, 0, -1)$ are equilibria of the modified system (60) corresponding to $(\omega, \lambda, u_w, u_i) = (0, 0, 0, 0)$ in (59) along trajectories with $u_i^2/u_w = u_i\nu \to 0$, which in turn correspond to $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_{w2}, \overline{d}_{i2}) = (0, 0, \pm\infty, \pm\infty)$ for the original system (52) along a trajectory with $d_w/d_i \to 0$, that is to say, with the debt of investors growing faster than the debt of workers.

Finally, yet another type of long-term steady state can be reached of the form $(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_{w3}, \overline{d}_{i3}) = (\overline{\omega}_3, 0, \overline{d}_{w3}, \overline{d}_{i3})$ where

$$\overline{\omega}_3 = \frac{\Phi(0) - \alpha}{m(1-\gamma)\eta_p} + \frac{1}{m}, \tag{61}$$

and $\overline{d}_{w3}$ is either a finite solution of

$$d\left[r + \delta - \frac{1-c)}{\nu} - i\right] = \overline{\omega}_3 - c_w, \tag{62}$$

or else $\overline{d}_{w3} = \pm\infty$, whereas $\overline{d}_{i3}$ is either a finite solution of

$$d\left[r(1-\Theta) + \delta - \frac{1-c}{\nu} - i\right] = \Theta(1 - \overline{\omega}_3 + r\overline{d}_{w3} - \delta\nu) - c_i \tag{63}$$

for finite $\overline{d}_{w3}$, or else $\overline{d}_{i3} = \pm\infty$.

The stability of these classes of equilibrium is discussed in Appendix C and numerical simulations are shown in Fig. 2.

3.4. Asymptotic inequality

Using the definitions in Table 2, we see that the total nominal income for the two types of households and retained profits of the firm sector are given by

$$\begin{aligned}
Y_w^n &= \mathrm{w}\ell - rD_w \\
Y_i^n &= r_k pK - rD_i = \Theta(pY - \mathrm{w}\ell - rD_f - p\delta K) - rD_i \\
\Pi_r &= (1-\Theta)(pY - \mathrm{w}\ell - rD_f - p\delta K),
\end{aligned}$$

so that, using again $D_f = -(D_w + D_i)$, we find that their sum equals total income, that is

$$Y_w^n + Y_i^n + \Pi_r = (pY - \delta pK). \tag{64}$$

Moreover, since investors are the private owners of the firms, we follow the terminology adopted in Piketty (2014), according to which retained profits should be added to dividends to obtain the total income arising from capital. In other words, the total income from capital corresponds to the sum $(Y_i^n + \Pi_r)$.

Accordingly, the shares of output corresponding to the income of workers, income of investors, and retained profits are given by

$$y_w = \frac{Y_w^n}{pY} = \omega - rd_w \tag{65}$$

$$y_i = \frac{Y_i^n}{pY} = r_k\nu - rd_i = \Theta(1 - \omega + rd_w - \delta\nu) - (1-\Theta)rd_i \tag{66}$$

$$\pi_r = \frac{\Pi_r}{pY} = (1-\Theta)(1 - \omega - rd_f - \delta\nu), \tag{67}$$

whereas the share of output corresponding to total capital income is

$$y_c = y_i + \pi_r = 1 - \omega + rd_\omega - \delta\nu = 1 - y_\omega - \delta\nu, \tag{68}$$

in accordance with (64).

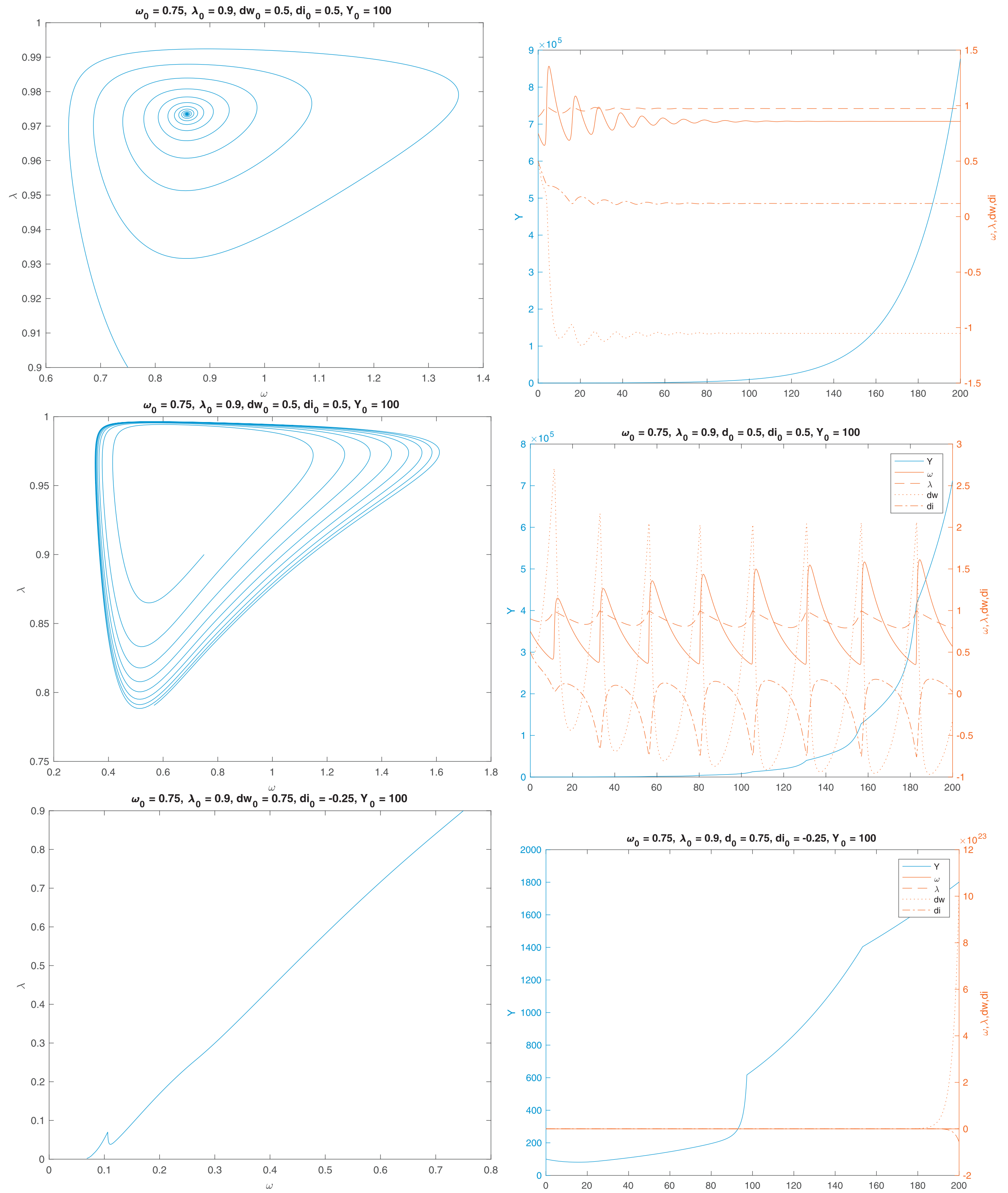

Fig. 2. Simulations of the dual Keen model with two classes of households using parameter values as in Table 3. Top row shows convergence to an interior equilibrium with $\overline{d}_w > 0$ and $\overline{d}_i < 0$ using all baseline parameters. Middle row shows business cycles with slowly increased oscillations when $\eta_p = 0.45$ and $\gamma = 0.96$. Bottom row shows convergence to the explosive equilibrium with $d_w \to +\infty$ and $d_i \to -\infty$ when $\nu = 15$.

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3.4.1. When inequality remains constant in the long-run

Let us now examine whether a divergence between investors' and workers' real incomes can occur in the long-run. The growth rate of total real income for workers $Y_w^n/p = (\omega - rd_w)Y$ is

$$g_w(\omega, \lambda, d_w, d_i) = \frac{\left(\dot{\omega} - r\dot{d}_w\right)}{\omega - rd_w} + \frac{\dot{Y}}{Y}.$$

At equilibrium, we have that $\dot{d}_w = \dot{\omega} = 0$, so that

$$g_w(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_{w1}, \overline{d}_{i1}) = \frac{\dot{Y}}{Y} = \alpha + \beta, \tag{69}$$

provided $\overline{\omega}_1 \neq r\overline{d}_{w1}$.

Similarly, the growth rate of total real income for investors $Y_i^n/p = \left(r_k\nu - rd_i\right)Y$ is

$$\begin{aligned} g_i(\omega, \lambda, d_w, d_i) &= \frac{\dot{r}_k\nu - r\dot{d}_i}{r_k\nu - rd_i} + \frac{\dot{Y}}{Y} \\ &= \frac{-\Theta\dot{\omega} + \Theta r\dot{d}_w - (1-\Theta)r\dot{d}_i}{\Theta(1 - \omega + rd_w - \delta\nu) - (1-\Theta)rd_i} + \frac{\dot{Y}}{Y}. \end{aligned}$$

where we have used (46) in the last equation. At equilibrium, we again have that $\dot{\omega} = \dot{d}_w = \dot{d}_i = 0$, so that

$$g_i(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_{w1}, \overline{d}_{i1}) = \alpha + \beta, \tag{70}$$

provided $r_k(\overline{\omega}_1, \overline{d}_{w1}, \overline{d}_{i1})\nu \neq r\overline{d}_{i1}$.

Finally, the growth rate of real retained profits $\Pi_r/p = (1-\Theta)(1 - \omega - rd_f - \delta\nu)Y$ is

$$g_r(\omega, \lambda, d_w, d_i) = \frac{-\dot{\omega} + r(\dot{d}_w + \dot{d}_i)}{(1-\Theta)[1 - \omega + r(d_w + d_i) - \delta\nu]} + \frac{\dot{Y}}{Y}.$$

Once more, at equilibrium $\dot{\omega} = \dot{d}_w = \dot{d}_i = 0$, so that

$$g_r(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_{w1}, \overline{d}_{i1}) = \alpha + \beta, \tag{71}$$

provided $1 - \overline{\omega}_1 + r(\overline{d}_{w1} + \overline{d}_{i1}) - \delta\nu \neq 0$.

In other words, in this case, both real income from labour and from capital grow at the same pace, namely the real growth rate of the economy. Furthermore, at the interior equilibrium, the rate of return on capital r_k and the income shares y_w, y_i and π_r all converge towards constant levels, fully justifying its characterization as a "balanced-growth path".

3.4.2. Inequality as a hallmark of inefficiency

Next, what happens at one of the undesirable equilibria? As shown in Section 3.3.2, there are four possible asymptotic crisis states corresponding to infinite levels of net debt for workers and investors and vanishing wage share and employment, namely: $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_{w2}, \overline{d}_{i2}) = (0, 0, \pm\infty, \pm\infty)$.

Let us start with the two cases where the debt ratio of workers increases without bound. In the case $d_w \to +\infty$ and $d_i \to -\infty$, observe first that

$$d_f = -(d_w + d_i) = -(\nu + 1)d_i \to +\infty,$$

since this corresponds to an equilibrium with $\nu \to 0$. We therefore see that (65), (66) and (67) with $r > 0$ (as we assume throughout) give $y_w \to -\infty$, whereas

$$y_i = \Theta(1 - \delta\nu) + \Theta rd_w - (1-\Theta)rd_i \to +\infty,$$

and $\pi_r \to -\infty$. In other words, the income share of investors explodes while profits and the income of workers plunges, showing that the distributional conflict here does not pit workers against investors, but rather banks (and their owners) against firms and workers. Nevertheless, (68) shows that $y_c \to +\infty$, that is, the share of total capital income still explodes, despite the fall in profits π_r and return on capital r_k, courtesy of interest on debt paid by workers.

On the other hand, in the case $d_w \to +\infty$ and $d_i \to +\infty$, observe that we now have

$$d_f = -(d_w + d_i) = -(\nu + 1)d_i \to -\infty.$$

We then obtain that $y_w \to -\infty$ and $\pi_r \to +\infty$, whereas, provided $0 \leq \Theta < 1$, one has

$$y_i = \Theta(1 - \delta\nu) + \Theta(rd_w + rd_i) - rd_i = \Theta(1 - \delta\nu) + rd_i(\Theta\nu + \Theta - 1) \to -\infty,$$

where we again used $\nu \to 0$. In other words, the income shares of both investors and workers plunges, while the net profit share, which depends on $d_f = -(d_w + d_i)$, explodes. The corner case of $\Theta = 1$ restores the situation where $y_i \to +\infty$, since in this case net profits are distributed in full to investors, and the share of profits is zero. For all $0 \leq \Theta \leq 1$, however, we have that $y_c \to +\infty$, still courtesy of the interest on debt paid by workers.

Conversely, consider now the cases where workers have positive cash balances, that is to say negative debt, growing faster than income. In the case $d_w \to -\infty$ and $d_i \to +\infty$ we find that

$$d_f = -(d_w + d_i) = -(\nu + 1)d_i \to -\infty.$$

We then have $y_w \to +\infty$, whereas

$$y_i = \Theta(1 - \delta\nu) + \Theta r d_w - (1 - \Theta) r d_i \to -\infty,$$

and $\pi_r \to +\infty$. In other words, the income of workers soars together with profits, whereas the income of investors plunges. We also have that $y_c \to -\infty$, so total capital income also collapses, despite increased profits, this time courtesy of the interest paid by investors (who are the owners of the banks) to workers. Similarly, in the case $d_w \to -\infty$ and $d_i \to -\infty$ we find that

$$d_f = -(d_w + d_i) = -(\nu + 1)d_i \to +\infty,$$

so that we have $y_w \to +\infty$ and $\pi_r \to -\infty$, whereas, provided $0 \le \Theta < 1$, we have that

$$y_i = \Theta(1 - \delta\nu) + \Theta(r d_w + r d_i) - r d_i = \Theta(1 - \delta\nu) + r d_i(\Theta\nu + \Theta - 1) \to +\infty.$$

In other words, the income share of both investors and workers soars, while this time the net profit share plunges. The corner case of $\Theta = 1$ restores the situation where $y_i \to -\infty$, since in this case the (negative) net profits of firms are distributed in full to investors, and the share of profits is zero. In either case, $y_c \to -\infty$, still courtesy of the interest paid by investors to workers.

We therefore see that in all four cases where $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_{w2}, \overline{d}_{i2}) = (0, 0, \pm\infty, \pm\infty)$, income inequality between workers and investors grows without bound. In the two cases where $d_w \to +\infty$, such extreme inequality favours capitalists, whereas in the two cases when $d_w \to -\infty$ the converse is true. This result sharply contrasts with those of Piketty (2014), according to which any efficient economy (where it is likely that the condition $r \ge g$ holds) should experience an ever growing inequality. We obtain exactly the opposite conclusion: an "inefficient" economy (i.e., one that converges to a disastrous equilibrium) experiences an ever growing inequality, while an efficient economy (one that ultimately follows the balanced-growth path) converges to a constant state of inequality.

Finally, consider the deflationary equilibrium $(\overline{\omega}_3, 0, \overline{d}_{w3}, \overline{d}_{i3})$. The situation when $(\overline{d}_{w3}, \overline{d}_{i3}) \to (\pm\infty, \pm\infty)$ is entirely analogous to the four cases analyzed above. In the case of finite $\overline{d}_{w3}$ and $\overline{d}_{i3}$, however, the income share of both populations will be respectively $r_k\nu - r\overline{d}_{i3}$ and $\overline{\omega}^3 - r\overline{d}_{w3}$ and therefore both finite. This is, however, an artifact of the fact that prices are falling faster than real output. In other words, nominal income ratios remain artificially constant, but the real income of both populations collapse. In other words, this time, there is no divergence because *both* types of households end up ruined! Appendix B.3. provides the necessary and sufficient conditions for the local stability of such a liquidity trap at large time scale. As before, it turns out that increasing the capital-income ratio, ν, makes it more likely for the asymptotic liquidity trap to be locally stable. As a consequence, a good policy to drive an actual economy out of the basin of attraction of such a catastrophic steady-state consists in reducing ν.

The analysis of wealth inequality is straightforward, since we have $X_w = -D_w$ for the wealth of workers and $X_i = pK + D_w$ for the wealth of investors. Namely, at the interior equilibrium the wealth-to-income ratios $x_w = X_w/(pY)$ and $x_i = X_i/(pY)$ converge to $\overline{x}_w = -\overline{d}_{w1}$ and $\overline{x}_i = \nu + \overline{d}_{w1}$ respectively. As discussed in Section (3.3.1), we have that $\overline{d}_{w1} \ge 0$ provided $c_w(\overline{\omega}_1, 0) \ge \overline{\omega}_1$, that is to say, when consumption for workers with zero debt is larger than wage income, which is likely to hold in reality, leading to an equilibrium situation where $\overline{x}_i > \overline{x}_w$, but with constant ratios. Even when workers have positive cash balances at equilibrium (that is, $\overline{d}_{w1} < 0$), we could still have $\overline{x}_i > \overline{x}_w$ provided $\nu > -2\overline{d}_{w1}$.

At the explosive equilibria, it is clear that $x_w \to -\infty$ and $x_i \to +\infty$ whenever $d_w \to +\infty$, so that we observe extreme levels of wealth inequality in favour of investors, with the reverse situation when $d_w \to -\infty$. Finally, at the deflationary equilibrium $(\overline{\omega}_3, 0, \overline{d}_{w3}, \overline{d}_{i3})$ with finite $\overline{d}_{w3}$ and $\overline{d}_{i3}$ we have finite wealth-to-income ratios of the form $\overline{x}_w = -\overline{d}_{w3}$ and $\overline{x}_i = \nu + \overline{d}_{w3}$, but only because prices are falling faster than real income in this collapsing economy.

4. Discussion and future work

In the previous sections we introduced a monetary stock-flow consistent model to analyze the interplay between household debt, inequality and growth. The version of the model with an aggregate household sector presented in Section 2 displays the characteristic behaviour of Keen-type models analyzed in Grasselli and Costa Lima (2012), namely a multiplicity of equilibria, one of them exhibiting an ever growing debt-to-income ratio and accompanying decrease in employment rate and wage share. As observed in Section 2.4, the conditions for stability of this type of equilibrium are likely to be violated for currently observed parameter values, therefore favouring the existence and stability of an interior equilibrium with a finite debt ratio and non-zero employment instead. In addition, we have shown that this interior long-run steady state can be seen as an extension of Solow's famous balanced-growth path, as it satisfies all of its properties. Finally, employment along this balanced-growth path turns out to converge towards a positive NAIRU, thus providing an out-of-equilibrium foundation for Tobin's concept.

This can be reversed, however, if some of the model parameters change in the way that is predicted in Piketty (2014), notably by an increase in the capital-income ratio ν and a decrease in the long-run "natural" growth rate $\alpha + \beta$, both of which would improve the stability of the deflationary equilibrium with explosive debt levels. Since both trends are associated with increasing income and wealth inequality in Piketty (2014), we then considered a version of the model with two classes of households, namely workers and investors, in Section 3 to investigate the connection between the two phenomena. As shown in Section 3.4, we indeed observe that the income shares of workers and investors diverge without bounds whenever the economy approaches any of the possible deflationary equilibria with explosive debt-to-income ratios for households. Among these catastrophic long-run steady states, some of them exhibit a debt-deflationary trend, typical of what has been described by Irving Fisher and that several papers recently tried to capture (see, e.g., Eggertsson and Krugman, 2011 and Giraud and Pottier (2016)). Both papers, however, confined themselves to an equilibrium approach of the liquidity trap, neglecting the transitional dynamics that may lead to it. Moreover, Eggertsson and Krugman (2011) considers the effect of debt-overhang in a real set-up, where the interaction between money and deflation cannot be tackled. Here, by contrast, we provide a monetary characterization of the road that leads to such a trap, and its hallmark turns out to be the ultimate convergence of real income of both workers and investors because all of them end up ruined. In most cases, including the asymptotic liquidity trap just mentioned, it turns out that increasing the capital-to-income ratio, ν, reinforces the stability of these catastrophic long-run steady states. As a consequence, we reach the rather unexpected conclusion that a good policy to escape from the current savings glut that seems to condemn the world economy to stagnation would consist in boldly reducing the capital-to-income ratio. More generally, our findings shed light on the old dichotomy between efficiency and fairness: the pace at which the size of the pie increases cannot be considered in isolation from the manner it is divided within a given society. Distribution (of income and debt) does have an impact on growth, and more egalitarian economies are more likely to be efficient in the long-run. One reason why this conclusion has hardly been reached in earlier analyses lies in the neglect of the macroeconomic impact of debt. Indeed, it has long been believed that, one debt being just the counterpart of someone else's liability, private debt has little consequence, if any, on the macroeconomic trajectory. This intuition turns out to be wrong, as we have shown in this paper. One first consequence is that long-term growth cannot be understood within a framework with a single "representative" consumer (who, by construction, cannot exhibit debt). In addition, capital and income taxation can become tools of paramount importance in order to foster the growth of national income, or at least to prevent it from declining.

We kept the model deliberately simple to highlight the interplay between debt and inequality in a straightforward way. Having already hinted at some possible extensions of our setting in Remark 2.2, we now briefly discuss the likely effect of some of them on our results. Let us begin with the assumption of a constant capital-to-output ratio in (1) and purely accommodating investment function of the form $(1 - c(y))$ in (25). It is clear that these simplifications are overoptimistic, in the sense that investment demand keeps aggregate demand artificially high even at times of very low consumption. A more realistic setting, for example along the Steindlian lines adopted in Dutt (2006), would assume that firms keep excess capacity and adjust output to meet aggregate demand, with investment demand being a function of utilization. This modification would result in reduced output when the debt ratio of workers increases (because of lower total consumption) therefore reducing employment and consequently wages, leading to even lower consumption. It is therefore clear that, absent other mechanism, a more realistic production side would strengthen the link between household debt and inequality in our model.

A second modification pertains the inclusion of capital markets in the model. Following Skott (2013), one can consider ownership of firms by investors through an active stock market where share prices are determined by equilibrium (as opposed to the prices of goods, which are assumed to follow the dynamics (6) in an imperfectly competitive market). As shown in Skott (2013), when the demand for shares depends on expected returns, which in turn follow an adaptive dynamics based on observed returns, the stock market is prone to inequality-induced bubbles. This mechanism is also likely to exacerbate the divergence of incomes in our set-up.

Third, in order to model in more detail the Veblenian effect of positional goods and expenditure cascades in the sense of Frank et al. (2014), one would need to replace the two-class representation adopted here by a continuum of households, where continuous deformations of income distribution could be captured. This would require extending our finite-dimensional dynamical-system approach to a partial differential equation framework where "space" would be given by the scale of income or wealth.

Two final modifications, nevertheless, do have the potential to alter the conclusions of the model: default and government intervention. As mentioned in Pottier and Nguyen-Huu (2017), the introduction of bankruptcy by firms in the original Keen (1995) model can prevent debt ratios from increasing indefinitely and lead to limit cycles instead. The same mechanism can prevent households from accumulating increasingly large debt ratios. As default also implies losses to asset holders, the corresponding redistribution of wealth and income can decrease inequality, much as the losses caused by the turmoil of wars and depression did in the first half of the last century, as documented in Piketty (2014). Moreover, here the dynamics of debt is exclusively determined by the borrowers and not by the lenders. Adding credit rationing, for example along the lines of Bovari et al. (2019) or Kapeller and Schütz (2014b) should again enable us to get rid of the unrealistic feature of collapsing equilibria with unbounded debt. As shown there, however, the endogenous upper-bound on private debt induced by some prudential constraints on the banking sector's ability to provide credit does not prevent catastrophic equilibrium to be asymptotically stable. Finally, as shown in Costa Lima et al. (2014), government intervention, both in the form of taxes and government spending, can destabilize the explosive equilibrium in the original Keen (1995) model, as well as prevent employment from remaining arbitrarily low, provided the intervention is responsive enough at periods of crisis. We expect that a similar stabilization role can be played by income and wealth taxes levied in a sufficiently progressive way. Moreover, the level of the short-run nominal interest rate turns out to have a long-lasting impact on the economy's trajectory. It is therefore to be expected that a countercyclical monetary policy (the Taylor rule being just one example aiming at stabilizing inflation) might help avoid a catastrophic steady-state. And since we have observed that, at large time scale, a trade-off between employment and inflation emerges along the trajectories that lead to the balanced-growth path, such monetary policy might as well try to favour employment.

Last but not least, an important test for the relevance of our framework will consist in bringing it to empirical data. This has been done with some success for the primal Keen model (see Grasselli and Maheshwari (2018) and Bovari et al. (2018)). It remains to be checked whether this can be done for our dual version, and its extension with two classes of households.

Appendix A. Intermediary results

Proof of Lemma 2.1: If $i(\overline{\omega}_1) \geq 0$, then the inequality $\Phi(0) < \alpha \leq \alpha + (1-\gamma)i(\overline{\omega}_1)$ implies that (32) admits a solution $\overline{\lambda}_1 > 0$. On the other hand, if $i(\overline{\omega}_1) < 0$, then (29) implies that

$$\alpha + (1-\gamma)i(\overline{\omega}_1) > \gamma(\alpha+\beta) - \beta > \Phi(0),$$

provided $\alpha + \beta + i(\overline{\omega}_1) > 0$, from which it follows again that $\overline{\lambda}_1 > 0$ exists. The second statement in the lemma, namely that equilibrium household debt is positive if, and only if, equilibrium consumption exceeds equilibrium disposable income, follows directly from the facts that

$$c(\overline{\omega}_1 - r\overline{d}_1) = 1 - \nu(\alpha+\beta+\delta)$$

and $\overline{\omega}_1 - r\overline{d}_1 = \eta_1$.

Proof of Lemma 3.1: Consider the nonlinear equation satisfied by a candidate for equilibrium net debt of workers:

$$d_w = \frac{c_w(\omega - rd_w) - \omega}{(\alpha+\beta) + i(\omega) - r}. \tag{72}$$

For a fixed $\omega > \omega_0$, define

$$\Gamma_w(d) := c_w(\omega - rd) - \omega - d[(\alpha+\beta) + i(\omega) - r]$$

as a function of d. It then follows from the definition of ω_0 that $\Gamma_w(d)$ is strictly decreasing, so that (72) admits a solution $d_w(\omega)$ with $d_w(\omega) \to d_{w0}$ as $\omega \to \omega_0$ and $d_w(\omega) \to -1/\eta_p$ as $\omega \to +\infty$.

Consider next the nonlinear equation satisfied by a candidate for the net debt of investors:

$$d_i = \frac{c_i(r_k\nu - rd_i) - \Theta(1 - \omega + rd_w + rd_i - \delta\nu)}{(\alpha+\beta) + i(\omega) - r}. \tag{73}$$

For fixed $\omega > \omega_0$ and $d_w(\omega)$ as determined above, define

$$\Gamma_i(d) := H(\omega, d_w(\omega), d) - G(\omega, d_w(\omega), d) - dF(\omega)$$

as a function of d, where

$$\begin{aligned} H(\omega, d_w(\omega), d) &= c_i(\Theta(1 - \omega + rd_w(\omega) - \delta\nu) - (1-\Theta)rd) \\ G(\omega, d_w(\omega), d) &= \Theta(1 - \omega + rd_w(\omega) + rd - \delta\nu) \\ F(\omega) &= (\alpha+\beta) + i(\omega) - r > 0 \end{aligned}$$

It then follows again that $\Gamma_i(d)$ is strictly decreasing and (73) admits a solution $d_i(\omega)$ with $d_i(\omega) \to d_{i0}$ as $\omega \to \omega_0$ and $d_i(\omega) \to \Theta/\eta_p$ as $\omega \to +\infty$.

Finally, for $d_w(\omega)$ and $d_i(\omega)$ determined as above, we see that total consumption as a function of ω is given by

$$c(\omega) = c_w(\omega - rd_w(\omega)) + c_i(\Theta(1 - \omega + rd_w(\omega) - \delta\nu) - (1-\Theta)rd_i(\omega)). \tag{74}$$

Observe that, if $\omega \to +\infty$, then Assumption 3.1 (a)-(i) implies that $c_i \to 0$, so that

$$c \to \lim_{\omega \to +\infty} c_w(\omega - rd_w(\omega)) = c_{w^+} > 1 - \nu(\alpha+\beta+\delta),$$

according to Assumption 3.1 (a)-(iii). Therefore, provided we have that

$$c(\omega_0) = c_w(\omega_0 - rd_{w0}) + c_i(\Theta(1 - \omega + rd_{w0} - \delta\nu) - (1-\Theta)rd_i(\omega)) < 1 - \nu(\alpha+\beta+\delta), \tag{75}$$

then we can conclude that there exists some $\omega_0 < \overline{\omega} < \infty$ such that

$$c(\overline{\omega}) = 1 - \nu(\alpha+\beta+\delta).$$

But it is easy to see that (75) is equivalent to (58) and the conclusion holds.

We then define an interior equilibrium as the point $\overline{\omega}_1 = \overline{\omega}$, $\overline{d}_{w1} = d_w(\overline{\omega})$ and $\overline{d}_{i1} = d_i(\overline{\omega})$ just obtained and

$$\overline{\lambda}_1 = \Phi^{-1}\big(\alpha + (1-\gamma)i(\overline{\omega}_1)\big), \tag{76}$$

which exists because of Assumption 3.1 (b).

Appendix B. Local stability analysis for (28)

B1. Interior equilibria

Due to the similarity with the monetary version of Keen's model, studied by Grasselli and Nguyen-Huu (2015), the local stability turns out to be quite similar. Assume the existence of an interior equilibrium, as defined by (31), (32), (33). The Jacobian matrix of the system (28) is

$$\begin{bmatrix} \Phi(\lambda)-\alpha+(1-\gamma)\eta_p(1-2\omega m) & \omega\Phi'(\lambda) & 0 \\ -\lambda\frac{c'(\omega-rd_h)}{\nu} & \frac{1-c(\omega-rd_h)}{\nu}-(\alpha+\beta+\delta) & \lambda r\frac{c'(\omega-rd_h)}{\nu} \\ c'(\omega-rd_h)(\frac{d_h}{\nu}+1)-1-d\eta_p m & 0 & K_4 \end{bmatrix}$$

with $K_4 := r+\delta-\frac{1-c(\omega-rd_h)}{\nu}-i(\omega)-rc'(\omega-rd_h)(1+\frac{d_h}{\nu})$. At the equilibrium point, $(\overline{\omega}_1,\overline{\lambda}_1,\overline{d}_1)$, this matrix becomes

$$J(\overline{\omega}_1,\overline{\lambda}_1,\overline{d}_1)=\begin{bmatrix} K_0 & K_1 & 0 \\ -K_2 & 0 & rK_2 \\ K_3-\eta_p m\overline{d}_1 & 0 & K_4 \end{bmatrix},$$

with $K_0 := (\gamma-1)\eta_p m\overline{\omega}_1 < 0$, $K_1 := \overline{\omega}_1\Phi'(\overline{\lambda}_1) > 0$ and $K_2 := \frac{\overline{\lambda}_1}{\nu}c'(\eta) > 0$ having obvious signs, and

$$K_3 := c'(\eta)\Big(\frac{\overline{d}_1}{\nu}+1\Big)-1 \quad \text{and } K_4 := r\Big[1-c'(\eta)(1+\frac{\overline{d}_1}{\nu})\Big]-\big(\alpha+\beta+i(\overline{\omega}_1)\big).$$

With these notations at hand, the characteriztic polynomial of $J(\overline{\omega}_1,\overline{\lambda}_1,\overline{d}_1)$ is quite similar to the one studied by Grasselli and Nguyen-Huu (2015), and the discussion of conditions for local stability are parallel. Details are left to the reader.

B2. Explosive equilibria

Next, the Jacobian associated to modification of (28) after the change in variable $u := 1/d_h$ at the equilibrium (0,0,0) is given by:

$$\begin{bmatrix} \Phi(0)-\alpha+(1-\gamma)\eta_p & 0 & 0 \\ 0 & \frac{1-c_\pm}{\nu}-(\alpha+\beta+\delta) & 0 \\ 0 & 0 & \frac{1-c_\pm}{\nu}-\delta-\eta_p-r \end{bmatrix},$$

where we used the fact that $c(-1/u)\to c_- > 0$ if $u\to 0^+$ (that is, $d_h\to+\infty$) and $c(-1/u)\to c_+\leq 1$ if $u\to 0^-$ (that is, $d_h\to-\infty$). The necessary and sufficient condition for such equilibrium to be locally stable are therefore

$$\Phi(0) < \alpha-(1-\gamma)\eta_p, \quad 1-\nu(\alpha+\beta+\delta) < c_\pm, \quad \frac{1-c_\pm}{\nu}-\delta-\eta_p < r. \tag{77}$$

B3. Deflationary equilibria

Finally, when applied to the deflationary equilibrium $(\overline{\omega}_3,0,\overline{d}_3)$ with $\overline{d}^3 < \infty$ described in subsection 2.4, the Jacobian becomes

$$\begin{bmatrix} (\gamma-1)\eta_p m\overline{\omega}_3 & \overline{\omega}_3\Phi'(0) & 0 \\ 0 & \frac{1-c(\overline{\omega}_3-r\overline{d}_3)}{\nu}-(\alpha+\beta+\delta) & 0 \\ \tilde{K}_3-\eta_p m\overline{d}_3 & 0 & \tilde{K}_4 \end{bmatrix},$$

where

$$\tilde{K}_3 := c'(\overline{\omega}_3-r\overline{d}_3)(\frac{\overline{d}_3}{\nu}+1)-1$$

and

$$\tilde{K}_4 := r+\delta-\frac{1-c(\overline{\omega}_3-r\overline{d}_3)}{\nu}-i(\overline{\omega}_3)-rc'(\overline{\omega}_3-r\overline{d}_3)(1+\frac{\overline{d}_3}{\nu}).$$

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The determinant of this matrix readily yields its eigenvalues, and therefore the necessary and sufficient conditions for local stability:

$$(\gamma-1)\eta_p\overline{\omega}_3 < 0\,,\ \frac{1-c(\overline{\omega}_3 - r\overline{d}_3)}{\nu} < \alpha+\beta+\delta \ \text{ and } \widetilde{K}_4 < 0.$$

While the first condition is always met, the two others are imposing non-trivial restrictions. Since the real growth rate of output at this equilibrium is given by

$$g = \frac{1-c(\overline{\omega}_3 - r\overline{d}_3)}{\nu} - \delta,$$

the second condition says that the real growth rate at this long-run new equilibrium must be lower than the growth rate at an interior equilibrium, namely $\alpha+\beta$.

Lastly, when $\overline{d}_3 = \pm\infty$ we have the following Jacobian matrix at equilibrium:

$$\begin{bmatrix} \Phi(0)-\alpha+(1-\gamma)\eta_p & \overline{\omega}_3\Phi'(0) & 0 \\ 0 & \frac{1-c_\pm}{\nu} - (\alpha+\beta+\delta) & 0 \\ 0 & 0 & \frac{1-c_\pm}{\nu} - \delta + i(\overline{\omega}_3) - r \end{bmatrix}.$$

We therefore see that this equilibrium is locally stable if an only if the following variant of (77) holds:

$$\Phi(0) < \alpha-(1-\gamma)\eta_p, \quad 1-\nu(\alpha+\beta+\delta) < c_\pm, \quad \frac{1-c_\pm}{\nu} - \delta + i(\overline{\omega}_3) < r. \tag{78}$$

Appendix C. Local stability analysis for (52)

C1. Interior equilibrium

First note that our standing assumption on the marginal propensities of consumption by workers and investors, namely $c_w'(y_w) > c_i'(y_i)$, implies that

$$\frac{\partial c}{\partial \omega} = c_w'(y_w) - \Theta c_i'(y_i) > 0$$
$$\frac{\partial c}{\partial d_w} = r\big[\Theta c_i'(y_i) - c_w'(y_w)\big] < 0$$
$$\frac{\partial c}{\partial d_i} = -r(1-\Theta)c_i'(y_i) < 0$$

Next, let $x \in \mathbb{R}^4$ denote the point $(\omega, \lambda, d_w, d_i)$, so that the interior equilibrium defined in Section (3.3.1) is denoted by $\bar{x}_1$. In addition, we write

$$\overline{y}_{w1} = \overline{\omega}_1 - r\overline{d}_{w1}, \quad \overline{y}_{i1} = \Theta(1-\overline{\omega}_1 + r\overline{d}_{w1} - \delta\nu) - r(1-\Theta)r\overline{d}_{i1}$$

for the disposable income of workers and investors at this equilibrium. The Jacobian matrix of system (52) at this interior equilibrium is then

$$J_1 = J\big(\overline{\omega}_1, \overline{\lambda}_1, \overline{d_{w1}}, \overline{d_{i1}}\big) = \begin{bmatrix} K_0 & K_1 & 0 & 0 \\ K_2 & 0 & K_3 & K_4 \\ K_5 & 0 & K_6 & K_7 \\ K_8 & 0 & K_9 & K_{10} \end{bmatrix} \tag{79}$$

with the terms

$$K_0 = -(1-\gamma)\eta_p m\overline{\omega}_1 < 0, \quad K_1 = \overline{\omega}_1\Phi'(\overline{\lambda}_1) > 0, \quad K_2 = -\frac{\overline{\lambda}_1}{\nu}\frac{\partial c}{\partial \omega}(\bar{x}_1) < 0$$
$$K_3 = -\frac{\overline{\lambda}_1}{\nu}\frac{\partial c}{\partial d_w}(\bar{x}_1) > 0, \quad K_4 = -\frac{\overline{\lambda}_1}{\nu}\frac{\partial c}{\partial d_i}(\bar{x}_1) > 0, \quad K_7 = -\frac{\overline{d}_{w1}}{\nu} r(1-\Theta)c_i'(y_i) < 0$$

having well-defined signs, and the following terms

$$
\begin{aligned}
&K_5 = \bar{d}_{w1}\left[\frac{1}{\nu}(c_w'(\bar{y}_{w1}) - \Theta c_i'(\bar{y}_{i1})) - \eta_p m\right] + c_w'(\bar{y}_{w1}) - 1,\\
&K_6 = \delta - i(\bar{\omega}_1) - \frac{1-c(\bar{x}_1)}{\nu} + r\left[1 + \frac{\bar{d}_{w1}}{\nu}\left(\Theta c_i'(\bar{y}_{i1}) - c_w'(\bar{y}_{w1})\right) - c_w'(\bar{y}_{w1})\right]\\
&K_8 = \bar{d}_{i1}\left[\frac{1}{\nu}(c_w'(\bar{y}_{w1}) - \Theta c_i'(\bar{y}_{i1})) - \eta_p m\right] + \Theta[1 - c_i'(\bar{y}_{i1})]\\
&K_9 = r\left[\frac{\bar{d}_{i1}}{\nu}\left(\Theta c_i'(\bar{y}_{i1}) - c_w'(\bar{y}_{w1})\right) + c_i'(\bar{y}_{i1}) - \Theta\right]\\
&K_{10} = \delta - i(\bar{\omega}_1) - \frac{1-c(\bar{x}_1)}{\nu} + (1-\Theta)r\left[1 - \frac{\bar{d}_{i1}}{\nu}c_i'(y_i) - c_i'(\bar{y}_{i1})\right]
\end{aligned}
\tag{80}
$$

having signs that depend on the parameters and the form of the consumption function. One can then find the characteristic polynomial for J_1 in terms of the K_i above and obtain the corresponding Routh–Hurwitz conditions for stability, which then need to be checked numerically for given values of the parameters.

C2. Local stability of income divergence

The Jacobian of (59) associated with the equilibrium $(\omega, \lambda, u_w, u_i) = (0, 0, 0, 0)$ is

$$
\begin{bmatrix}
\Phi(0) - \alpha + (1-\gamma)\eta_p & 0 & 0 & 0\\
 & \frac{1-c_0}{\nu} - (\alpha+\beta+\delta) & 0 & 0\\
(*) & & \frac{1-c_0}{\nu} + i - r - \delta & 0\\
 & & & \frac{1-c_0}{\nu} + i - r(1-\Theta) - \delta
\end{bmatrix}
$$

where we used the fact that $c(\omega, d_w, d_i) \to c_0$ depending on which combination of $u_w \to 0^{\pm}$ and $u_i \to 0^{\pm}$ is considered. Namely, recalling that we are considering trajectories where d_i grows faster than d_w, we have that

$$
c_0 = \begin{cases}
c_{w^-} + c_{i^+} & \text{if } u_w \to 0^+, u_i \to 0^-\\
c_{w^-} + c_{i^-} & \text{if } u_w \to 0^+, u_i \to 0^+\\
c_{w^+} + c_{i^-} & \text{if } u_w \to 0^-, u_i \to 0^+\\
c_{w^+} + c_{i^+} & \text{if } u_w \to 0^-, u_i \to 0^-
\end{cases}
\tag{81}
$$

Consequently, the necessary and sufficient conditions for stability of the corresponding equilibrium are

$$
\Phi(0) < \alpha - (1-\gamma)\eta_p, \quad 1 - \nu(\alpha+\beta+\delta) < c_0, \quad \text{and} \quad \frac{1-c_0}{\nu} - \delta - \eta_p < r(1-\Theta).
\tag{82}
$$

Finally, the Jacobian of (60) at the equilibria $(\omega, \lambda, u_w, v) = (0, 0, 0, v)$, with $v = 0$ and $v = -1$, is identical to (C.2) except for the last row, which now has $(1+2v)r\Theta$ as the diagonal entry. We therefore see that, provided the conditions in (82) hold, the equilibrium with $v = 0$ is unstable, whereas the equilibrium with $v = -1$ is stable.

The analysis of stability of the deflationary equilibria of the type $(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_{w3}, \bar{d}_{i3}) = (\bar{\omega}_3, 0, \bar{d}_{w3}, \bar{d}_{i3})$ follows similarly to that of Appendix B.3 above and is not included here for space reasons.

Appendix D. Parameters and functions for numerical simulations

The baseline parameters for our simulations are provided in Table 3. Alternative values for some specific parameters are provided in the legend of each figure. We use a Phillips curve of the form

$$
\Phi(\lambda) = \frac{\phi_1}{(1-\lambda)^2} - \phi_0,
\tag{83}
$$

with parameters specified in Table 3. For Fig. 1, which illustrates the dual Keen model of Section 2, we used a consumption function for the aggregate household sector of the form

$$
c(y) = \max\left\{c_-, A_c + \frac{K_c - A_c}{(C_c + Q_c e^{-B_c y})^{1/\nu_c}}\right\}
\tag{84}
$$

that is to say, a generalized logistic function truncated at c_- on the negative half-line, with parameters given in Table 3.

Table 3
Baseline parameter values.

Symbol	Value	Description
ν	3	capital-to-output ratio
α	0.025	productivity growth rate
β	0.02	population growth rate
δ	0.05	depreciation rate
r	0.03	real interest rate
η_p	0.35	adjustment speed for prices
m	1.6	markup factor
γ	0.8	inflation sensitivity in the bargaining equation
ϕ_0	0.0401	Phillips curve parameter
ϕ_1	6.41×10^{-5}	Phillips curve parameter
c_-	0.03	hard lower bound for consumption function
A_c	0	asymptotic lower bound for consumption function
B_c	5	growth rate for consumption function
K_c	0.9	upper bound for consumption function
C_c	1	consumption function parameter
ν_c	0.2	consumption function parameter
Q_c	0.9743505	consumption function parameter

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